From configuration to dynamics

Emergence of Lorentz signature in classical field theory

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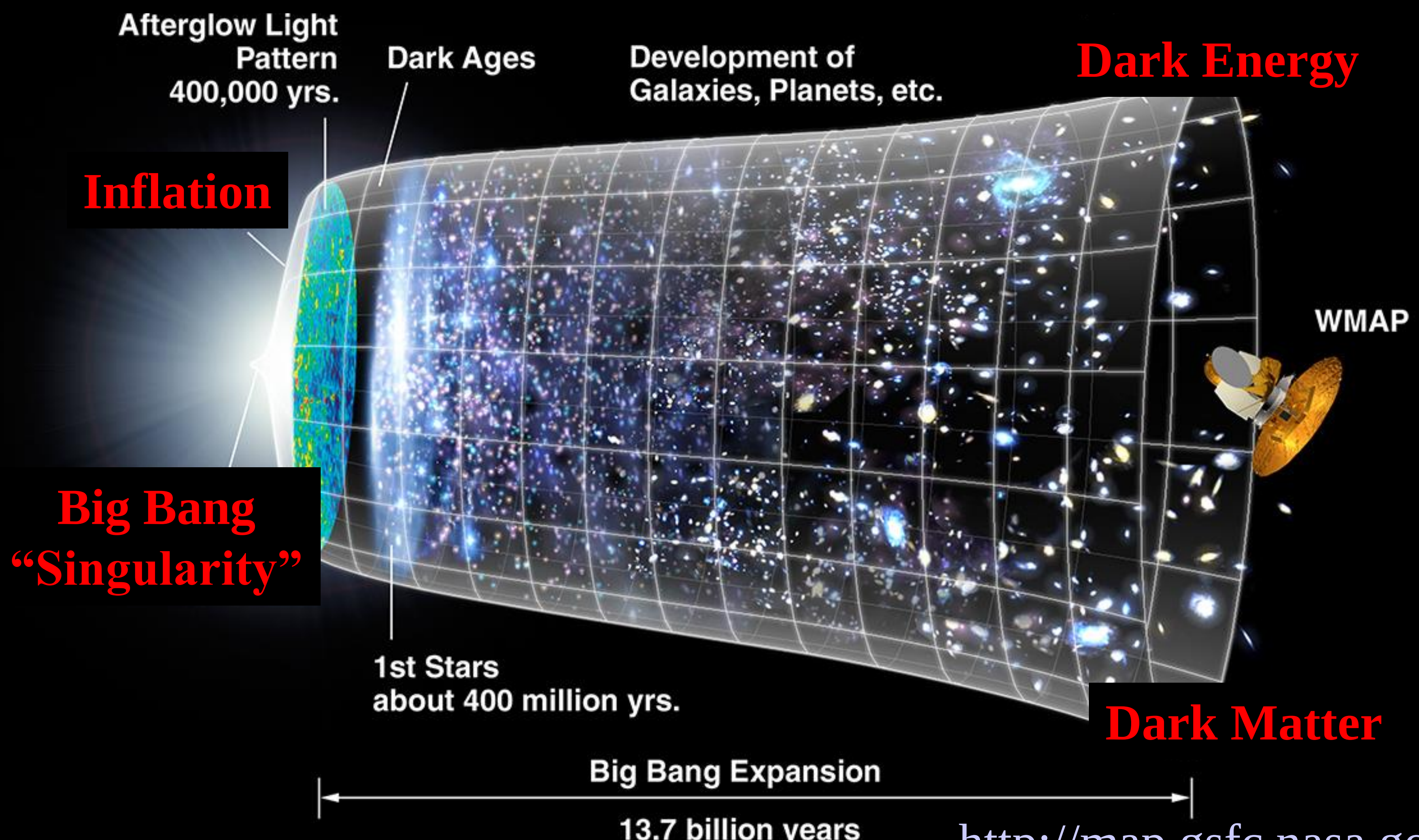
INTRODUCTION

History of the universe



http://map.gsfc.nasa.gov/

History of the universe

- History = dynamics = sequence of configurations parameterized by time
- Beginning of the hot universe @ reheating
- Geometrical description of the universe breaks down @ initial singularity
- Space may be emergent
- How about time? Can time be emergent?

