Aspects of aethereal gravity

1. Action terms of expansion, shear twist and acceleration
2. Horava as a limit of Einstein-aether
3. Aetheron
4. Positive energy theorem
5. Neutron stars
6. Aether waves & supernova core collapse
7. Cosmic alignment of the aether
8. Hamiltonian Horava Constraint equation in closed universe

$$S = \frac{-1}{16\pi G} \int d^4x \sqrt{-g} \left(R + 2\Lambda + K^{ab}{}_{mn} \nabla_a u^m \nabla_b u^n \right)$$

$$K^{ab}{}_{mn} = c_1 g^{ab} g_{mn} + c_2 \delta^a_m \delta^b_n + c_3 \delta^a_n \delta^b_m + c_4 u^a u^b g_{mn},$$

$$g_{ab} = u_a u_b - h_{ab}$$

$$\begin{array}{rrcl} expansion & \theta & = & \nabla_a u^a \\ shear & \sigma_{ab} & = & h^m{}_{(a} h^n{}_{b)} (\nabla_m u_n - \frac{1}{3} \theta h_{mn}) \\ twist & \omega_{ab} & = & h^m{}_{[a} h^n{}_{b]} \nabla_m u_n \\ acceleration & a_a & = & u^m \nabla_m u_a \end{array}$$

$$K^{ab}{}_{mn}\nabla_a u^m \nabla_b u^n = c_{14}a^2 + c_+\sigma^2 + (c_2 + \tfrac{1}{3}c_+)\theta^2 + c_-\omega^2$$

