IPMU Focus week 2013.2.21

A brief comment on rotational symmetry violation

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Ref. **A.Maleknejad, M.M.Sheikh-Jabbari and J.Soda, ``Gauge Fields and Inflation,'' arXiv:1212.2921 [hep-th].**

Anisotropic Scaling in Cosmology

Watanabe, Kanno, Soda, PRL, 2009

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\left(\partial_\mu\phi\right)^2 - e^{\lambda\varphi} - \frac{1}{4}e^{2\rho\phi}F_{\mu\nu}F^{\mu\nu}\right]$$

power-law isotropic inflation

$$ds^2 = -dt^2 + t^{\frac{4}{\lambda^2}}\left(dx^2 + dy^2 + dz^2\right)$$

$$\phi = -\frac{2}{\lambda}\log t \qquad A_\mu = 0$$

power-law anisotropic inflation

For the parameter region $\lambda^2 + 2\rho\lambda - 4 > 0$, we found the following new solution

$$ds^2 = -dt^2 + t^{2\omega}\left[t^{-4\zeta}dx^2 + t^{2\zeta}\left(dy^2 + dz^2\right)\right]$$

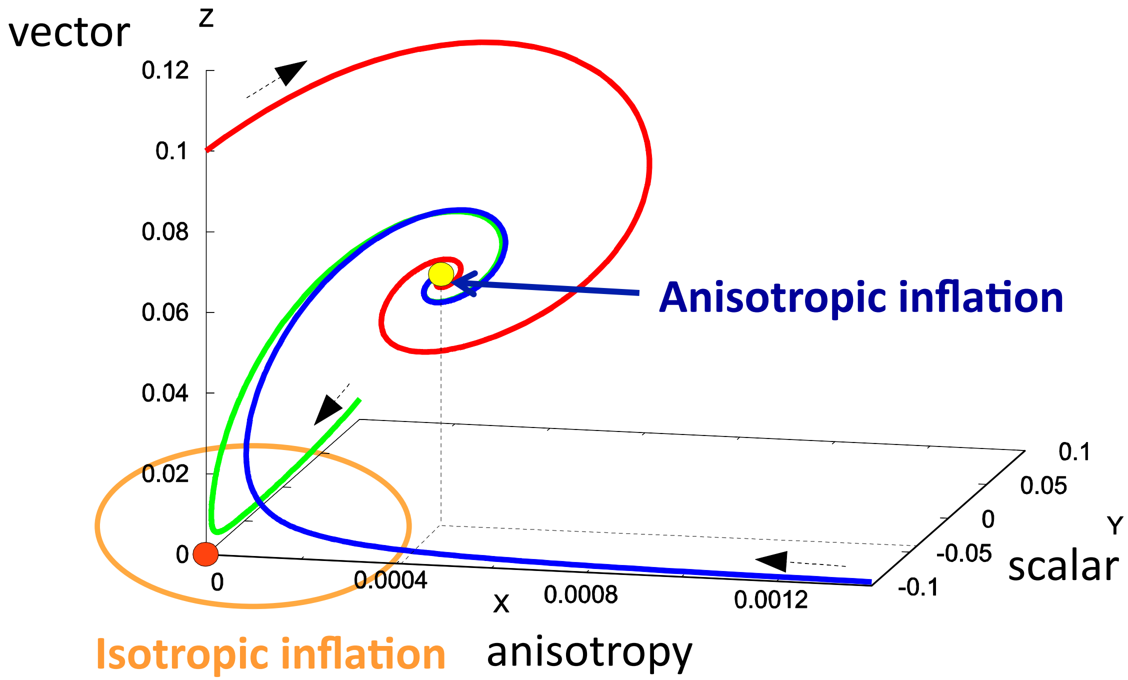
$$\phi = -\frac{2}{\lambda}\log t \qquad \dot{A}_x(t) = t^\gamma$$

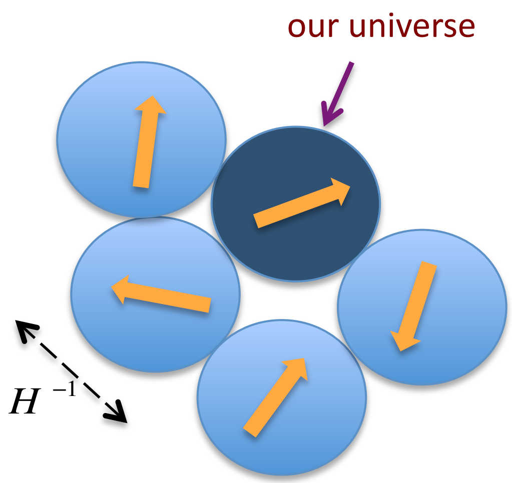
$$\omega = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)} \qquad \zeta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)} \qquad \gamma = 4\frac{\rho}{\lambda} - \omega - 4\zeta$$

Spontaneous Rotational Symmetry breakdown

Kanno, Watanabe, Soda, JCAP, 2010

After a transient isotropic inflationary phase,
the universe enter into an anisotropic inflationary phase.





Quantum fluctuations generate seeds of coherent vector fields.

