FEATURE

Kavli IPMU Assistant Professor **Masahito Yamazaki**
Research Area: Theoretical Physics

A Theoretical Physicist's Take on Spacetime Dimensions

Privilege of a Theoretical Physicist

When you hear the word "physicist," what kind of person comes to mind? Is he or she a crazy person dressed in a white lab coat, stuck in a messy lab and playing around with fancy equipment? "Doc" from the movie "*Back to the Future*" is one famous manifestation of such a wide-spread stereotype.

While I am a physicist, I am not a physicist of that kind: I have not worn a lab coat in the last ten years, and I almost never deal with reagents in the lab (in fact, I am bad at these things). I spend the bulk of my daily life working on a computer or talking with my fellow researchers, and I really am not doing experiments in the lab—yes, you might by now realize that I am a theoretical physicist.

As the name suggests, the work of a theoretical physicist is to come up with a theory in physics. A theory in physics, roughly speaking, is a theoretical framework which naturally and uniformly explains the essence of a variety of natural phenomena.

Theoretical physicists like me cannot do experiments ourselves: even if I come up with an ingenious theory, I myself cannot do the final verification of the theory, so I turn to my experimental colleagues for help. This is what is meant by the familiar expression "experiment and theory develop hand in hand in the research of physics."

While it is often the case that theoretical physicists need to rely on experimental physicists, we theoretical physicists have our own strengths. We can use our minds to ponder a wide variety of natural phenomena, deepen our understanding and sometimes even make new predictions, without ever realizing the phenomena in the lab with complicated experimental apparatus (which often requires a lot of labor and money, by the way). With only paper and pen (and sometimes a handy laptop), we can indulge in deep thought, on topics ranging from the beginning of the Universe to the behavior of tiny elementary particles, all in a tiny corner of a coffee shop! I would say it is a rare privilege of a theoretical physicist to have such absolute freedom in our thinking.

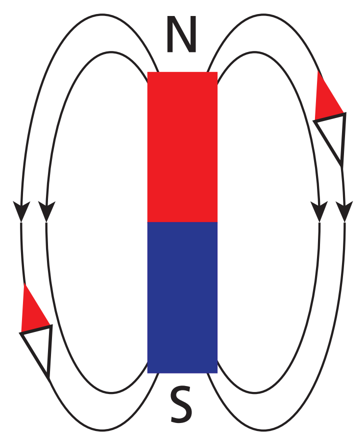

Figure 1. A magnet creates a magnetic field around it. This in turn affects the surrounding matter, e.g., the direction of the compass.

Let me give you an example of such a line of thinking, coming from my research experience—I can promise that this will be fun!

Let us begin with a simple example. You might have learned this at school: when we place iron scraps near a magnet, the iron scraps align in a beautiful pattern. Since I am a physicist, I can restate this phenomenon in physics jargon. First, when we place the magnet, it creates a magnetic field in the space around it, where a "field" can be thought of as a set of arrows at each point of the space, specifying the strength and orientations of the magnetic field (see Figure 1). Then, when we place iron scraps around the magnet, the iron scraps "feel" the magnetic field, and align in the directions dictated by the field. When we move the magnet, the magnetic field created from the magnet also changes as a result, which in turn causes a change in the patterns of the iron scraps. To state this in a fancier way, we can say that the two physical systems, namely the magnetic field and the iron scraps, are not independent and "interact" with each other.

What happens if we use another metal, or another material, instead of the iron scraps? We know that some materials are very strongly pulled by a magnet, while others are not pulled much. To restate this, the strength of the interaction depends on the types of materials we have.

With real-world materials it is often not easy to change the strength of interactions. However, a theorist has no trouble imaging a hypothetical material for which we can freely tune the strength of interactions. When the interaction length is zero, the magnetic field and the material are completely decoupled; as we increase the interactions the two physical systems begin to have more and more mutual effects on each other.

Theorists Like Generalization

Since a theorist often likes the idea of generalization, let us try to make this setup slightly more general. First, we assume that there are several different types of electromagnetic fields. Moreover, we assume that matter also comes in several different types. This is a natural generalization of electromagnetism.

For simplicity, let us assume that there are no direct interactions between electromagnetic fields of different types. Note that this does not exclude

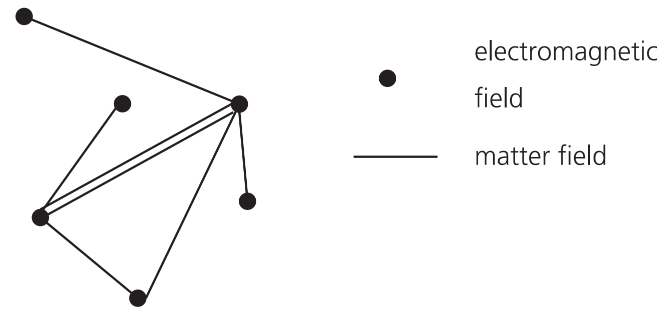

Figure 2. A graph (the so-called quiver diagram) representing the types of electromagnetic fields as well as matter interacting with them. Here a vertex represents a type of the electromagnetic field, and an edge the matter transforming under the electromagnetic field.

more indirect interactions mediated by matter. Namely, an electromagnetic field causes a change in the nearby matter, which then causes change in another electromagnetic field. In such a generalized setup, all the electromagnetic fields and matter eventually interact with each other in a complicated manner, and it becomes in general a difficult problem to figure out their behavior exactly.