Time and dynamics

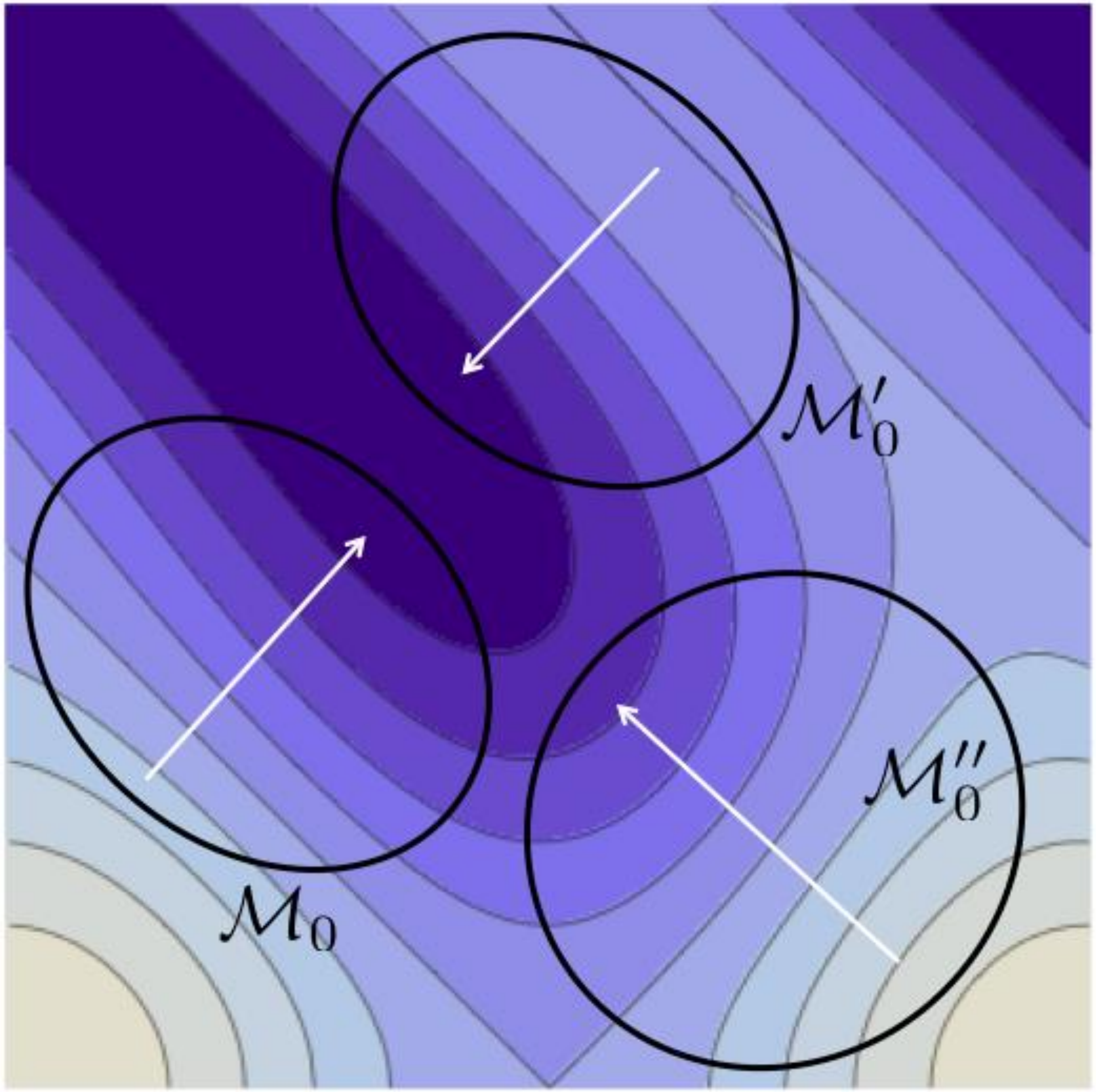
- In any diffeo-invariant theories of gravity,
  $H = \Sigma$ constraints $= 0$ (up to boundary terms)
  → no evolution for quantum state
- Dynamics should be encoded as correlations among various fields
  → one of the fields plays the role of time
- In this sense, concept of time and dynamics may be emergent

BASIC IDEA

Clock field

- Clock field = field playing the role of time
- It must carry at least one number
  $\rightarrow$ simplest: scalar field ϕ
- Time translational symmetry requires
  shift symmetry: $\phi \rightarrow \phi + c$
- Time reflection symmetry requires
  Z_2 symmetry: $\phi \rightarrow -\phi$

- Clock field does not have to be the same everywhere; multi-clock models also possible


$\mathcal{M}_0'$
$\mathcal{M}_0$
$\mathcal{M}_0''$

Effective metric

- If Lorentz symmetry can be emergent, how about Lorentz signature?
- Let's suppose that there is
  no concept of time @ fundamental level
- and start with 4D Riemannian (i.e. locally Euclidean) metric with (++++) signature.
- Physical fields couple to effective metric.
- Can effective metric have signature (-+++) ?

SIMPLE EXAMPLES

Scalar field χ in flat space

- Suppose that $\boxed{\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0}$
- Choose one of coordinates t so that $t \equiv \dfrac{\phi}{M^2}$
- Consider the Euclidean action

potential

$$S_\chi = \int \mathrm{d}^4 x \left[-\frac{1}{2}\delta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) + \frac{1}{M^4}\left(\delta^{\mu\nu}\partial_\mu\phi\partial_\nu\chi\right)^2 \right]$$

Euclidean kinetic term coupling to clock field

- This can be rewritten as

$$\boxed{S_\chi = \int \mathrm{d}t\mathrm{d}^3 x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V \right]}$$

- Lorentz signature emerges!

Vector field A_μ in flat space

- Suppose that $\boxed{\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0}$
- Choose one of coordinates t so that $t \equiv \dfrac{\phi}{M^2}$
- Consider the Euclidean action

$$S_A = \frac{1}{4}\int \mathrm{d}^4x \left[\underbrace{-F_{\mu\nu}F_{\mathrm{E}}^{\mu\nu}}_{\text{Euclidean kinetic term}} + \underbrace{\frac{4}{M^4}F_{\mathrm{E}}^{\mu\rho}F^{\nu}_{\mathrm{E}\rho}\partial_\mu\phi\partial_\nu\phi}_{\text{coupling to clock field}} \right]$$

- This can be rewritten as

$$\boxed{S_A = -\frac{1}{4}\int \mathrm{d}t\mathrm{d}^3x\, \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\alpha}F_{\nu\beta}}$$

- Lorentz signature emerges!