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Observer dependence of Schwinger pair production

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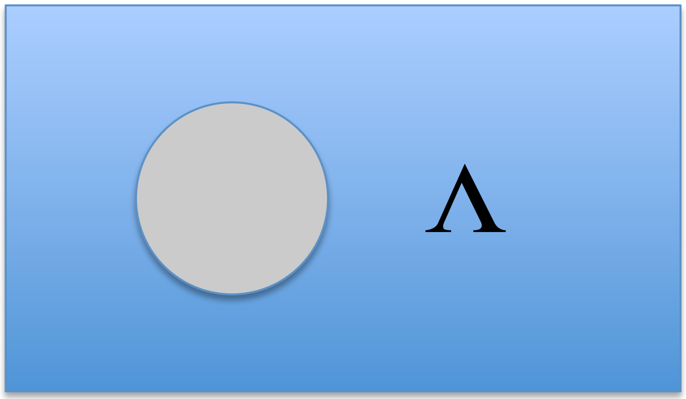
J.Garriga, S.Kanno, M.Sasaki, J.Soda, A.Vilenkin, JCAP 1212, 006 (2012) [arXiv:1208.1335 [hep-th]].

Introduction

In the picture of string theory landscape, there are many vacuum transitions.

Note that the vacuum energy $T_{\mu\nu} = -\Lambda \eta_{\mu\nu}$ is Lorentz invariant

If this is a meta stable state, a vacuum transition occurs and a bubble would nucleate.



In this picture, the whole bubble has nucleated at the same time.

If the vacuum transition process is Lorentz invariant, a different observer should look at a different one that would be weird.

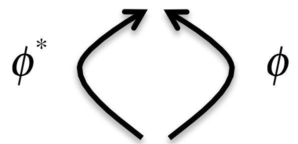
Question: In which frame does observer look at this particular picture?

This is an academic issue!

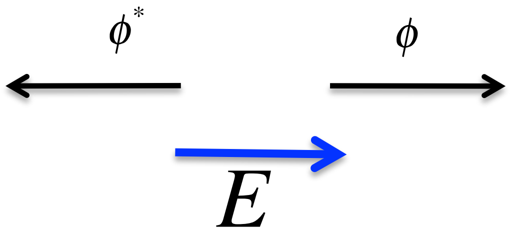
In this talk, I would like to discuss this issue with a 2-d toy model.

Schwinger pair production: A toy model

Vacuum polarization



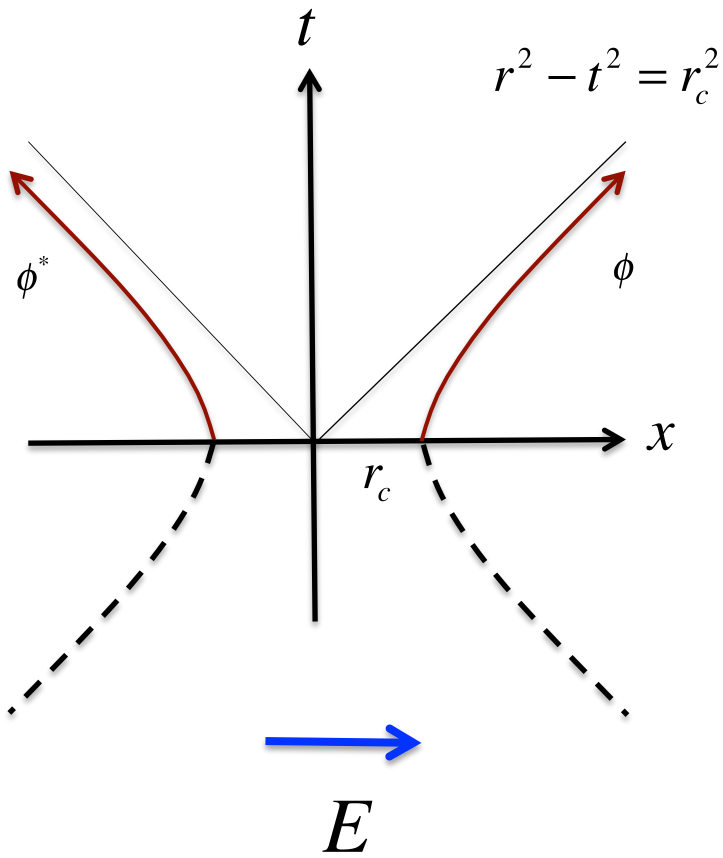
Let us consider a constant background electric field.



Pair creation occurs when enough energy is provided
by the constant electric field

$$eE \times L \sim 2m \quad \Rightarrow \quad r_c = \frac{m}{eE}$$

The pair is separated at a finite distance.



Schwinger pair production rate

In the first quantization picture, we can calculate transition rate via instanton method.