In order to concisely describe such a theory with interacting electromagnetic fields and matter, we introduce the concept of a graph,[1] which is often called a quiver diagram in the literature (see Figure 2). A vertex of this graph represents an electromagnetic field, whereas an edge connecting between two vertices represents matter interacting with the electromagnetic fields associated with the two vertices. This means in particular that a matter field always interacts with two electromagnetic fields, even when there are many electromagnetic fields. Given a graph, a theorist can in this manner consider a corresponding theory (often called the quiver gauge theory in the literature). A graph is something even a small child can draw. But a trained theoretical physicist can associate a complicated physical theory to a graph, and spend hours and hours thinking about the theory.

We have to introduce a graph as technical shorthand for a complicated theory. While this might be useful, in physics one often asks the "physical meaning" of various mathematical gadgets. Is the graph only a technical tool, or can

[1] In the literature we often consider an oriented graph. However, readers can ignore this feature for the purposes of this article.

we associate a more physical substance to it?

One might quickly conclude that this is a nonsense question. At least, even if a graph lives in some space, that space is completely different from our time and space, like another world. In fact, even when we draw an edge, we do not necessarily have in mind a particle moving in our spacetime; rather, an arrow is simply shorthand for the matter content of the theory we are interested in.

It is too early to give up, however. What happens if the “other world” really exists? What happens if a graph in that other world represents the choice of the theory in “our world” ?

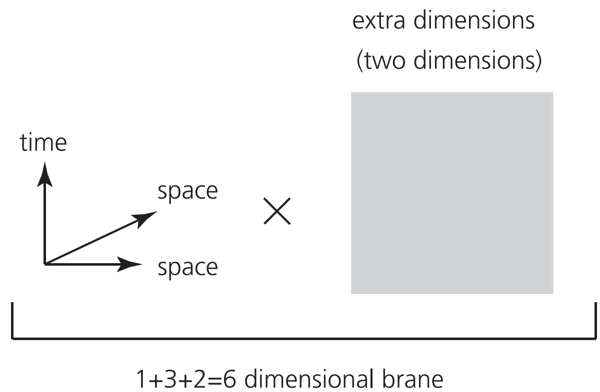

Figure 3. In addition to our one-dimensional time as well as three-dimensional spatial directions, we have two extra dimensions, where the graph of Figure 2 lives. We then have total of 1+3+2=6 dimensions, which in the superstring theory is realized on a 6-dimensional "brane."

Superstring Theory and Extra Dimensions

Very interestingly, in the field of superstring theory (which I have been working on), something as absurd-sounding as “the other world” is realized as “extra dimensions.” An extra dimension is a “rolled-up” dimension which is too small to observe, and is different from our familiar dimensions (one time and three spatial dimensions). In superstring theory, a number of properties of “our space,” such as the types of matter we have, can be translated into the properties of the extra dimensions. Since it is natural to draw graphs in two-dimensional planes, we should have (at least) two extra dimensions. We thus have total of 3+1+2=6 spacetime dimensions; we will live in a six-dimensional world (see Figure 3).

In superstring theory, there exists a natural six-dimensional brane (membrane).[2] If we wrap this brane along two spatial directions, we have four remaining directions, which gives us four spacetime dimensions (one for time, and three for spatial directions). The graph appears in two-dimensional spatial directions, where the graph represents how the branes spread in the two-dimensional extra dimensions (see Figure 4).[3] It turns out that this is correct in the precise technical sense, as shown by detailed studies of the shapes of the branes. This is what I worked on for my Master’s thesis many years ago.[4]

That we can speculate about six spacetime dimensions at the corner of a coffee shop is what makes the life of the theoretical physicist exciting. We can however go further and be even more brave. Suppose that we make the size of one of our spatial dimensions smaller and smaller, to the extent that it is eventually so small that nobody (or nothing) can see it. This means that we are eventually confined to a world with spatial two dimensions only, like on a piece of paper.

Having only two spatial dimensions causes all sorts of troubles for our daily lives (for example, there are no airplanes or pedestrian overpasses), but let us neglect these matters here, and ask what happens to the graph we introduced earlier. Recall that our graph was drawn in two spatial dimensions, which requires two extra dimensions. Now that our spatial directions are reduced from three to two, we have a new extra dimension, and hence in the end we have a total of three extra dimensions. This should mean that the two-dimensional graph should be replaced by a three-dimensional graph. We can formulate this statement in a mathematically more precise manner.[5]

[2] These are called D5-branes and NS5-branes in our jargon, where “5” here denotes the spatial dimensions.
[3] To be more precise, we have two different types of branes along the two extra dimensions, and the graph represents how these two branes intersect with each other.
[4] M. Yamazaki, Fortsch. Phys. **56** (2008) 555-686, arXiv:0803.4474 [hep-th].
[5] M. Yamazaki, JHEP **1205** (2012) 147, arXiv:1203.5784; Y. Terashima and M. Yamazaki, Phys. Rev. Lett. **109** (2012) 091602, arXiv:1203.5792.