Abelian Higgs field ω in flat space

- Suppose that $\boxed{\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0}$
- Choose one of coordinates t so that $t \equiv \dfrac{\phi}{M^2}$
- Consider the Euclidean action

$$D_\mu \equiv \partial_\mu - iqA_\mu$$

potential

$$S_\omega = \int \mathrm{d}^4x \left[\underbrace{-\frac{1}{2}\delta^{\mu\nu}(D_\mu\omega)^*(D_\nu\omega)}_{\text{Euclidean kinetic term}} - U(|\omega|^2) + \underbrace{\frac{1}{M^4}\delta^{\mu\nu}|\partial_\mu\phi D_\nu\omega|^2}_{\text{coupling to clock field}} \right]$$

- This can be rewritten as

$$\boxed{S_\omega = \int \mathrm{d}t\mathrm{d}^3x \left[-\frac{1}{2}\eta^{\mu\nu}(D_\mu\omega)^*(D_\nu\omega) - U \right]}$$

- Lorentz signature emerges!

Dirac spinor ψ in flat space

- Consider the Euclidean action

$$S_\psi = \int dx^4 \left\{ \bar{\psi} \left(\frac{i}{2} \gamma_{\mathrm{E}}^{\mu} \overleftrightarrow{\partial_\mu} - m \right) \psi \right.$$

Euclidean kinetic and mass terms

$$\left. + \frac{1}{2M^2} \delta^{\mu\nu} \left[(i\bar{\psi}\gamma_{\mathrm{E}}^{5} \overleftrightarrow{\partial_\mu} \psi) - (i\bar{\psi}\gamma_{\mathrm{E}}^{\rho} \overleftrightarrow{\partial_\mu} \psi)\partial_\rho \phi \right] \partial_\nu \phi \right\}$$

couplings to clock field

- This can be rewritten as

$$\boxed{S_\psi = \int dx^4 \bar{\psi} \left[\frac{i}{2} \gamma^0 \overleftrightarrow{\partial_0} + \frac{i}{2} \gamma^i \overleftrightarrow{\partial_i} - m \right] \psi}$$

- Lorentzian & Euclidean gamma matrices

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu} \qquad \gamma^5 \equiv -i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\gamma_{\mathrm{E}}^0 \equiv i\gamma^5 \qquad \gamma_{\mathrm{E}}^i \equiv \gamma^i \qquad \gamma_{\mathrm{E}}^5 \equiv \gamma_{\mathrm{E}}^0\gamma_{\mathrm{E}}^1\gamma_{\mathrm{E}}^2\gamma_{\mathrm{E}}^3 = \gamma^0$$

$$\{\gamma_{\mathrm{E}}^\mu, \gamma_{\mathrm{E}}^\nu\} = -2\delta^{\mu\nu} \qquad (\gamma_{\mathrm{E}}^5)^2 = 1 \qquad \{\gamma_{\mathrm{E}}^5, \gamma_{\mathrm{E}}^\mu\} = 0$$

Gravity (tensor sector)

- Consider the Riemannian action

couplings to clock field

$$I_g = -M^2 \int dx^4 \sqrt{g_E} \left[\frac{\kappa_g}{2} R_E + \frac{\alpha_g}{M^4} G_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

Euclidean Einstein-Hilbert

- Adopt ADM in "unitary gauge" $\boxed{t \equiv \frac{\phi}{M^2}}$

$$g_{\mu\nu}^E dx^\mu dx^\nu = \boxed{N_E^2 dt^2} + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Use formulas

$$\begin{aligned} \frac{1}{2} \int dx^4 \sqrt{g_E} R_E &= \frac{1}{2} \int dt dx^3 N_E \sqrt{\gamma} (-K_E^{ij} K_{ij}^E + K_E^2 + R^{(3)}) \\ \frac{1}{M^4} G_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi &= \frac{1}{2N_E^2} (-K_E^{ij} K_{ij}^E + K_E^2 - R^{(3)}) \end{aligned}$$

- The action is rewritten as

$$\boxed{I_g = \frac{M^2}{2} \int dt dx^3 N_E \sqrt{\gamma} \left[\left(\frac{\alpha_g}{N_E^2} + \kappa_g \right) (K_E^{ij} K_{ij}^E - K_E^2) + \left(\frac{\alpha_g}{N_E^2} - \kappa_g \right) R^{(3)} \right]}$$

- This describes Lorentzian gravity if $\frac{\alpha_g}{N_E^2} > |\kappa_g|$ (but only for tensor sector)

GRAVITY

Model of clock field and gravity

- Shift symmetry: $\phi \rightarrow \phi + c$
- Z_2 symmetry: $\phi \rightarrow - \phi$
- Minimal # of d.o.f. → 2nd-order EOM →
  Riemannian version of covariant Galileon

$$S_g = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \Big\{ G_4(X_{\mathrm{E}}) R_{\mathrm{E}} - g_5 G_{\mathrm{E}}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \mathcal{K}(X_{\mathrm{E}}) \\ -2G_4'(X_{\mathrm{E}}) \left[(\nabla_{\mathrm{E}}^2 \phi)^2 - (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)^2 \right] \Big\}$$

$$X_{\mathrm{E}} \equiv g_{\mathrm{E}}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \qquad (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)^2 \equiv g_{\mathrm{E}}^{\nu\rho} g_{\mathrm{E}}^{\sigma\mu} (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)(\nabla_\rho^{\mathrm{E}} \nabla_\sigma^{\mathrm{E}} \phi)$$

