

Primordial Black Holes in the Inflationary Universe

In September 2015, for the first time in history, gravitational waves from the coalescence of binary black holes were detected by the Laser Interferometer Gravitational-Wave Observatory (LIGO) in Livingston, Louisiana and Hanford, Washington. Afterward, more gravitational wave events were discovered and the third event was announced this year. These three events are caused by binaries of black holes whose masses are from about 10 to 30 solar mass. This historical discovery also means that humankind has obtained a new observational tool for exploring the universe.

Usually, black holes are formed when massive stars collapse due to their own gravity at the end their lifetime. However, the masses of the black holes observed by LIGO are larger than those expected for the gravitational collapse of massive stars, and it is in dispute how such heavy black hole binaries are formed by stellar evolution. Therefore, we are interested in another idea on the formation of black holes; that is, primordial black holes produced in the early universe form black hole binaries and account for the gravitational events detected by LIGO.

Primordial Black Holes

Primordial black holes are formed by gravitational collapse of high density fluctuations if they exist in

the very early universe at the cosmic time of less than 1 second. The possibility of the existence of black holes in the early universe was first considered by Zeldovich and Novikov in 1966, and Hawking in 1971, who presented the modern picture of primordial black holes forming from the collapse of high density regions. Since then, primordial black holes have been discussed from various points of view by physicists and astronomers.

The mass of a primordial black hole is determined by the formation epoch. Suppose that there are large density fluctuations in the early universe. At that time, the density fluctuations have various spatial sizes. Having an image that those fluctuations are something like waves which have crests and troughs in density, we call the spatial sizes of the fluctuations “wavelengths.” In this picture, large fluctuations correspond to waves with large amplitudes. Let us consider a wave with a certain wavelength. The evolution of the fluctuation depends on whether its wavelength is larger or smaller than the horizon length. Here the horizon length represents the maximum distance that can be reached by light. Roughly, the horizon length is given by $(\text{horizon length}) = (\text{light speed}) \times (\text{cosmic time})$ and increases with time. Since no information travels faster than the speed of light, the causal relationship does not exist beyond the horizon. First, the wavelength of the fluctuation

is larger than the horizon. In this case, nothing happens to the fluctuation. The wavelength of the fluctuation increases by the cosmic expansion but the horizon length grows faster. Thus, it becomes equal to the horizon length after some time. When the fluctuation enters the horizon, a black hole is formed by gravitational collapse of the large density region if the amplitude of the fluctuation is large enough for its gravity to overcome the pressure by the radiation that dominates the universe. So the mass of the black hole formed is almost equal to the total energy of radiation inside the horizon. For example, a primordial black hole has a mass of about 10^{-5} solar mass $\sim 10^{28}$ grams if it is formed at the cosmic temperature of 10^{15} degrees (cosmic time $\sim 10^{-11}$ sec) and it has a mass of about 10 solar mass if it is formed at the cosmic temperature of 10^{12} degrees (cosmic time $\sim 10^{-4}$ sec). Thus, primordial black holes have various masses depending on the formation time.

The gravity of black holes is so strong that even light cannot escape from them. Thus, black holes had been considered stable once they were formed. However, in 1974 Hawking applied quantum theory to black holes and showed that they evaporate by emitting particles. This is called Hawking radiation. The effect of Hawking radiation is more significant for smaller black holes and black holes with mass less than about 10^{15} grams have evaporated by now.

Therefore, primordial black holes existing now have masses larger than about 10^{15} grams.

Density Fluctuations in the Inflationary Universe

Let us consider how large density fluctuations required for primordial black hole formation are produced. It is known that large scale structures of the universe such as galaxies and clusters of galaxies are formed from tiny density fluctuations in the early universe which grow through gravitational instability. In fact, the existence of density fluctuations has been confirmed by the COBE satellite launched by NASA. The COBE observed the cosmic microwave background (CMB). CMB is relic light existing when the universe is hot and in thermal equilibrium and is presently observed as microwaves with a wavelength of several millimeters. CMB comes from all directions of the sky almost isotropically, but in 1992 the precise observation performed by the COBE revealed that there are small anisotropies. This implies that there existed small density fluctuations in the universe at a cosmic time of about 0.38 million years when the CMB light was emitted. Those anisotropies have been further studied in detail by the WMAP and Planck satellites, which have shown that our universe has density fluctuations with an initial amplitude of about 10^{-5} .