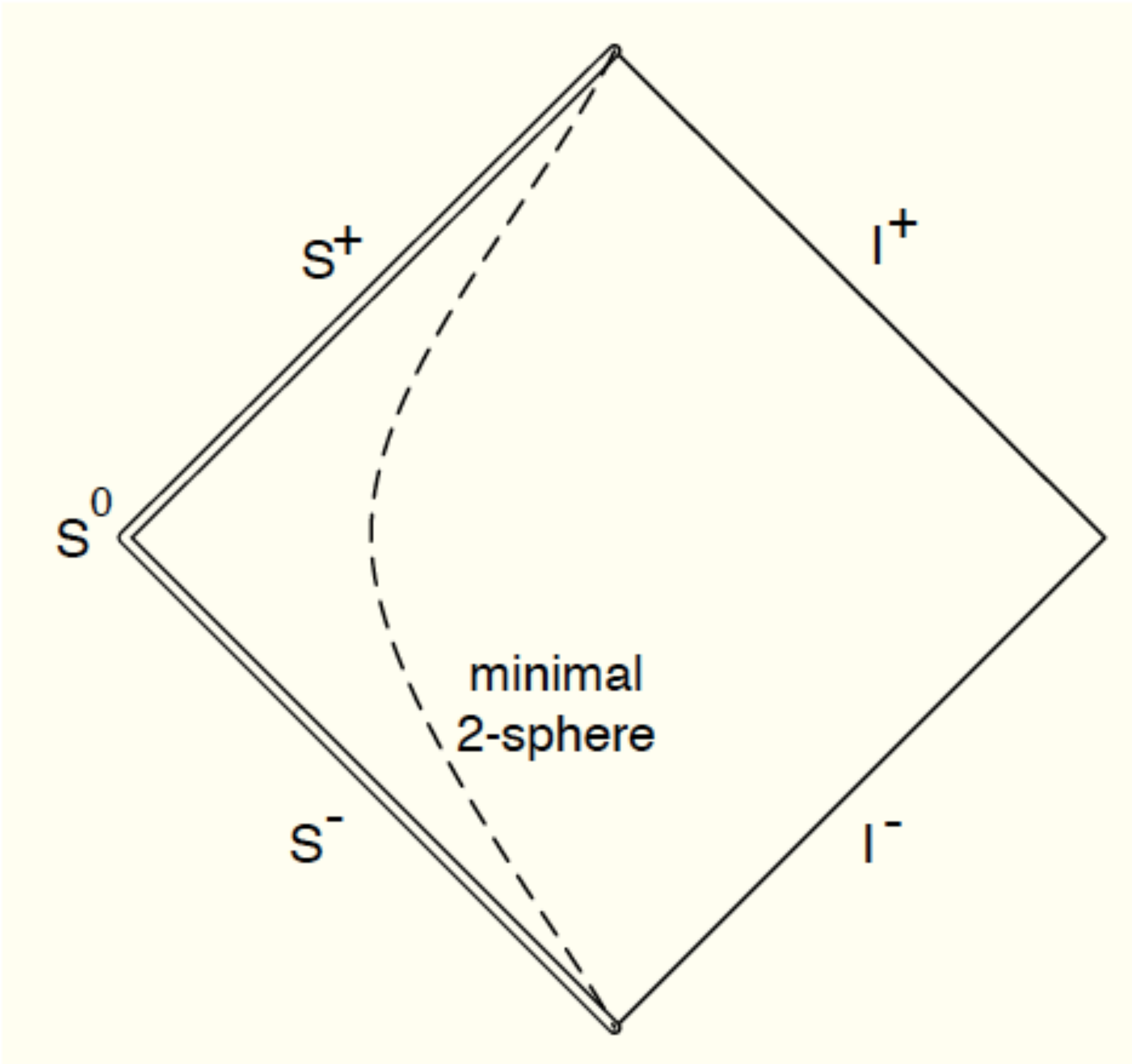
$$c_\pm = c_1 \pm c_3$$

$$\boxed{Hořava = \lim_{c_- \to \infty} Aether}$$

<u>PPN constraints $\alpha_{1,2} = 0$:</u>

$$\begin{array}{rcl} Hořava & & c_{14}a^2 + c_+\sigma^2 - \tfrac{1}{3}c_{14}\theta^2 \\ \\ Aether & & c_{14}a^2 + c_+\sigma^2 - \tfrac{1}{3}c_{14}\theta^2 + \tfrac{1}{3}c_{14}\omega^2 \\ & = & 3c_-a^2 + c_+\sigma^2 - c_-\theta^2 + c_-\omega^2 \end{array}$$