- Redefinition of $G_4(X_E)$ → $g_5 = 0$

$$\boxed{S_g = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \Big\{ G_4(X_{\mathrm{E}}) R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}}) \left[(\nabla_{\mathrm{E}}^2 \phi)^2 - (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)^2 \right] \Big\}}$$

Riemannian action in unitary gauge

- Riemannian metric in unitary gauge

$$g^{\mathrm{E}}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{N_{\mathrm{E}}^2\mathrm{d}t^2} + \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

$$\boxed{t \equiv \frac{\phi}{M^2}} \quad N_{\mathrm{E}} \equiv \frac{1}{\sqrt{g^{tt}_{\mathrm{E}}}} = \frac{M^2}{\sqrt{X_{\mathrm{E}}}} \quad N^i \equiv \gamma^{ij} g^{\mathrm{E}}_{tj} \quad \gamma_{ij} \equiv g^{\mathrm{E}}_{ij}$$

- Extrinsic curvature $K^{\mathrm{E}}_{ij} \equiv \frac{1}{2N_{\mathrm{E}}}(\partial_t \gamma_{ij} - D_i N_j - D_j N_i)$
- Useful formulas

$$\begin{aligned}
\sqrt{g_{\mathrm{E}}} R_{\mathrm{E}} &= N_{\mathrm{E}}\sqrt{\gamma}(-K^{ij}_{\mathrm{E}} K^{\mathrm{E}}_{ij} + K_{\mathrm{E}}^2 + R^{(3)}) \\
&\quad -2\partial_i(\sqrt{\gamma}\gamma^{ij}\partial_j N_{\mathrm{E}}) - 2\partial_t(\sqrt{\gamma} K_{\mathrm{E}}) + 2\partial_i(\sqrt{\gamma} N^i K_{\mathrm{E}})
\end{aligned}$$

$$\begin{aligned}
\phi^{\mathrm{E}}_{;ij} &\equiv \nabla^{\mathrm{E}}_i \nabla^{\mathrm{E}}_j \phi = \sqrt{X_{\mathrm{E}}} K^{\mathrm{E}}_{ij} \\
\phi^{\mathrm{E}}_{;\perp i} &\equiv \phi^{\mathrm{E}}_{;i\perp} \equiv n^\mu_{\mathrm{E}} \nabla^{\mathrm{E}}_\mu \nabla^{\mathrm{E}}_i \phi = \frac{1}{2}\sqrt{X_{\mathrm{E}}}\partial_i \ln X_{\mathrm{E}} \\
\phi^{\mathrm{E}}_{;\perp\perp} &\equiv n^\mu_{\mathrm{E}} n^\nu_{\mathrm{E}} \nabla^{\mathrm{E}}_\mu \nabla^{\mathrm{E}}_\nu \phi = \frac{1}{2}\sqrt{X_{\mathrm{E}}}\partial^{\mathrm{E}}_\perp \ln X_{\mathrm{E}}
\end{aligned}$$

$$\partial^{\mathrm{E}}_\perp \equiv n^\mu_{\mathrm{E}}\partial_\mu \equiv \frac{1}{N_{\mathrm{E}}}(\partial_t - N^i \partial_i)$$

- Riemannian action is rewritten as

$$\boxed{S_g = \int \mathrm{d}t\mathrm{d}x^3 N_{\mathrm{E}}\sqrt{\gamma}\left\{(2G_4' X_{\mathrm{E}} - G_4)(K^{ij}_{\mathrm{E}} K^{\mathrm{E}}_{ij} - K_{\mathrm{E}}^2) + G_4 R^{(3)} + \mathcal{K}(X_{\mathrm{E}})\right\}}$$

Apparent Lorentzian structure

- Lorentzian metric

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{-N^2\mathrm{d}t^2} + \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(dx^j + N^j\mathrm{d}t)$$

$$N\mathrm{d}N = -N_{\mathrm{E}}\,\mathrm{d}N_{\mathrm{E}} \qquad \boxed{N = \sqrt{N_c^2 - N_{\mathrm{E}}^2}}$$

- Extrinsic curvature $K_{ij} \equiv \frac{1}{2N}(\partial_t\gamma_{ij} - D_iN_j - D_jN_i)$
- Apparent Lorentzian structure

$$\boxed{\begin{aligned} S_g = \int \mathrm{d}t\mathrm{d}x^3 N\sqrt{\gamma}\,\Big\{&[f(X) - 2Xf'(X)]\,(K^{ij}K_{ij} - K^2) \\ &+ f(X)R^{(3)} + P(X)\Big\}\end{aligned}}$$

$$f(X) \equiv \frac{N_{\mathrm{E}}}{N}G_4(X_E) \qquad f'(X) \equiv \frac{\mathrm{d}f(X)}{\mathrm{d}X} \qquad P(X) \equiv \frac{N_{\mathrm{E}}}{N}\mathcal{K}(X_{\mathrm{E}})$$

$$X \equiv \frac{M^4}{N^2}$$

Undo unitary gauge

- Useful formulas

$$\sqrt{-g}R = N\sqrt{\gamma}\left[K^{ij}K_{ij} - K^2 + R^{(3)}\right] - \Delta$$

$$\Delta = 2\partial_i\left(\sqrt{\gamma}\gamma^{ij}\partial_j N\right) - 2\partial_t\left(\sqrt{\gamma}K\right) + 2\partial_i\left(\sqrt{\gamma}N^i K\right)$$