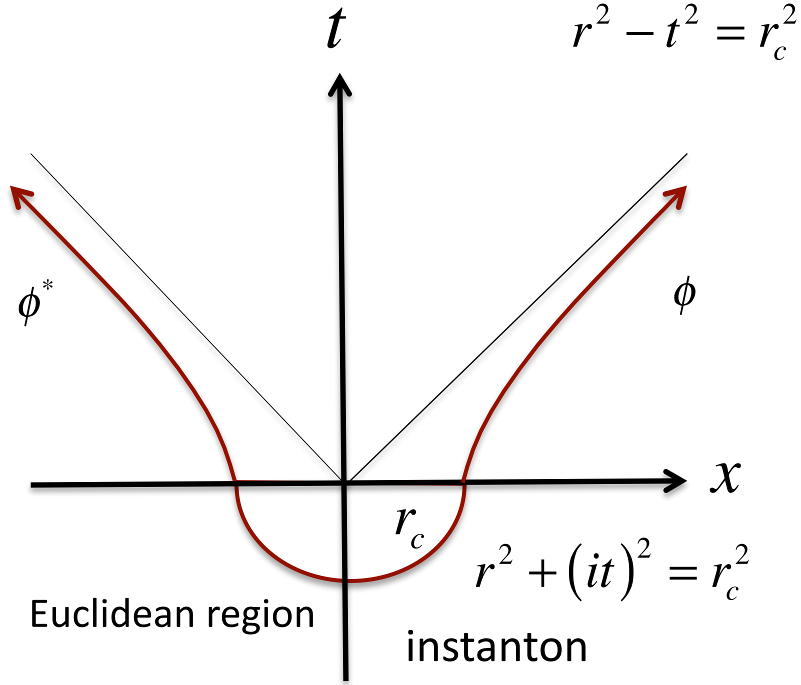
$$S_E = m\int ds + \frac{1}{4}\int d^2x\sqrt{g}F^{\mu\nu}F_{\mu\nu}$$

$$P \approx \exp\left(-\left\{S_E[\text{instanton}] - S[\text{background}]\right\}\right)$$

$$S_E[\text{instanton}] - S[\text{background}] = 2\pi m r_c - eE\times\pi r_c^2$$

$$= \pi m r_c = \frac{\pi m^2}{eE}$$

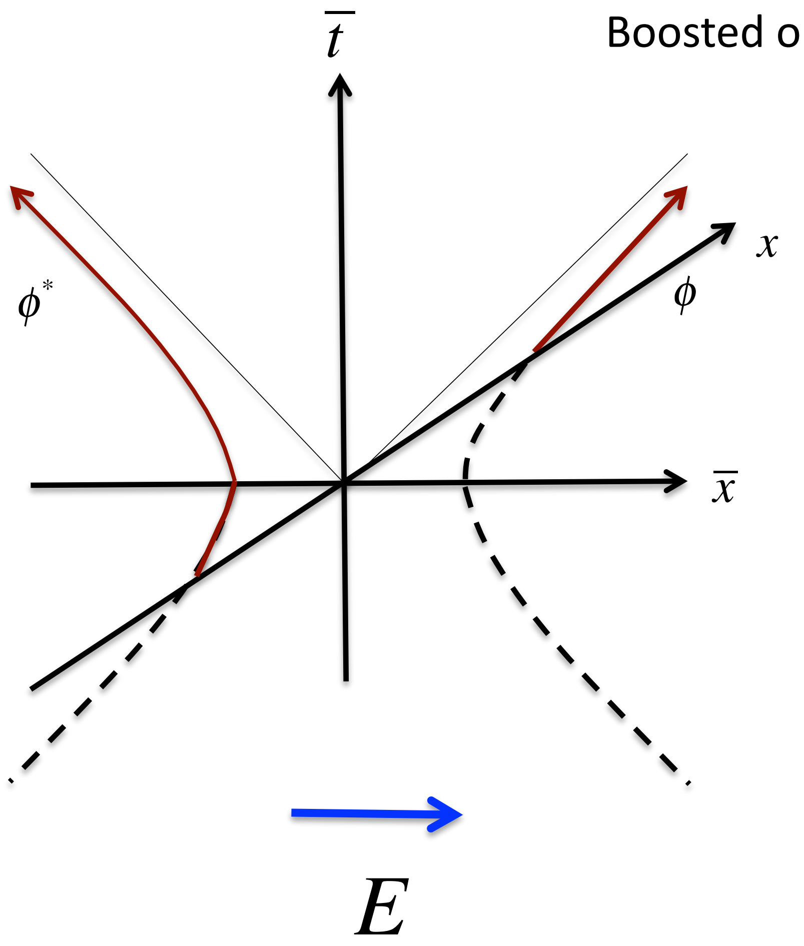
$$\therefore \quad P \approx e^{-\frac{\pi m^2}{eE}}$$



Of course, this calculation does not tell us anything about the nucleation frame.

Can we see the boosted nucleation?

Now, let me go back to our problem.