The Aetheron: static, spherical vacuum



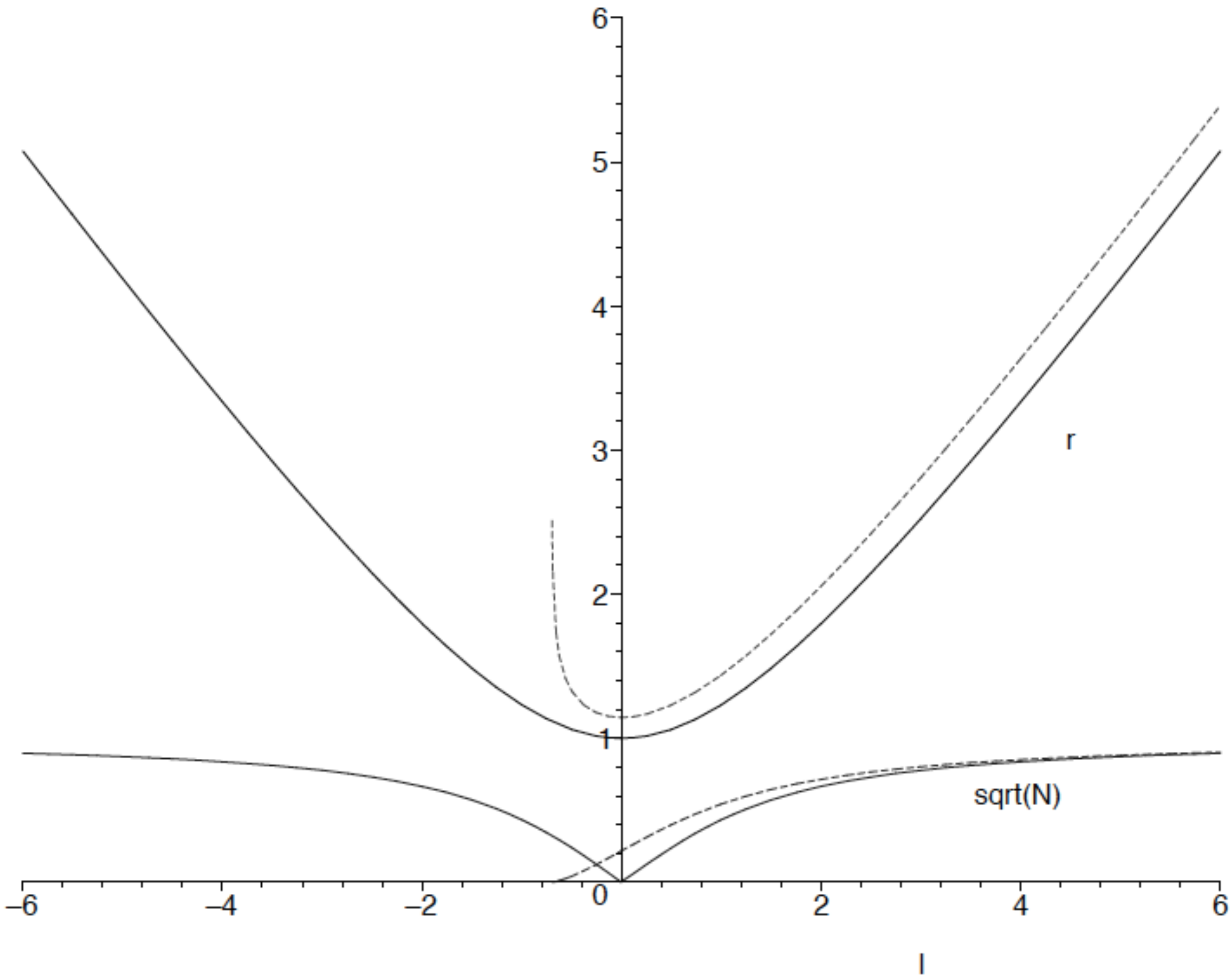


FIG. 4: Plot of $r(l)$ and the norm of the Killing vector $\sqrt{N(l)}$ for GR and for $c_1 = 0.5$, for the solution with the same value of the total mass M, in units with $2M = 1$. In GR N vanishes at the bifurcation sphere at the center of the Einstein-Rosen bridge. In the ae-theory solution the Killing vector remains timelike at the throat, but at the internal $r = \infty$ curvature singularity both the norm and its slope vanish, indicating the presence of a singular extremal Killing horizon.