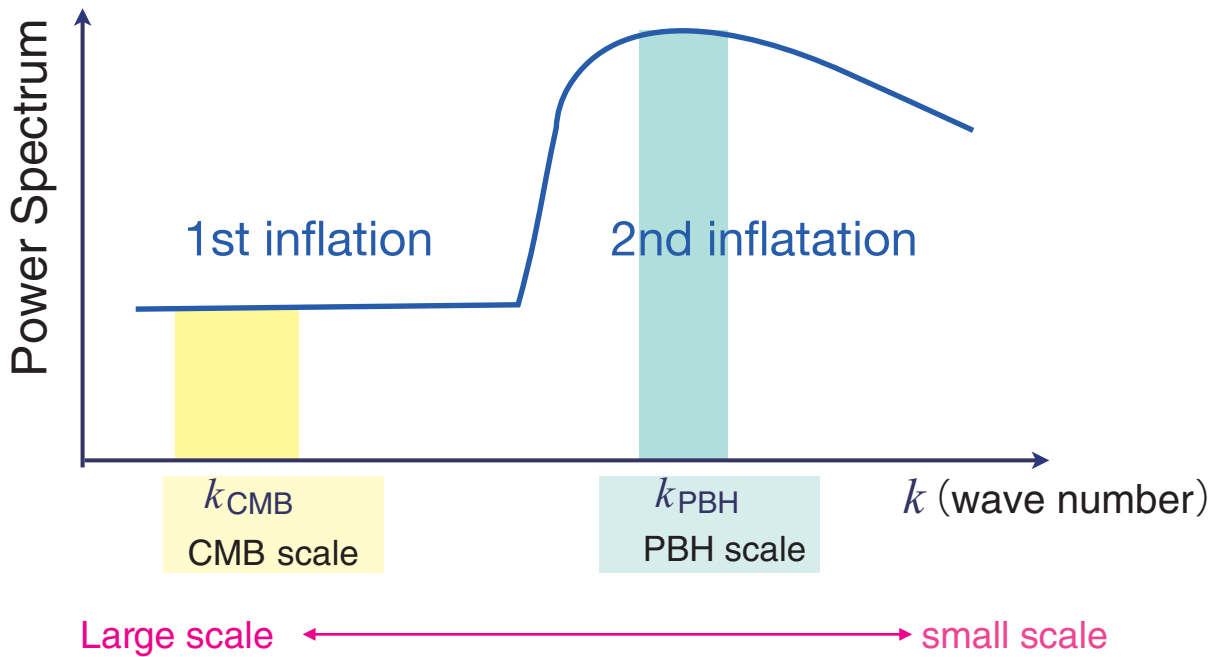


Figure 1: Schematic figure of the power spectrum produced by the double inflation model. The power spectrum represents the square of the amplitude of fluctuations with wavenumber k . The wavenumber is inversely proportional to wavelength, that is $k \sim 1/(\text{wavelength})$. The first inflation produces fluctuations on a large scale (small k) while the second inflation produces fluctuations on a small scale (large k). PBH means primordial black hole.

Now the problem is how the density fluctuations observed by the COBE are produced. Almost all researchers believe that those density fluctuations are produced by the accelerated cosmic expansion called inflation which is considered to have taken place at about 10^{-36} sec after the birth of the universe. Inflation is driven by some scalar field called an inflaton whose potential energy dominates the energy density of the very early universe. During inflation, the inflaton field has quantum fluctuations which are stretched by inflation into fluctuations with cosmological scale. The fluctuations produced in the inflationary universe have the characteristic property of being almost scale-invariant. The scale invariant fluctuations mean that various waves composing the fluctuations have the same amplitude independently of their wavelengths. This property of the fluctuations produced by inflation

has been confirmed by CMB observations together with other predictions, which provides a reason for us to believe inflation.

Primordial Black Hole Formation in Double Inflation

Almost scale-invariant fluctuations are produced in the inflationary universe and the CMB observation shows that the amplitude of the fluctuations is about 10^{-5} for large scale wavelengths (longer than about 100 Mpc). On the other hand, in order to form primordial black holes, the density fluctuations should have amplitudes as large as 10^{-1} when they enter the horizon. However, if the scale-invariance is satisfied, the fluctuations always have amplitudes of about 10^{-5} when they enter the horizon so it is difficult to produce black holes without breaking the scale-invariance.