$$\phi_{;ij} \;\equiv\; \nabla_i\nabla_j\phi = -\sqrt{X}K_{ij}$$

$$\phi_{;\perp i} \;\equiv\; \phi_{;i\perp} \equiv n^\mu\nabla_\mu\nabla_i\phi = \frac{1}{2}\sqrt{X}\partial_i \ln X$$

$$\phi_{;\perp\perp} \;\equiv\; n^\mu n^\nu\nabla_\mu\nabla_\nu\phi = \frac{1}{2}\sqrt{X}\partial_\perp \ln X$$

$$\partial_\perp = n^\mu\partial_\mu = \frac{1}{N}(\partial_t - N^i\partial_i)$$

- Lorentzian action

$$\boxed{\begin{aligned} S_g \;=\; \int \mathrm{d}x^4\sqrt{-g}\,\{f(X)R + 2f'(X)\left[(\nabla^2\phi)^2\right. \\ \left. -(\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)\right] + P(X)\}\end{aligned}}$$

- "Covariantization"

$$X \;\equiv\; \frac{M^4}{N^2} \;\Rightarrow\; X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Correspondence

$$\boxed{S_g = \int dx^4 \sqrt{g_{\mathrm{E}}}\left\{G_4(X_{\mathrm{E}})R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}})\left[(\nabla_{\mathrm{E}}^2\phi)^2 - (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2\right]\right\}}$$

$$\boxed{g_{\mu\nu}} = \boxed{g^{\mathrm{E}}_{\mu\nu}} - \frac{\partial_\mu\phi\partial_\nu\phi}{X_c}$$

$$\boxed{g^{\mu\nu}} = \boxed{g_{\mathrm{E}}^{\mu\nu}} + \frac{\boxed{g_{\mathrm{E}}^{\mu\rho}g_{\mathrm{E}}^{\nu\sigma}}\partial_\rho\phi\partial_\sigma\phi}{X_c - \boxed{X_{\mathrm{E}}}}$$

$$\frac{1}{\boxed{X}} = \frac{1}{X_c} - \frac{1}{\boxed{X_{\mathrm{E}}}} \qquad X_c = \frac{M^4}{N_c^2}$$

$$\boxed{\frac{f(X)}{\sqrt{X}}} = \boxed{\frac{G_4(X_{\mathrm{E}})}{\sqrt{X_{\mathrm{E}}}}} \qquad \boxed{\frac{P(X)}{\sqrt{X}}} = \boxed{\frac{\mathcal{K}(X_{\mathrm{E}})}{\sqrt{X_{\mathrm{E}}}}}$$

$$\boxed{\begin{aligned} S_g = \int dx^4\sqrt{-g}\,\{f(X)R + 2f'(X)\left[(\nabla^2\phi)^2\right.\\ \left. -(\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)\right] + P(X)\}\end{aligned}}$$

COSMOLOGICAL SOLUTION

Cosmological solution

- Flat (K=0) FLRW

$$N = 1 \quad N_i = 0 \quad \gamma_{ij} = a(t)^2\delta_{ij} \qquad \phi = \phi_0(t)$$

- EOM for ϕ = shift charge conservation

$$\dot{J}_\phi + 3HJ_\phi = 0 \quad \Rightarrow \quad \boxed{J_\phi \propto 1/a^3}$$

$$J_\phi \equiv \left[P_0' + 6H^2(2X_0 f_0'' + f_0')\right]\dot{\phi}_0$$

- Metric EOM

$$3M_{\rm eff}^2 H^2 = 2J_\phi\dot{\phi}_0 - P_0 \qquad M_{\rm eff}^2 \equiv 2(f_0 - 2X_0 f_0')$$

- P'(X) and f'(X) near a local minimum of P(X)

$$P'(X) = p_2\delta + \mathcal{O}(\delta^2) \quad f'(X) = \frac{f_1 + f_2\delta}{M^2} + \mathcal{O}(\delta^2) \quad \delta \equiv \frac{X}{M^4} - q$$

$$J_\phi \propto 1/a^3 \Rightarrow \quad \delta + \mathcal{O}(H^2/M^2) \propto 1/a^3 \to 0$$

$$\Rightarrow \quad \boxed{3M_{\rm eff}^2 H^2 = \text{DM}\ (\propto 1/a^3) + \text{DE}\ (\sim \text{const})}$$

Stability of tensor perturbation

- Tensor-type perturbation

$$N = 1 \quad N_i = 0 \quad \gamma_{ij} = a(t)^2 \left[\mathrm{e}^h\right]_{ij}$$

$$\phi = \phi_0(t) \qquad \boxed{\partial_i h^i_k = 0 = \delta^{ij} h_{ij}}$$

- Quadratic action in Fourier space

$$\delta S^{(2)}_{\mathrm{T},\boldsymbol{k}} = \frac{1}{8}\int \mathrm{d}t a^3 \left[\boxed{M^2_{\mathrm{eff}}} \dot{h}^2_{\boldsymbol{k}} - 2\boxed{f_0}\frac{\boldsymbol{k}^2}{a^2} h^2_{\boldsymbol{k}}\right]$$

$$M^2_{\mathrm{eff}} \equiv 2(f_0 - 2X_0 f_0')$$

- Stability condition

$$\boxed{M^2_{\mathrm{eff}} > 0 \qquad f_0 > 0}$$

Stability of scalar perturbation

- Scalar-type perturbation in unitary gauge

$$N = 1 + \alpha \quad N_i = \partial_i \beta \quad \gamma_{ij} = a(t)^2 \boxed{\mathrm{e}^{2\zeta}} \delta_{ij}$$