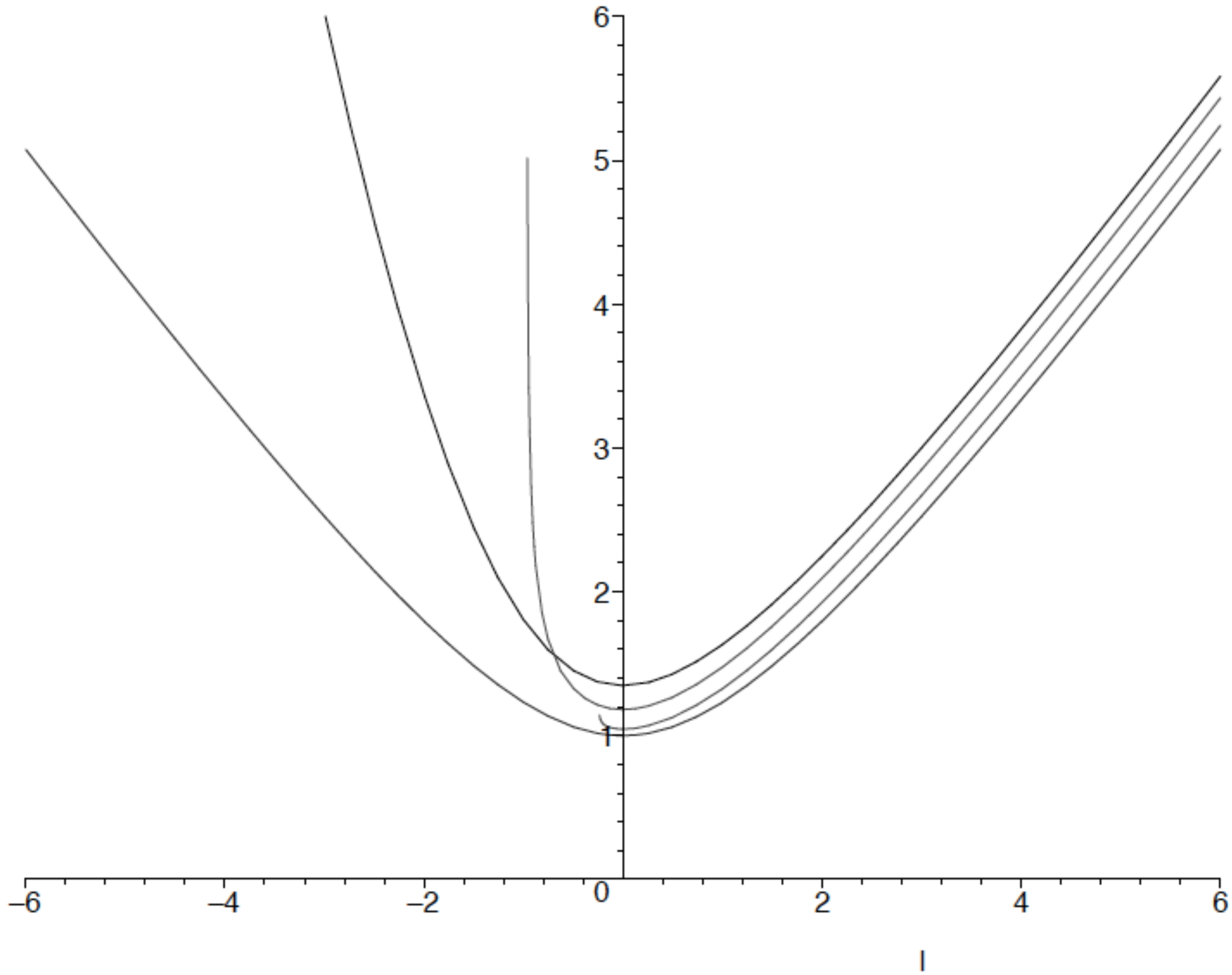


FIG. 3: Plot of area radius r vs. proper length l for fixed mass M, in units with $2M = 1$, for $c_1 =$ 0, 0.1, 0.7, and 1.9. In the GR case $c_1 = 0$ this is the Einstein-Rosen bridge. For $c_1 = 0.1$ the radius flares out to infinity so quickly that the code used to make the plot halted at small radius. With increasing c_1 the throat widens, the flare-out inside is slower, and the proper length to the curvature singularity increases, becoming infinite for $c_1 \geq 3/2$.

Static aether ISCO properties

$$r_{\rm ISCO} \simeq 6(1 + [\ln(3/2) - 1/6]\nu) \simeq 6(1 + 0.030\, c_{14})$$

$$\omega_{\rm ISCO} \simeq \frac{1}{6\sqrt{6}}(1 + [-2\ln(3/2) + 1/2]\nu) \simeq \frac{1}{6\sqrt{6}}(1 - 0.039\, c_{14})$$

- This aetheron has negative energy density everywhere, but positive mass.

- It is stable to spherical perturbations (M. Siefert), and nonlinearly in numerical simulations (D. Garfinkle, C. Eling, TJ) .

- Not known if it is stable beyond spherical symmetry.

- Its positive mass strongly hints at a positive energy theorem.

Such has been proved (D. Garfinkle & TJ) for hypersurface orthogonal aether with vanishing expansion ($K = 0$).

The proof uses a Schoen-Yau result:

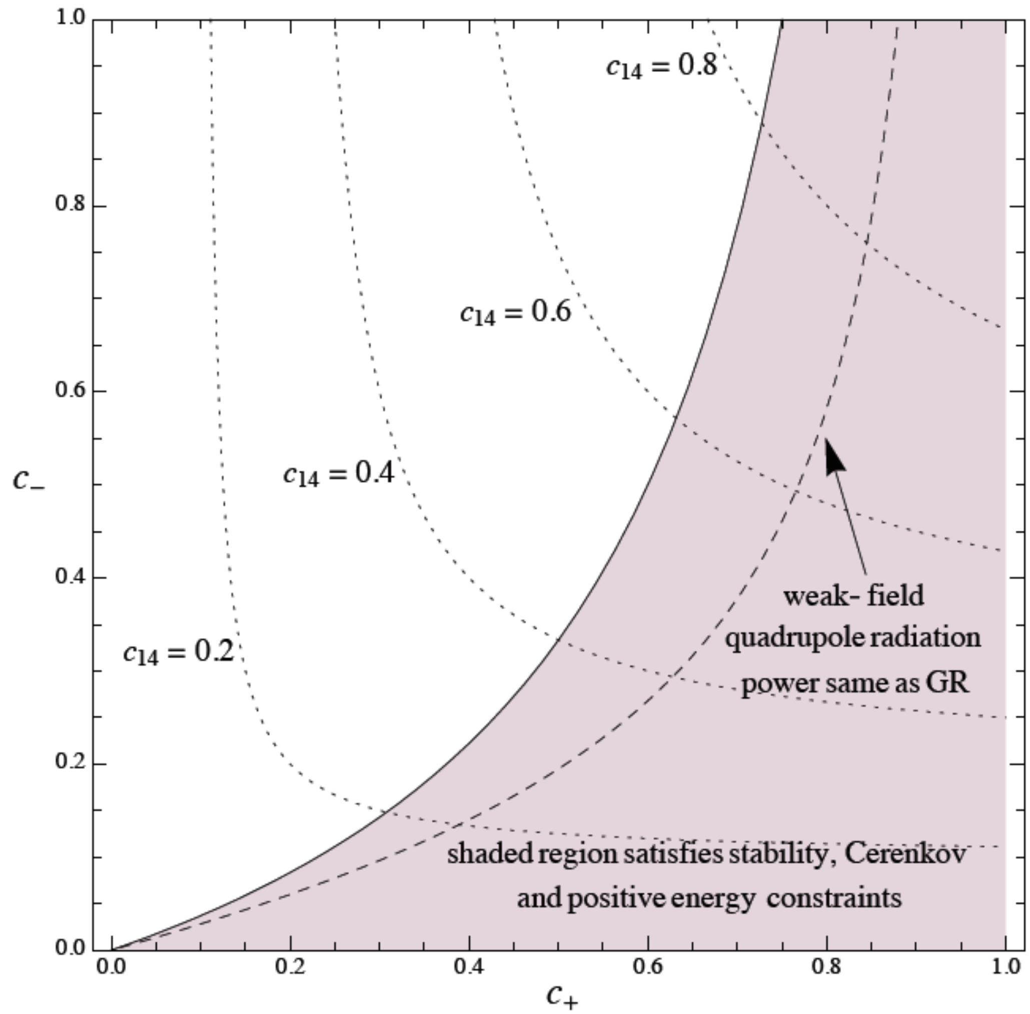
ADM mass of an orientable 3-manifold is nonnegative
if the Ricci scalar is nonnegative.