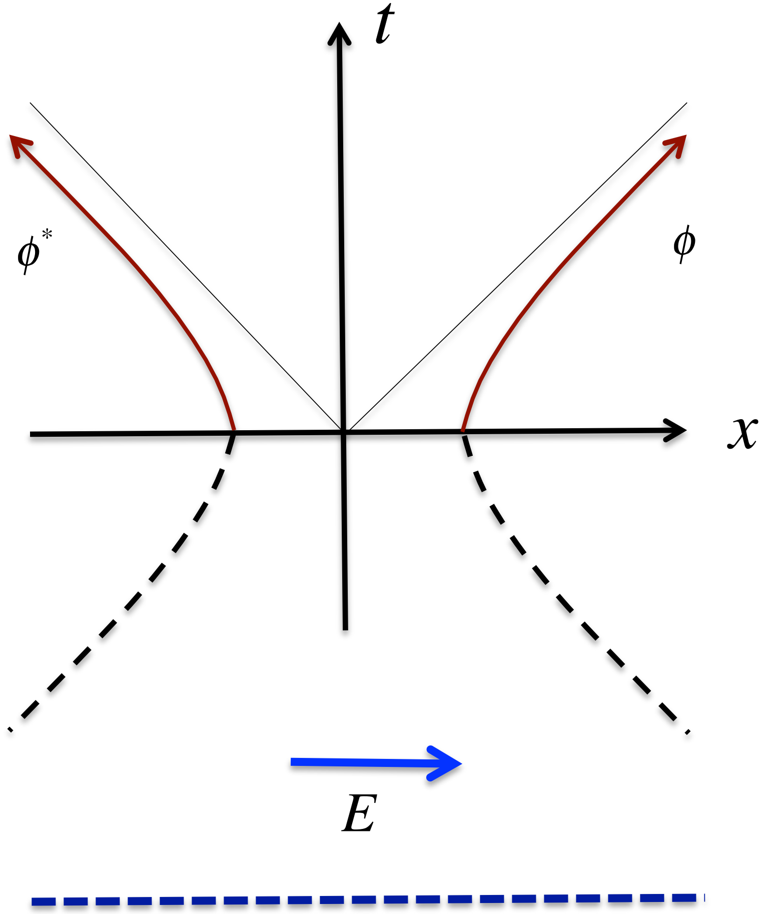
Boosted observer should look at a different picture.

First, an anti-particle appears abruptly.
Then, it bounces back to the left.
After a while, a particle appears.

If the nucleation process is Lorentz invariant, we must find this type of nucleation.

If not, the nucleation frame must be determined by the observer or others.

Initial conditions determine the nucleation frame?



initial condition hypersurface

What is other possibility?

There may be a preferred frame determined by the initial conditions.

In that frame, the pair of particles appear at the same time.

Strategy

There are at least the following options for the answer to our problem.

(A) The frame of nucleation is determined by initial conditions.

(B) The frame of nucleation is determined by the rest frame of the observer

(C) Others

To settle the issue, we move on to the second quantization picture.

Schwinger pair production (1)

charged scalar field

$$S = \int d^2x \left[-\eta^{\mu\nu} \left(\partial_\mu \phi^* + ieA_\mu \phi^* \right) \left(\partial_\mu \phi - ieA_\mu \phi \right) - m^2 \phi^* \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right]$$

constant electric field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = -\varepsilon_{\mu\nu} E$

It is clear that a constant electric field in (1+1)-dimensions is Lorentz invariant.

Take a gauge $A_\mu = (0, -E\,t)$

$$\left[-\partial_t^2 + \left(\partial_x - ieA_x \right)^2 - m^2 \right] \phi = 0$$

$$\phi(t, x) = \int \frac{dk}{(2\pi)^{1/2}} \left[a_k \phi_k(t) + b_{-k}^\dagger \phi_k^*(t) \right] e^{ikx}$$

$$\left[a_k, a_{k'}^\dagger \right] = \delta(k - k') \qquad \left[b_k, b_{k'}^\dagger \right] = \delta(k - k')$$

$$\left[\frac{d^2}{dt^2} + m^2 + \left(k + eEt \right)^2 \right] \phi_k(t) = 0$$

vacuum $a_k |0\rangle = 0$ $b_k |0\rangle = 0$

Schwinger pair production (2)

in vacuum

$$\phi_k(z) = \frac{1}{(2eE)^{1/4}} e^{i\frac{\pi}{4}\nu^*} D_{\nu^*}\left[-(1-i)z\right]$$

$$D_\Lambda(Z) = \frac{e^{-\frac{Z^2}{4}}}{\Gamma(-\Lambda)} \int_0^\infty dw\, e^{-Zw-\frac{w^2}{2}} w^{-\Lambda-1} \qquad \mathrm{Re}(\Lambda) < 0$$

parabolic cylinder function

$$z = \sqrt{eE}\left(t + \frac{k}{eE}\right) \qquad \nu = -\frac{1}{2} - i\frac{\lambda}{2} \qquad \lambda = \frac{m^2}{eE}$$

$$\phi_k \approx \frac{1}{(2eE)^{1/4}} \left(\sqrt{2}|z|\right)^{\nu^*} e^{\frac{i}{2}z^2} \qquad z \ll -|\nu| \qquad i\partial_t \phi_k \sim -eEt\,\phi_k \quad at\ t \to -\infty$$

out vacuum

$$\phi_k^{out}(z) = \frac{1}{(2eE)^{1/4}} e^{-i\frac{\pi}{4}\nu} D_\nu\left[(1+i)z\right]$$

Bogoliubov trans.

$$\phi_k = \alpha_k \phi_k^{out} + \beta_k \phi_k^{out*} \qquad \alpha_k = \frac{\sqrt{2\pi}}{\Gamma(-\nu^*)} e^{i\frac{\pi}{4}(\nu^* - \nu)} \qquad \beta_k = e^{i\pi\nu^*}$$

$$\frac{dn}{dk} = \frac{1}{2\pi}|\beta_k|^2 \qquad |\beta_k|^2 = e^{-\pi\lambda} = e^{-\frac{\pi m^2}{eE}} \quad \longrightarrow \quad n = \frac{1}{2\pi}\int_{-eEt}^{-eEt_0} dk |\beta_k|^2$$

Thus, we obtain Schwinger formula

$$\frac{dn}{dt} = \left(\frac{eE}{2\pi}\right) e^{-\frac{\pi m^2}{eE}}$$

First, we check the option (A).
Namely, we examine if the initial state breaks Lorentz invariance.