$$\phi = \phi_0(t)$$

- Quadratic action after eliminating α and β

$$\delta S^{(2)}_{\mathrm{S},\boldsymbol{k}} = \frac{1}{2}\int \mathrm{d}t a^3 \left[\boxed{\mathcal{A}}\dot{\zeta}_{\boldsymbol{k}}^2 - \boxed{\mathcal{B}}\frac{\boldsymbol{k}^2}{a^2}\zeta_{\boldsymbol{k}}^2\right]$$

$$\mathcal{A} = \frac{M_{\mathrm{eff}}^2}{H^2\mathcal{G}^2}\left(6 + M_{\mathrm{eff}}^2\mathcal{F}\right) \qquad \mathcal{B} = \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{aM_{\mathrm{eff}}^4}{H\mathcal{G}^2}\right) + 4f_0$$

$$\mathcal{F} = P_0'' X_0^2 + \frac{1}{2}J_\phi\dot{\phi}_0 + 3H^2\left[4f_0''' X_0^3 + 14 f_0'' X_0^2 + 6 f_0' X_0 - f_0\right]$$

$$\mathcal{G} = 4f_0'' X_0^2 + 4f_0' X_0 - f_0 \qquad M_{\mathrm{eff}}^2 \equiv 2(f_0 - 2X_0 f_0')$$

- Stability condition

$$\boxed{\mathcal{A} > 0 \qquad \mathcal{B} > 0}$$

PHENOMENOLOGY

Free functions/parameters

- Gravity sector: $G_4(X_E)$, $K(X_E)$ [or $f(X)$, $P(X)$]

$$S_g = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \Big\{ \boxed{G_4(X_{\mathrm{E}})} R_{\mathrm{E}} - g_5 G_{\mathrm{E}}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \boxed{\mathcal{K}(X_{\mathrm{E}})} - 2\boxed{G_4'(X_{\mathrm{E}})} \left[(\nabla_{\mathrm{E}}^2 \phi)^2 - (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)^2 \right] \Big\}$$

- Matter sector: $(\kappa_\chi , \alpha_\chi)$, (κ_A , α_A), $(\kappa_\omega , \alpha_\omega)$

$$S_\chi = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \left[-\frac{\boxed{\kappa_\chi}}{2} g_{\mathrm{E}}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \tilde{V}(\chi) + \frac{\boxed{\alpha_\chi}}{2M^4} (g_{\mathrm{E}}^{\mu\nu} \partial_\mu \phi \partial_\nu \chi)^2 \right]$$

$$S_A = \frac{1}{4} \int dx^4 \sqrt{g_{\mathrm{E}}} \left[-\boxed{\kappa_A} F_{\mathrm{E}}^{\mu\nu} F_{\mu\nu} + 2 \frac{\boxed{\alpha_A}}{M^4} F_{\mathrm{E}}^{\mu\rho} F_{\mathrm{E}\rho}^{\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$S_\psi = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \Big\{ -\frac{\boxed{\kappa_\psi}}{2} g_{\mathrm{E}}^{\mu\nu} (\partial_\mu + iqA_\mu) \psi^* (\partial_\nu - iqA_\nu) \psi + \frac{\boxed{\alpha_\psi}}{2M^4} |g_{\mathrm{E}}^{\mu\nu} \partial_\mu \phi (\partial_\nu - iqA_\nu) \psi|^2 - \tilde{U}(|\psi|^2) \Big\}$$

- Clock field configuration: $\phi(x)$

$$X_{\mathrm{E}} \equiv g_{\mathrm{E}}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Constraints

- Stability of clock field/gravity sector

$$M_{\rm eff}^2 > 0 \qquad f_0 > 0 \qquad \mathcal{A} > 0 \qquad \mathcal{B} > 0$$

- Amount of DE/DM

$$P_0 \sim -3\Omega_{\Lambda 0} M_{\rm eff}^2 H_0^2 \sim -2.1 M_{\rm eff}^2 H_0^2$$

$$\frac{2}{3}\frac{J_{\phi_0}}{M_{\rm eff}^2}\sqrt{q}\frac{M^2}{H_0^2} \leq \Omega_{\rm m0} \sim 0.3$$

- Stability of matter sector

$$\frac{\alpha_\chi}{N_{\rm E}^2} > \kappa_\chi > 0 \qquad \frac{\alpha_A}{N_{\rm E}^2} > \kappa_A > 0$$

- Coincidence of speed limits in matter sector

$$\frac{\kappa_A}{\alpha_A} = \frac{\kappa_\chi}{\alpha_\chi}$$ independently from clock field configuration

- Avoidance of gravi-Cerenkov radiation

$$\frac{c_\gamma - c_{\rm GW}}{c_\gamma} < 2\times 10^{-15} \qquad c_\gamma^2 = \left[\frac{\alpha_A X_{\rm E}}{\kappa_A M^4} - 1\right]^{-1} \qquad c_{\rm GW}^2 = \left[\frac{2G_4' X_{\rm E}}{G_4} - 1\right]^{-1}$$