Positive energy theorem for aether

$$
\begin{aligned}
M_{ae} &= M_{ADM} - \frac{c_{14}}{8\pi G}\oint_\infty \hat{r}^a a_a \qquad (= \frac{G}{G_N} M_{ADM} \text{ for weak field, slow motion}) \\
&= M_{ADM} - \frac{c_{14}}{8\pi G}\oint_\infty \hat{r}^i \partial_i N \qquad (N = \text{lapse}) \\
&= M_{ADM} - \frac{1}{4\pi G}\oint_\infty \hat{r}^i \partial_i N^{c_{14}/2} \\
&= M_{ADM}[N^{c_{14}} h_{ab}]
\end{aligned}
$$

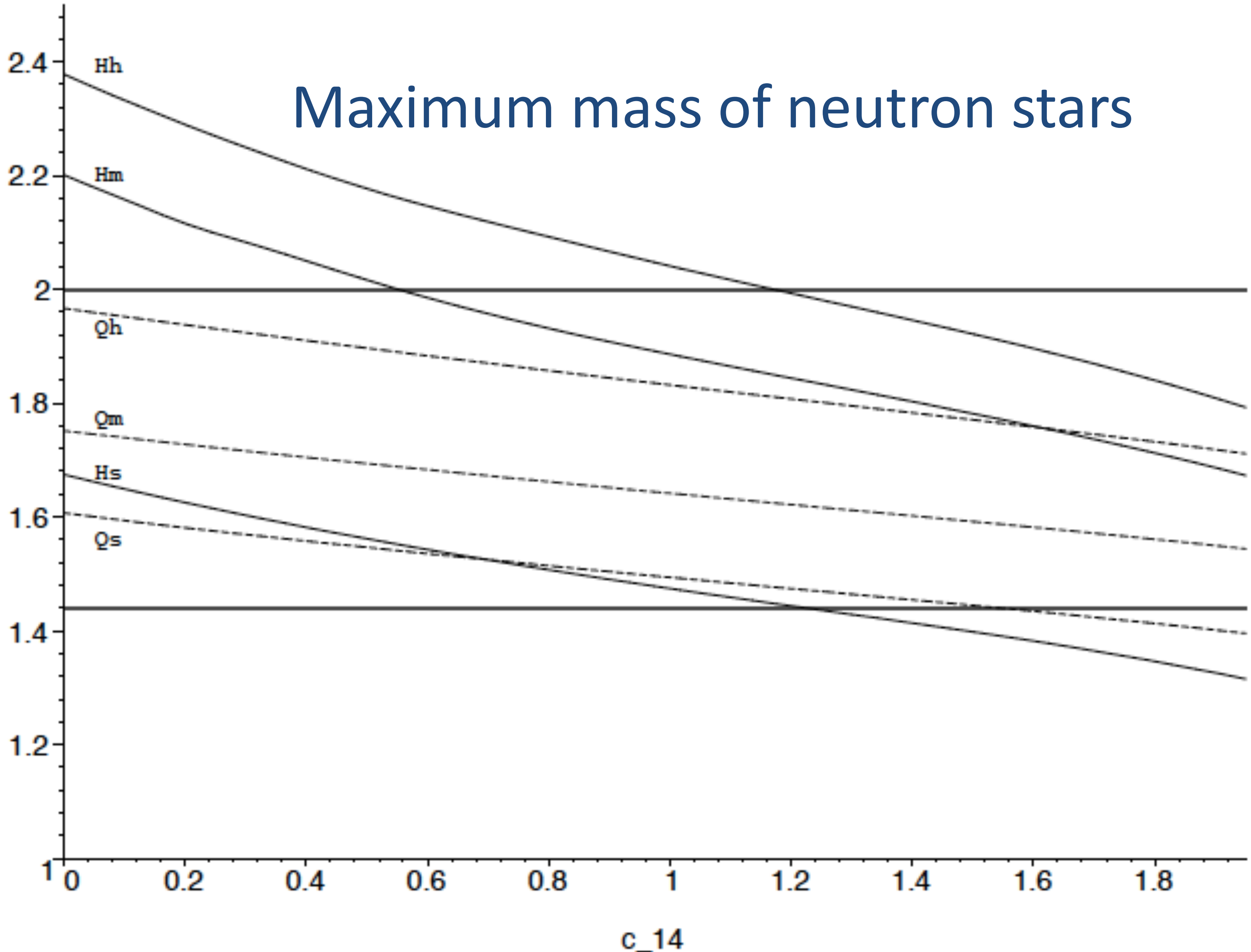
$$
\begin{aligned}
{}^{(3)}R &= 16\pi G\rho + c_{14}(2D_a a^a + a_a a^a) \\
&+ (1 - c_{13})K_{ab}K^{ab} - (1 + c_2)K^2
\end{aligned}
$$