Lorentz invariance of the in-vacuum (1)

Recall the positive frequency mode functions

$$\phi_k(z)e^{ikx} \qquad z=\sqrt{eE}\left(t+\frac{k}{eE}\right) \qquad A_0=0 \quad \text{gauge}$$

In a different frame, we have mode functions

$$\phi_{\bar{k}}(\bar{z})e^{i\bar{k}\bar{x}} \qquad \bar{z}=\sqrt{eE}\left(\bar{t}+\frac{\bar{k}}{eE}\right) \qquad \bar{A}_{\bar{0}}=0 \quad \text{gauge}$$

We need to calculate mixing

$$K\equiv\int_{t=const.}dx\left[\phi_k\partial_t\bar{\phi}_{\bar{k}}-\bar{\phi}_{\bar{k}}\partial_t\phi_k\right]e^{-ikx+i\bar{k}\bar{x}}$$

Do Lorentz trans. $\bar{t}=\gamma\left(t-vx\right)$ $\bar{x}=\gamma\left(x-vt\right)$ $\gamma=\frac{1}{\sqrt{1-v^2}}$

Do gauge trans. $A_0=\frac{\partial\bar{x}}{\partial t}\bar{A}_{\bar{1}}-\partial_0\Omega=0$ $A_1=\frac{\partial\bar{x}}{\partial x}\bar{A}_{\bar{1}}-\partial_1\Omega=-Et$ $\longrightarrow$ $\Omega=\frac{E\gamma^2v}{2}\left(t^2-2vtx+x^2\right)$

Thus, we obtain mode functions in the same coordinate system as the original one

$$\bar{\phi}_{\bar{k}}(t,x)=\phi_{\bar{k}}(\bar{z})e^{-ie\Omega(t,x)} \qquad \bar{z}=\sqrt{eE}\gamma\left(t-vx\right)+\frac{\bar{k}}{\sqrt{eE}}$$

Lorentz invariance of the in-vacuum (2)

$$\phi_k(t,x) = e^{i\frac{z^2}{2}} \int_0^\infty dw\, e^{(1-i)zw - \frac{w^2}{2}} w^\nu \qquad \bar{\phi}_{\bar{k}}(t,x) = e^{-ie\Omega} e^{i\frac{\bar{z}^2}{2}} \int_0^\infty dw'\, e^{(1-i)\bar{z}w' - \frac{w'^2}{2}} w'^\nu$$

$$K = \int_0^\infty dw\, w^\nu e^{-\frac{w^2}{2}} \int_0^\infty dw' w'^\nu e^{-\frac{w'^2}{2}} I\left(k, \bar{k}, w, w', t\right)$$

$$I = i\sqrt{eE} \int_{t=const.} dx \left[(1-v)\gamma\bar{z} - z - (1+i)(\gamma w' - w)\right] f_k \bar{f}_{\bar{k}}$$

$$= \frac{2\sqrt{\pi} i}{\gamma\sqrt{(1-v)v}} e^{i\frac{\bar{k}^2}{2veE} - \frac{v w'^2}{1-v}} \left[w - \gamma(1+v)w'\right]$$

$$f_k = e^{-ikx} e^{i\frac{z^2}{2}} e^{(1-i)zw}$$

$$\bar{f}_{\bar{k}} = e^{i\bar{k}\bar{x} - ie\Omega} e^{i\frac{\bar{z}^2}{2}} e^{(1-i)\bar{z}w'}$$

Now, it is easy to get

$$K \equiv \int_{t=const.} dx \left[\phi_k \partial_t \bar{\phi}_{\bar{k}} - \bar{\phi}_{\bar{k}} \partial_t \phi_k\right] e^{-ikx + i\bar{k}\bar{x}} = 0$$

It turns out that there is no mixing between the positive and negative mode functions.
Hence, both vacua are equivalent.

Now, the initial state is Lorentz invariant,
it seems we excluded the option (A).
However, there are subtle features …

Two-point function is not Hadamard

$$G^{(1)} = G^{+} + G^{-}$$

with

$$G^{+}(x^{\mu}, y^{\mu}) = \left\langle 0 \middle| \phi^{\dagger}(x^{\mu}) e^{-ie\int_x^y A_\mu dx^\mu} \phi(y^{\mu}) \middle| 0 \right\rangle \qquad G^{-}(x^{\mu}, y^{\mu}) = \left\langle 0 \middle| \phi(y^{\mu}) e^{-ie\int_x^y A_\mu dx^\mu} \phi^{\dagger}(x^{\mu}) \middle| 0 \right\rangle$$

$$\Delta s^2 = -(\Delta t)^2 + (\Delta x)^2$$

Whittaker function

$$G^{+}(\Delta x^{\mu}) = \frac{|\alpha|^2}{4\pi} \Gamma(-\nu^{*}) \frac{W_{i\lambda/2,0}\left(ieE\Delta s^2/2\right)}{\sqrt{ieE\Delta s^2/2}} \qquad \text{for} \quad \Delta t - \Delta x > 0$$

$$G^{+}(\Delta x^{\mu}) = \frac{|\alpha|^2}{4\pi} \Gamma(-\nu) \frac{W_{i\lambda/2,0}\left(-ieE\Delta s^2/2\right)}{\sqrt{-ieE\Delta s^2/2}} \qquad \text{for} \quad \Delta t - \Delta x < 0$$