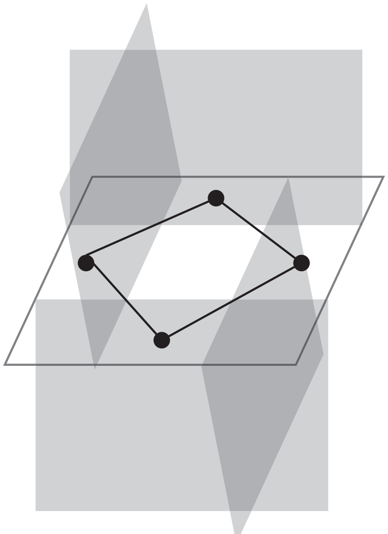

Figure 4. In superstring theory, a 6-dimensional brane (represented here as a white plane in the horizontal direction) intersects in a complicated manner with a different type of branes (represented here as gray planes in vertical directions). The graph drawn on the two-dimensional extra dimensions represents this intersection pattern of the two different types of branes.

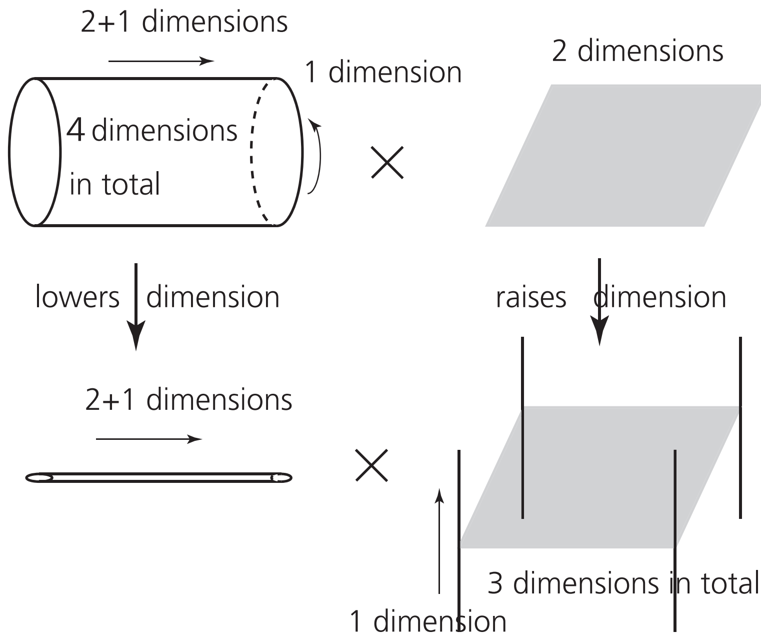

Figure 5. In the starting setup (above), we start with four-dimensional spacetime (which we know well), as well as 2 extra dimensions, leading to a total of 6 dimensions. Let us now choose one of the 3 spatial dimensions, curl it up, and make it very small. The spacetime as we know it then reduces to the total of 3 dimensions, with 1 time direction and 2 spatial directions. All is not lost, however; the dimension of the extra dimension goes up, leading to three spatial dimensions (below). Note that the total number of dimensions is always 6, and does not change. Note that we find the following curious phenomenon: in the left of this figure the dimension decreases by one, whereas on the side of extra dimensions (right hand side) the dimension goes up.

Joy of Theoretical Physicists

If you think about it, this is a rather dramatic and counterintuitive conclusion. In one side of the six-dimensional theory, namely in “our” world, the number of spacetime dimensions decreases from four to three, and we have less space for our lives. However, on the other sides of the six-dimensional theory (i.e. in the directions of the “extra dimensions”), the number of spatial dimensions increases from two to three, and an inhabitant of the extra dimensions have more directions to go for a walk! In the world of superstring theory, a number of surprising things happen, even to the fundamental concept of dimensions of spacetime.

Starting with the familiar electromagnetism, we arrived at graphs and extra dimensions, all the way to the dramatic insights concerning the dimensions of our spacetime and extra dimensions. This is one illustration of the joy of research in theoretical physics, where a theoretical physicist talks to himself/herself, discusses with his/her fellow researchers, and after hours of work finally arrives at fascinating conclusions. Any physicist knows well that it is a challenging problem to better understand Nature, and quite often he or she spends days and months being stuck in research. Despite numerous failures and frustrations, however, theoretical physicists never stop thinking and doing research. By being ambitious and patient and sometimes thinking outside the box to eventually contribute to mankind’s progress in better understanding Nature—however small the result may be—we can make such contributions, and this fact is a source of pleasure and pride for theoretical physicists, including myself.