SUMMARY & DISCUSSIONS

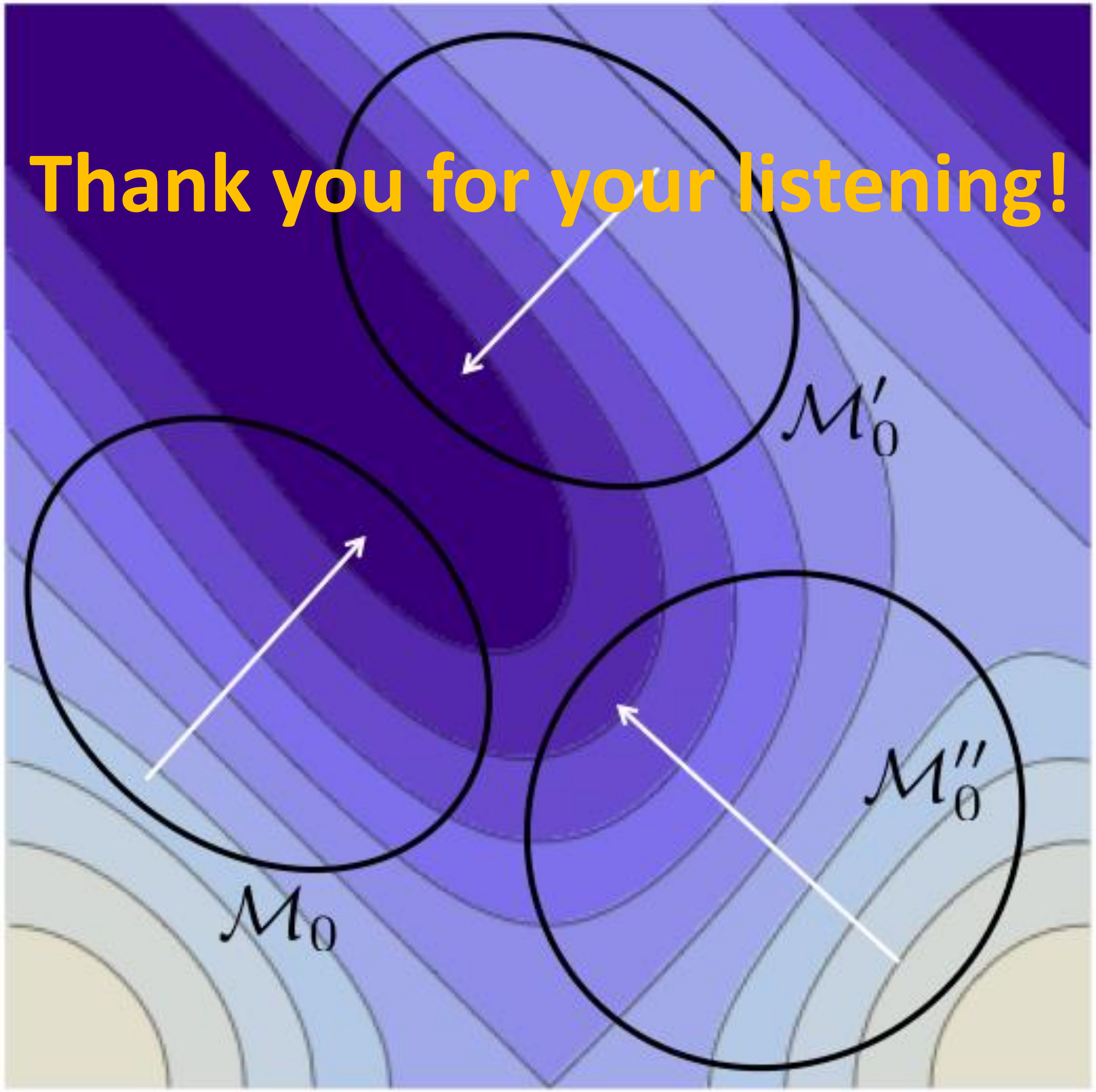
Summary

- Lorentzian dynamics can emerge as an effective property of a fundamentally Riemannian theory.
- This requires introduction of a field playing the role of time, a clock field.
- This idea was applied to scalar, vector, Dirac fields and gravity as explicit examples.
- In our simple realization, the clock field/gravity sector is described by the Riemannian version of a shift- and Z_2-symmetric covariant Galileon.
- We obtained the dictionary for the mapping from Riemannian Galileon to Lorentzian Galileon.
- We found a FLRW solution and analyzed stability of scalar- and tensor- perturbations.

Future works

- Development of quantum theory
- Construction of Majorana and Weyl spinors
- CPT-invariant construction of Dirac spinor
- Understanding of black holes and singularities
- Possibility of embedding Lorentzian dS/CFT inside Euclidean AdS/CFT
- Emergence of Lorentz symmetry at low energy
- …
- Multi-clock models
- Time emergence & compactification → landscape with various signatures & dimensions?

Thank you for your listening!



BACKUP SLIDES

MATTER FIELDS IN CURVED SPACE

Scalar field χ in curved space

- ADM decomposition in unitary gauge $t \equiv \frac{\phi}{M^2}$

$$g^{\mathrm{E}}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{N_{\mathrm{E}}^2 \mathrm{d}t^2} + \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

- Riemannian action

$$\begin{aligned} S_\chi &= \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}}\left[-\frac{\kappa_\chi}{2} g_{\mathrm{E}}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \tilde{V}(\chi) + \frac{\alpha_\chi}{2M^4}(g_{\mathrm{E}}^{\mu\nu}\partial_\mu\phi\partial_\nu\chi)^2\right] \\ &= \int \mathrm{d}t\mathrm{d}x^3 N_{\mathrm{E}}\sqrt{\gamma}\left[\frac{1}{2}\left(\frac{\alpha_\chi}{N_{\mathrm{E}}^2} - \kappa_\chi\right)(\partial_\perp^{\mathrm{E}}\chi)^2 - \tilde{V}(\chi) - \frac{\kappa_\chi}{2}\gamma^{ij}\partial_i\chi\partial_j\chi\right] \end{aligned}$$