Again, we have Lorentz invariance, however, it is not Hadamard.

$$G^{+}(\Delta x^{\mu}) = -\frac{|\alpha|^2}{4\pi} \left[\log\left(ieE\Delta s^2/2\right) + \psi\left(\frac{1}{2} - i\frac{\lambda}{2}\right) + 2\gamma \right]$$

There must be something strange …

Lorentz breaking current

gauge invariant current

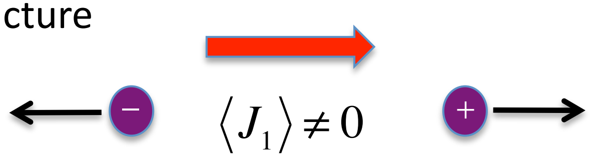
$$\langle J_\mu \rangle = \frac{ie}{2} \lim_{x \to y} \left(\frac{\partial}{\partial y^\mu} - \frac{\partial}{\partial x^\mu} \right) G^{(1)}\left(y^\nu - x^\nu \right)$$

One can show $\langle J_0 \rangle = 0$

Let us evaluate $\langle J_1 \rangle = 2e \int \frac{dk}{2\pi} (k + eEt) |\phi_k|^2$

$$\left\langle \frac{\partial J_1}{\partial t} \right\rangle = 2e^2 E \int \frac{dk}{2\pi} \frac{d}{dk} \left[(k + eEt) |\phi_k|^2 \right] = \frac{e^2 E}{2\pi} \left[|\alpha|^2 + |\beta|^2 - 1 \right] = \frac{e^2 E}{\pi} e^{-\frac{\pi m^2}{eE}}$$

This is consistent with an intuitive picture



$$\langle J_1 \rangle \neq 0$$

There may be no Lorentz invariance.

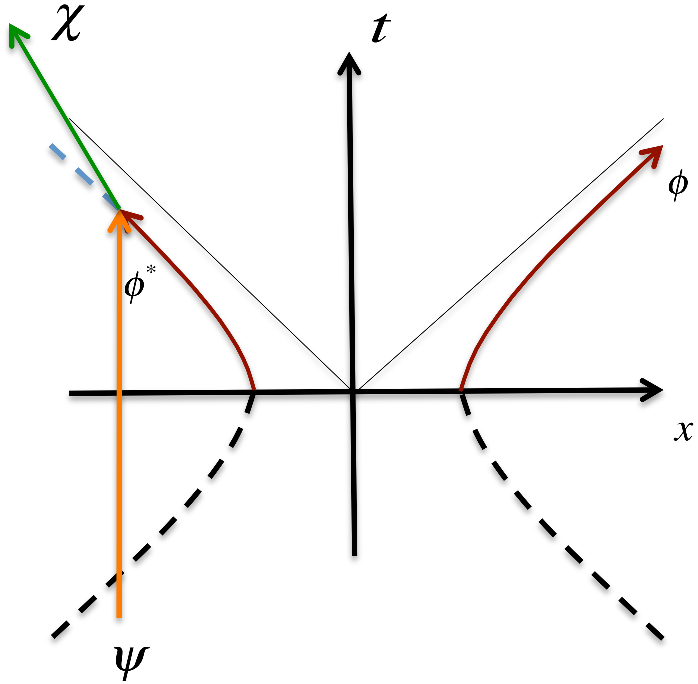
Although the initial state is Lorentz invariant,
it seems there exists quantum Lorentz violation…
So, somehow there might be a preferred nucleation frame!

Observe nucleation frame via a detector

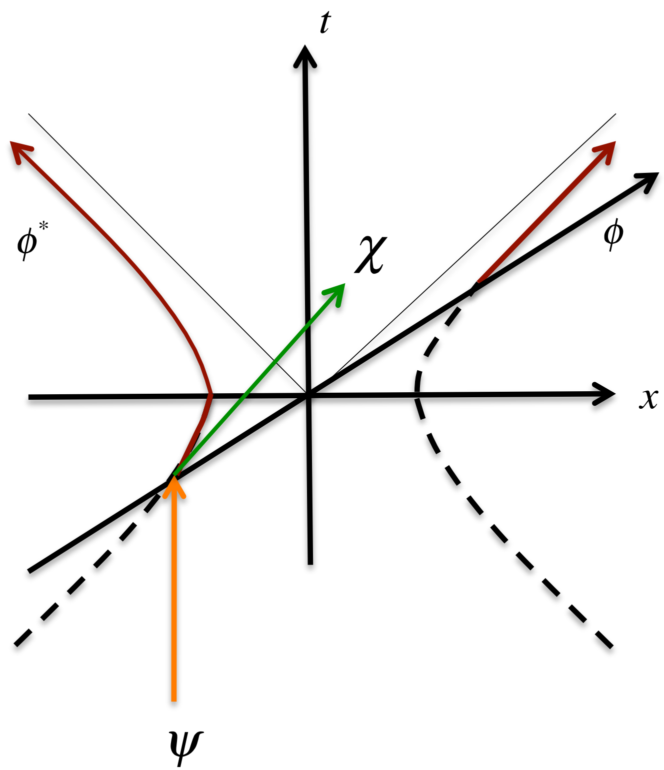
In order to get the information about nucleation frame, let us introduce a detector.

$$S_{\text{int}} = -g\int d^2x\left[\phi^\dagger\psi\chi + \phi\psi^\dagger\chi\right] = -\int dt\ H_{\text{int}}$$

There are two qualitatively different scatterings.