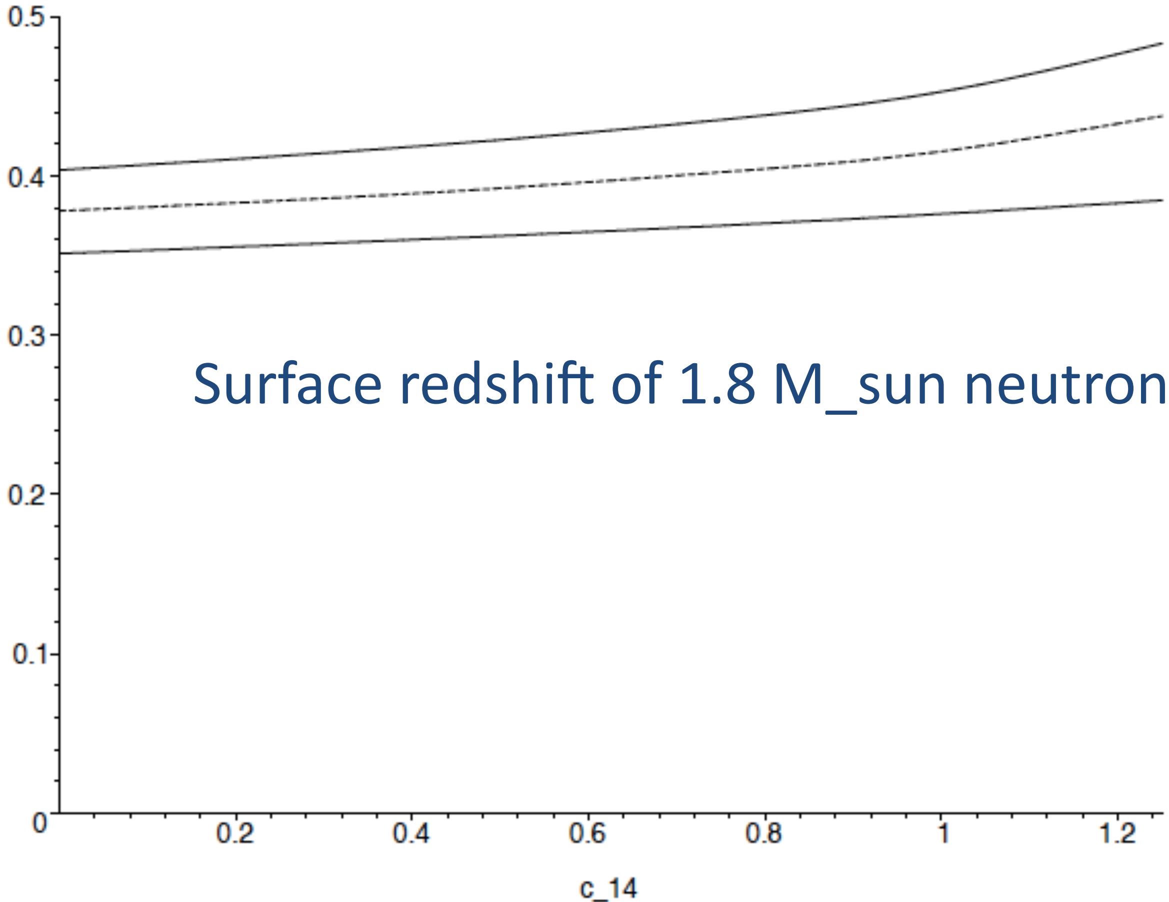
$$
\begin{aligned}
{}^{(3)}\tilde{R} &= N^{-c_{14}}\Big(16\pi G\rho + c_{14}(1 - c_{14}/2)a_a a^a \\
&+ (1 - c_{13})K_{ab}K^{ab} - (1 + c_2)K^2\Big)
\end{aligned}
$$



$$c_\pm = c_1 \pm c_3, \quad c_{2,4} \text{ chosen so that } \alpha_{1,2} = 0$$

Maximum mass of neutron stars





Surface redshift of 1.8 M_sun neutron star

Figure 4: Surface redshift factor z versus c_{14} for 1.8 $M_\odot$ neutron stars using the hardest equations of state. Hm (solid) is on top, Qh (dashed) is in the middle and Hh (solid) is on the bottom. Note that the GR value of $z = 0.35$ for the hardest EOS, Hh, is consistent with the proposed surface redshift of 0.35 for EXO 0748–676 [48]. The Hm and Qh lines begin to curve up near c_{14} =1.1-1.2 because the maximum mass for these equations of state is approaching 1.8 $M_\odot$.