- If $\boxed{\frac{\alpha_\chi}{N_{\mathrm{E}}^2} > \kappa_\chi > 0}$, then the action is rewritten as

$$S_\chi = -\int \mathrm{d}x^4 \sqrt{-g^\chi}\left[\frac{1}{2} g_\chi^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + V(\chi, X)\right]$$

$$V(\chi, X) = \tilde{V}(\chi)\left[\kappa_\chi^3\left(\frac{\alpha_\chi X_{\mathrm{E}}}{M^4} - \kappa_\chi\right)\right]^{-1/2}$$

with Lorentzian metric

$$g^\chi_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{-N_\chi^2 \mathrm{d}t^2} + \boxed{\Omega_\chi^2}\gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

$$N_\chi = N_{\mathrm{E}}\left[\frac{\kappa_\chi^3}{\frac{\alpha_\chi}{N_{\mathrm{E}}^2} - \kappa_\chi}\right]^{1/4} \qquad \Omega_\chi = \left[\kappa_\chi\left(\frac{\alpha_\chi}{N_{\mathrm{E}}^2} - \kappa_\chi\right)\right]^{1/4}$$

Vector field A_μ in curved space

- ADM decomposition in unitary gauge $\quad t \equiv \dfrac{\phi}{M^2}$

$$g^{\mathrm{E}}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{N_{\mathrm{E}}^2 \mathrm{d}t^2} + \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

- Riemannian action $\quad \tilde{F}_{\perp i} \equiv \dfrac{1}{N_{\mathrm{E}}}(F_{ti} - N^j F_{ji})$

$$S_A = \frac{1}{4}\int dx^4 \sqrt{g_{\mathrm{E}}}\left[-\kappa_A F_{\mathrm{E}}^{\mu\nu} F_{\mu\nu} + 2\frac{\alpha_A}{M^4} F_{\mathrm{E}}^{\mu\rho} F^{\nu}_{\mathrm{E}\rho}\partial_\mu\phi\partial_\nu\phi\right]$$

$$= \frac{1}{4}\int \mathrm{d}t\mathrm{d}x^3 N_{\mathrm{E}}\sqrt{\gamma}\left[2\left(\frac{\alpha_A}{N_{\mathrm{E}}^2} - \kappa_A\right)\gamma^{ij}\tilde{F}_{\perp i}\tilde{F}_{\perp j} - \kappa_A \gamma^{ik}\gamma^{jl}F_{ij}F_{kl}\right]$$

- If $\boxed{\dfrac{\alpha_A}{N_{\mathrm{E}}^2} > \kappa_A > 0}$, then the action is rewritten as

$$S_A = -\int \mathrm{d}x^4 \sqrt{-g^A}\frac{1}{4e^2} g_A^{\mu\rho} g_A^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \qquad e^2 = \left[\kappa_A\left(\frac{\alpha_A}{N_{\mathrm{E}}^2} - \kappa_A\right)\right]^{-1/2}$$

with Lorentzian metric

$$g^{A}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{-N_A^2 dt^2} + \boxed{\Omega_A^2}\gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

$$N_A = N_{\mathrm{E}}\Omega_A\left[\frac{\kappa_A}{\frac{\alpha_A}{N_{\mathrm{E}}^2} - \kappa_A}\right]^{1/2} \qquad \Omega_A > 0$$

Abelian Higgs field ψ in curved space

- ADM decomposition in unitary gauge $t \equiv \dfrac{\phi}{M^2}$

$$g^{\mathrm{E}}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{N_{\mathrm{E}}^2 \mathrm{d}t^2} + \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

- Riemannian action

$$S_\psi = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \left\{ -\frac{\kappa_\psi}{2} g_{\mathrm{E}}^{\mu\nu}(\partial_\mu + iqA_\mu)\psi^*(\partial_\nu - iqA_\nu)\psi + \frac{\alpha_\psi}{2M^4}\left|g_{\mathrm{E}}^{\mu\nu}\partial_\mu\phi(\partial_\nu - iqA_\nu)\psi\right|^2 - \tilde{U}(|\psi|^2) \right\}$$

- If $\boxed{\dfrac{\alpha_\psi}{N_{\mathrm{E}}^2} > \kappa_\psi > 0}$, then the action is rewritten as

$$S_\psi = -\int \mathrm{d}x^4 \sqrt{-g^\psi} \left[\frac{1}{2} g_\psi^{\mu\nu}(\partial_\mu + iqA_\mu)\psi^*(\partial_\nu - iqA_\nu)\psi + U(|\psi|^2, X) \right]$$

$$U(|\psi|^2, X) = \tilde{U}(|\psi|^2)\left[\kappa_\psi^3\left(\frac{\alpha_\psi X_{\mathrm{E}}}{M^4} - \kappa_\psi\right)\right]^{-1/2}$$

with Lorentzian metric

$$g^\psi_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \boxed{-N_\psi^2 \mathrm{d}t^2} + \boxed{\Omega_\psi^2}\gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

$$N_\psi = N_{\mathrm{E}}\left[\frac{\kappa_\psi^3}{\frac{\alpha_\psi}{N_{\mathrm{E}}^2} - \kappa_\psi}\right]^{1/4} \qquad \Omega_\psi = \left[\kappa_\psi\left(\frac{\alpha_\psi}{N_{\mathrm{E}}^2} - \kappa_\psi\right)\right]^{1/4}$$