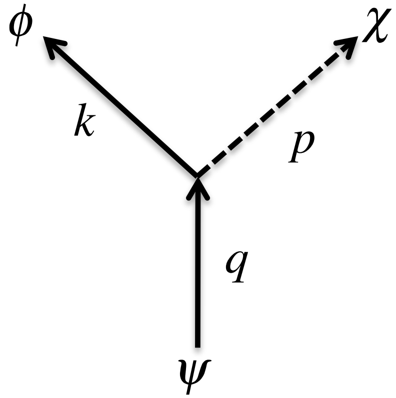
or

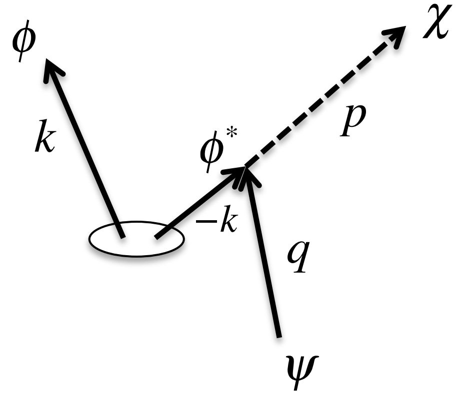


Hence, the momentum distribution of chi particles carry the information about the frame.

Kinematical conditions

Before going into details, let me comment on kinematical conditions.





In order to suppress the left process , we assume $m_\psi < m_\chi + m_\phi$

$\psi + \phi^* \to \chi$ $\qquad$ $m_\chi > m_\psi + m_\phi$

$q_{phys} + k_{phys} = p_{phys}$ $\qquad$ $q_{phys} = q - eEt$ $\quad$ $k_{phys} = k + eEt$ $\quad$ $p_{phys} = p$

$$\sqrt{q_{phys}^2 + m_\psi^2} + \sqrt{k_{phys}^2 + m_\phi^2} = \sqrt{p_{phys}^2 + m_\chi^2}$$

$$2m_\psi^2 k_{phys} = M^2 q_{phys} \pm \sqrt{q_{phys}^2 + m_\psi^2}\sqrt{M^4 - 4m_\phi^2 m_\psi^2} \qquad M^2 = m_\chi^2 - m_\psi^2 - m_\phi^2$$

Momentum distribution of chi particles

Let us derive momentum distribution of chi particles.

$$\phi = \int \frac{dk}{\sqrt{2\pi}} \left(a_k \phi_k + b_{-k}^\dagger \phi_k^* \right) e^{ikx} \qquad \left[a_k ,\, a_{k'}^\dagger \right] = \delta(k-k') \qquad \left[b_k ,\, b_{k'}^\dagger \right] = \delta(k-k')$$

$$\psi = \int \frac{dq}{\sqrt{2\pi}} \left(d_q \psi_q + f_{-q}^\dagger \psi_k^* \right) e^{iqx} \qquad \left[d_q ,\, d_{q'}^\dagger \right] = \delta(q-q') \qquad \left[f_q ,\, f_{q'}^\dagger \right] = \delta(q-q')$$

$$\chi = \int \frac{dk}{\sqrt{2\pi}} \left(c_p \chi_p + c_{-p}^\dagger \chi_p^* \right) e^{ipx} \qquad \left[c_p ,\, c_{p'}^\dagger \right] = \delta(p-p') \qquad \chi_p = \frac{1}{\sqrt{2\omega_p}} e^{ipx - i\omega_p t}$$

$$\frac{dN_\chi}{dp} = \frac{\langle \mathrm{f} | c_p^\dagger c_p | \mathrm{f} \rangle}{\langle \mathrm{q} | \mathrm{q} \rangle} \qquad |\mathrm{f}\rangle = \exp\left[-i \int_{-\infty}^{\infty} dt' H_{\mathrm{int}}(t') \right] |\mathrm{i}\rangle \simeq |\mathrm{i}\rangle - i \int_{-\infty}^{\infty} dt' H_{\mathrm{int}}(t') |\mathrm{i}\rangle$$

$$|\mathrm{q}\rangle = d_q^\dagger |0\rangle$$

After a short calculation, we obtain

$$\frac{dN_\chi}{dp} = \frac{1}{2\pi} \left| \int_{-\infty}^{\infty} dt \;\, \mathrm{g} \phi_{q-p}^* \psi_q \chi_p^* \right|^2 \equiv \frac{1}{2\pi} \left| \mathcal{A}_\chi(p;q) \right|^2$$

We have already excluded the option (A).
Now, we we want to examine the possibility
(B) The frame of nucleation is determined
by the rest frame of the observer.