Aether wave polarizations

$h_{0i} = 0 \quad v^i{}_{,i} = 0$ Gauge conditions

spin-2 $\quad h_{12}, \quad h_{11} = -h_{22}$

spin-1 $\quad v_I, \quad h_{3I} = \left[2c_{14}c_{13}^2/(2c_1 - c_1^2 + c_3^2)\right]^{1/2} v_I$

spin-0 $\quad v_0, \quad h_{00} = -2v_0, \quad h_{11} = h_{22} = -c_{14}v_0, \quad h_{33} = [2c_{14}(1+c_2)/c_{123}]\, v_0,$

Compute Riemann tensor to find gauge-invariant polarizations.

Cosmic alignment of the aether

Following KS, we specialize to Bianchi type I spacetimes, i.e. to metrics that are homogeneous and spatially flat, with three commuting translation symmetries,

$$\begin{aligned} ds^2 &= N^2(t)dt^2 - e^{2\alpha(t)}\big[e^{-4\sigma_+(t)}dx^2 \\ &\quad + e^{2\sigma_+(t)}(e^{2\sqrt{3}\sigma_-(t)}dy^2 + e^{-2\sqrt{3}\sigma_-(t)}dz^2)\big]. \end{aligned} \tag{3}$$

We also assume that the aether vector is tilted only in the x-direction,

$$u = \frac{1}{N(t)}\cosh\theta(t)\,\partial_t + e^{-\alpha(t)+2\sigma_+(t)}\sinh\theta(t)\,\partial_x. \tag{4}$$

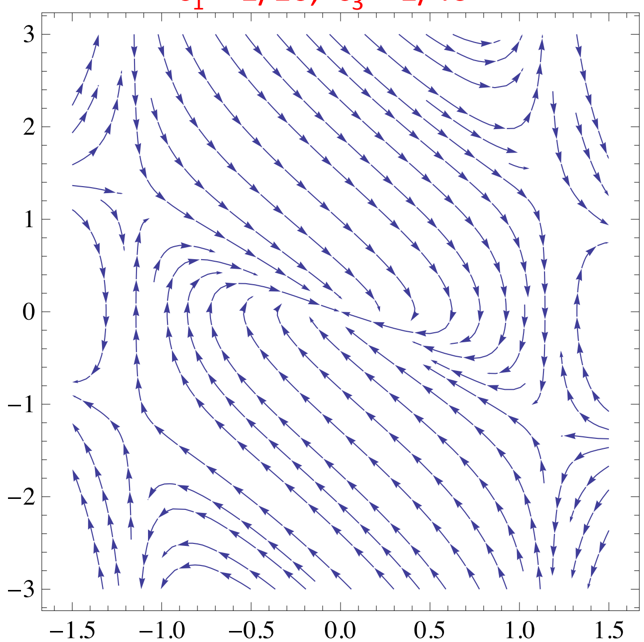
Isotropization of the aether during exponential expansion

(Nonlinear extension of Kanno & Soda; with Isaac Carruthers)

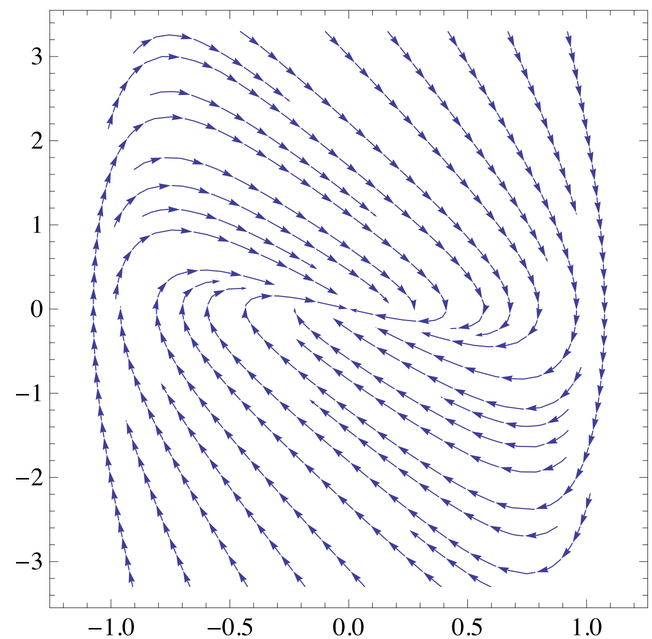
Homogeneous, initially anisotropic Bianchi I (Kasner) metric, with cosmological constant (cf. "tilted cosmology")