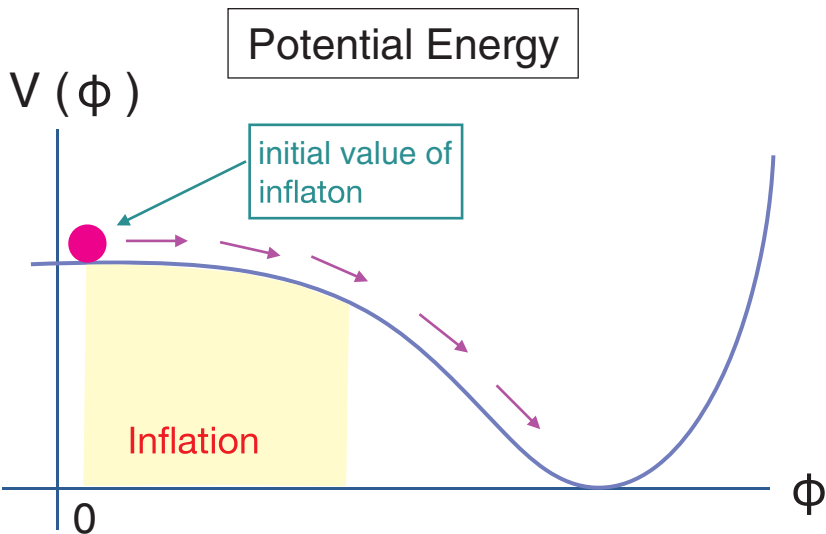


Figure 2: Potential of the scalar field (= inflaton) in the new inflation model. Inflation takes place when the inflaton has an initial value near zero and rolls down slowly toward the minimum of the potential.

Here I introduce a double inflation model as one of the interesting ideas for forming primordial black holes. The scale-invariance is hardly broken if a single scalar field causes inflation. However, this is not the case when inflation is derived by more than one field. Suppose that inflation takes place in two stages. Fluctuations with the smallest wavelengths are produced near the end of inflation, and the fluctuations produced earlier have larger wavelengths. This is because the fluctuations produced early are stretched by subsequent inflation. Therefore, in the scenario where inflation takes place in two stages, the first stage of inflation accounts for the fluctuations on a large scale observed by CMB, while the second stage of inflation produces large fluctuations on a small scale and hence primordial black holes. The model with two such stages of inflation is called the double inflation model. The schematic figure of the power spectrum (which is the square of the amplitude of fluctuations with wavenumber k or wavelength $1/k$) is shown in Figure 1.

To build a double inflation model which produces primordial black holes, the model for the second inflation needs some special properties. It is required to take place at low energy and produce large fluctuations. Concretely, the new inflation model with potential shown in Figure 2 is the most favorable. The new inflation model, contrary to its name, is one of the oldest inflation models. It is a successful inflation model but has an initial value problem; the initial value of the inflaton field should be near the maximal point of the potential for sufficient inflation (Figure 2). However, in the double inflation model, this initial value problem can be solved through the effect of the first inflation.^{*1}

Figure 3 shows the mass distribution (mass function) of primordial black holes produced in the double inflation model. If appropriate sets of model parameters are chosen, double inflation can produce black holes with various masses. In an

^{*1} This was my personal reason for considering the double inflation model 20 years ago (K.-I. Izawa, M. Kawasaki and T. Yanagida, *Physics Letters B* **411**, 249 (1997)).