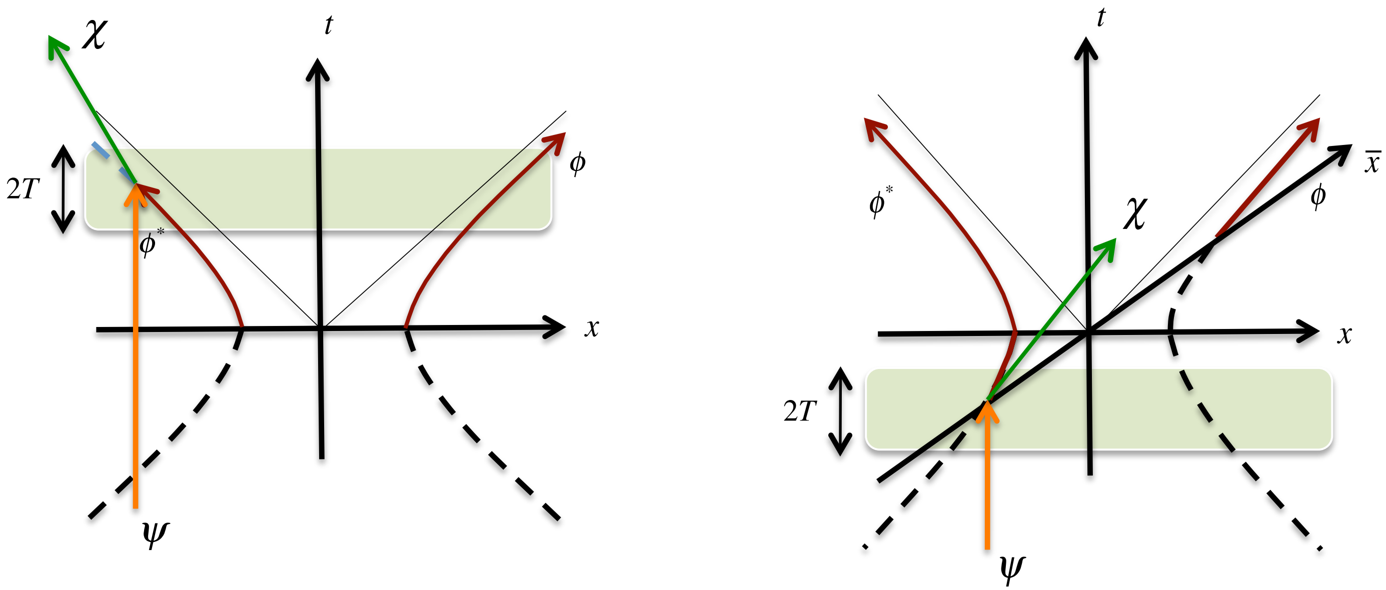
Introducing the observer

Let us turn on the detector only for a finite time interval 2T $g(t) = ge^{-t^2/T^2}$

This can be interpreted as an observer.

We also assume $m_\phi \ll eET \ll m_\psi$ so that psi detector particle remains unchanged.

Consider static detector $q = 0$, thus we have $p \approx k_{phys}$



If both cases are observed, we have to seek other possibility (C).

The answer

Now, the amplitude reads

$$\mathcal{A}_\chi(p;q=0) = \int_{-\infty}^{\infty} dt\ g(t)\ \phi^*_{-p}(t)\ \psi_0(t)\ \chi^*_p(t)$$

$$\psi_0(t) \approx \frac{1}{\sqrt{2m_\psi}} e^{-im_\psi t}$$

Then,

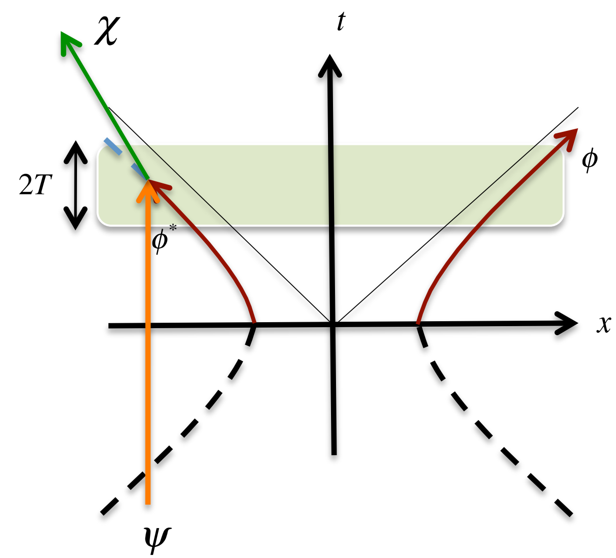
$$\frac{dN_\chi}{dp} \propto \frac{1}{2\pi} \left| \mathcal{A}_\chi(p;q=0) \right|^2 \propto \exp\left[-\frac{2(p+\omega_p - m_\psi)^2}{(eET)^2} \right]$$

$$\omega_p = \sqrt{p^2 + m_\chi^2}$$

The distribution has a peak at $p = -\frac{1}{2m_\psi}\left(m_\chi^2 - m_\psi^2\right)$ with width $\Delta p \approx eET$

Namely, we do not see the lower branch.

This supports the option (B).



Conclusion

- In each observers frame, particle and antiparticle nucleate at rest and are accelerated in opposite direction by the electric field.

- The close analogy between pair production in 2-d and bubble nucleation in 4-d lead us to expect that a similar picture should apply to observations of bubble nucleation.