Sample phase plots of aether rapidity & its time derivative:

$c_1 = 1/10$, $c_3 = 1/40$



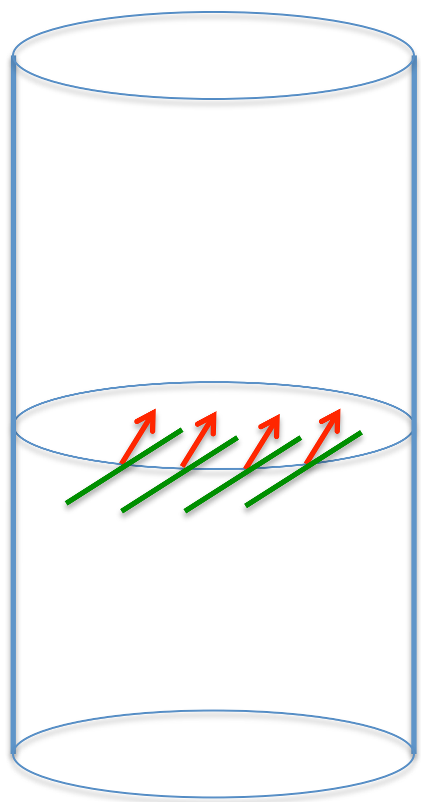
$c_1 = 1/2$, $c_3 = 1/4$



Generically runaway or singular outside an order unity neighborhood of aligned configuration.

If spatial sections are compact, locally hypersurface-orthogonal configurations may not be globally hypersurface-orthogonal ---

--- so Horava gravity is then more restricted than hypersurface-orthogonal Ae-theory.



Hamiltonian structure of Hořava gravity

William Donnelly* and Ted Jacobson†

$$\mathcal{L} = \frac{1}{16\pi G_H}\sqrt{g}N\left(K_{ij}K^{ij} - \lambda K^2 - V(g_{ij}, a_i)\right)$$

$$V(g_{ij}, a_i) = -\xi R - \alpha a_i a^i + \ldots$$

$$\begin{aligned}
\mathcal{H} &= N\mathcal{H}_t + N^i\mathcal{H}_i + v^i p_i + v p_N \\
\mathcal{H}_t &= \frac{1}{\sqrt{g}}\left(p^{ij}p_{ij} + \frac{\lambda}{1-3\lambda}p^2 + gV\right) \\
\mathcal{H}_i &= -2g_{ik}\nabla_j p^{jk}
\end{aligned}$$

Preservation of the primary constraints in time leads to secondary constraints

$$C = \delta H / \delta N = 0, \quad C_i = \delta H / \delta N^i = 0, \tag{15}$$

where

$$C = \mathcal{H}_t - \frac{1}{N} \nabla_i V^i, \tag{16}$$

$$C_i = \mathcal{H}_i, \tag{17}$$

and we have defined the vector density

$$V^i(x) = \frac{\delta}{\delta a_i(x)} \int \sqrt{g} N V. \tag{18}$$

In the IR limit,

$$V^i = -2\alpha\sqrt{g}\nabla^i N, \tag{19}$$

and the constraint $C = 0$ becomes

$$\frac{1}{g}\left(p^{ij}p_{ij} + \frac{\lambda}{1-3\lambda}p^2\right) - \xi R - \alpha\frac{(\nabla N)^2}{N^2} + 2\alpha\frac{\nabla^2 N}{N} = 0. \tag{20}$$

The constraint equation in the IR limit can be linearized by the change of variables[3] $N = n^2$, resulting in the equation

$$Ln = 0 \tag{21}$$

where L is the linear differential operator

$$L \equiv -4\alpha\nabla^2 - \frac{1}{g}\left(p^{ij}p_{ij} + \frac{\lambda}{1-3\lambda}p^2\right) + \xi R. \tag{22}$$

Such an equation admits a solution if and only if the spectrum of the Schrödinger-like operator L contains zero. Moreover, if the foliation by constant t surfaces is to be smooth the lapse must be positive everywhere, which requires that $n(x)$ is positive for all x. A solution with positive n exists if and only if zero is the least eigenvalue of L: this is the familiar statement that the Schrödinger equation admits a unique eigenstate with no nodes, and this state is a ground state [10]. Thus Eq. (21) contains a condition on the metric g_{ij} and its conjugate momentum p^{ij}. If this condition is met the constraint has a unique (up to rescaling) positive solution for n, and hence the lapse is determined up to a constant scaling.

This is a weird, totally nonlocal constraint, isn't it?