Feature

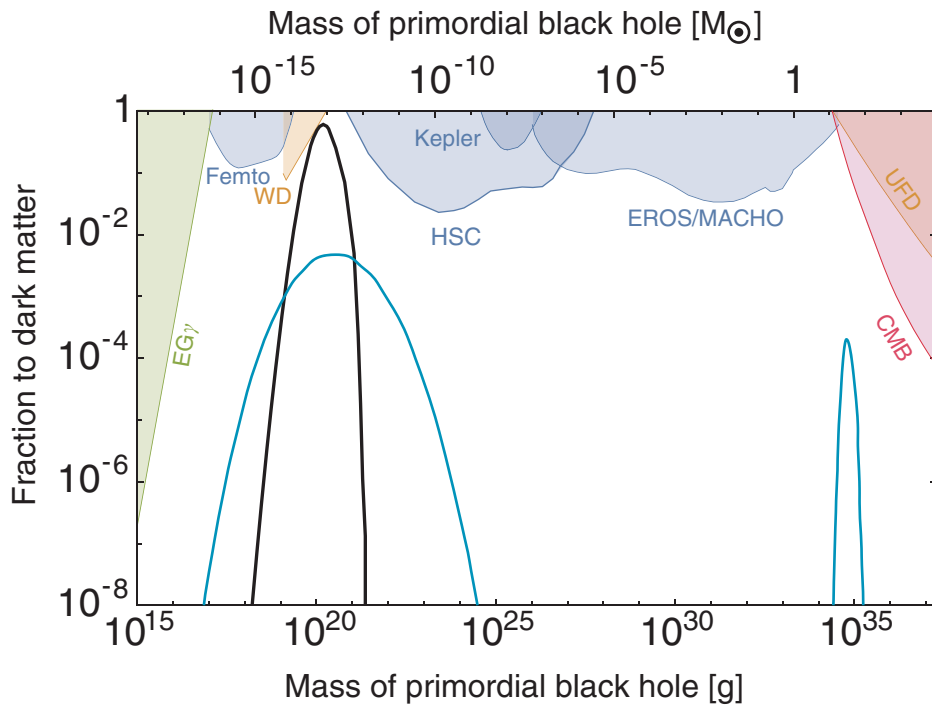


Figure 3: Mass distribution (mass function) of primordial black holes produced in the double inflation model. The solid black and blue lines represent the mass functions for different sets of the model parameters. Various observational constraints are shown by the shaded regions. M_{\odot} means the solar mass.

interesting case, the mass distribution of black holes has two peaks shown by the solid blue line in Figure 3.

Searching for Primordial Black Holes

As mentioned above, primordial black holes could have a very wide range of masses. Many attempts have been made to observe primordial black holes in various ways. Unfortunately, no evidence for primordial black holes has been obtained, from which stringent constraints on their abundance have been imposed. For masses of between about 10^{15} grams and 10^{17} grams, the abundance is constrained by observations of gamma rays from evaporating black holes. Observations of the interference of gravitationally lensed light (femto lensing) from gamma ray bursters give a constraint on black holes with masses of 10^{17} to 10^{19} grams. On the other

hand, the abundance in the mass range from 10^{20} to 10^{35} grams is constrained by another gravitational lensing effect (microlensing^{*2}) by which the luminosity of stars is enhanced when black holes cross the line of sight to those stars. There are also constraints from the heating of gravitational systems like dwarf galaxies by black holes and from effects on CMB due to accreting gas around black holes. Figure 3 shows the constraints on abundance (fraction to the dark matter density) of primordial black holes. From this figure, it is seen that abundance is stringently constrained in the wide range of mass.

Primordial Black Holes as Dark Matter

Can primordial black holes formed in the inflationary universe account for the dark matter?

*2 See page 22.

As shown in Figure 3, there are stringent constraints on black hole abundance. However, in the mass range of around 10^{20} grams the constraint is relatively weak, so it is possible for primordial black holes to account for the dark matter of the universe if they have a mass distribution with sharp peak at 10^{20} grams. In fact, such black holes can be formed in the double inflation model explained above. In Figure 3, the mass function denoted by the solid black line gives the observed dark matter density if it is integrated over mass, which means that the double inflation model can account for the all dark matter of the universe. However, the observational constraints are expected to be more stringent in future and the possibility of primordial black holes as dark matter might be excluded.

LIGO Gravitational Wave Events and Primordial Black Holes

As I mentioned at the beginning, the gravitational wave events detected by LIGO can be caused by primordial black holes. When black holes are formed in the early universe, it is still uncertain how abundant they should be in order to account for the LIGO events, but one analysis indicates that it is possible if primordial black holes with a mass of about 30 solar mass have about 10^{-3} of the dark matter density. The mass function of such black holes produced in the double inflation model is shown by the solid blue line in Figure 3.

In general scenarios where black holes are formed

by large fluctuations produced during inflation, gravitational waves are produced through the second order effect of the fluctuations (which is equal to the effect proportional to the fluctuation squared). These gravitational waves are different from those produced in binary systems of black holes, and have a frequency of nHz for the case of fluctuations which form primordial black holes and explain the LIGO events. The gravitational wave background with such frequencies is stringently constrained by observations of pulsar timings. Therefore, the density fluctuations should have a very sharp peak in order to avoid the pulsar timing constraint and account for the LIGO events. (The mass function in Figure 3 satisfies this condition.)

Summary

Primordial black holes have been attracting interest since LIGO detected the first gravitational wave event from coalescence of binary black holes. In addition, primordial black holes could contribute significantly to the dark matter of the universe. In this article, I introduce the double inflation model which can produce density fluctuations large enough to form primordial black holes, and show that the primordial black holes produced can account for the LIGO events and the dark matter of the universe. I hope that future observations such as microlensing searches will discover primordial black holes.