Black holes in Lorentz-violating gravity

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Why LV gravity?

Lorentz-violating effects are severely constrained in the matter sector. However,

- Observational constraints are far weaker in the more weakly coupled gravitational sector
- A low energy effective theory of Lorentz-violating gravity is needed for such tests (e.g. Einstein-aether theory)

The might be an additional pay-off:

- Recently it has been claimed that some models of Lorentz-violating gravity are power-counting renormalizable (Hořava-Lifshitz gravity)

Why black holes?

A black hole is essentially a singularity cloaked by an horizon, which acts as a causal boundary.

- Simpler to find than other solutions (e.g. stars), no matter
- Potentially a probe for observable deviations from GR
- Intrinsically interesting in theories with Lorentz violations

Why? The very existence of black holes appears to depend on the causal structure...

Einstein-aether theory

The action of the theory is

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int d^4x \sqrt{-g}(-R - M^{\alpha\beta\mu\nu} \nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).

Einstein-aether theory

- Extensively tested and still viable
- It propagates a spin-2, a spin-1 and spin-0 mode.
- Linear dispersion relations.
- These modes travel at different speeds.
- Requires a UV-completion (which would likely modify the dispersion relations and lead to arbitrarily higher speeds for all modes).

Hypersurface orthogonality

Now assume $$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T}(g^{TT})^{-1/2} = N\delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{æ}^{ho} = \frac{1}{16\pi G_H}\int dT d^3x N\sqrt{h}\left(K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta a^i a_i\right)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_{æ}} = \xi = \frac{1}{1-c_{13}} \qquad \lambda = \frac{1+c_2}{1-c_{13}} \qquad \eta = \frac{c_{14}}{1-c_{13}}$$

T. Jacobson, Phys. Rev. D 81, 101502 (2010).

Horava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H}\int dT d^3x\, N\sqrt{h}\left(L_2 + \frac{1}{M_\star^2}L_4 + \frac{1}{M_\star^4}L_6\right)$$

where

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi\,{}^{(3)}\!R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

P. Hořava, Phys. Rev. D 79, 084008 (2009)
D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Let. 104, 181302 (2010)

Horava-Lifshitz gravity

- Higher order terms contain higher order spatial derivatives: higher order dispersion relations!
- They modify the propagator and render the theory power-counting renormalizable
- All terms consistent with the symmetries will be generated by radiative corrections
- This version of the theory is viable so far
- "Low energy limit" is h.o. Einstein-aether theory!

LV and black hole structure

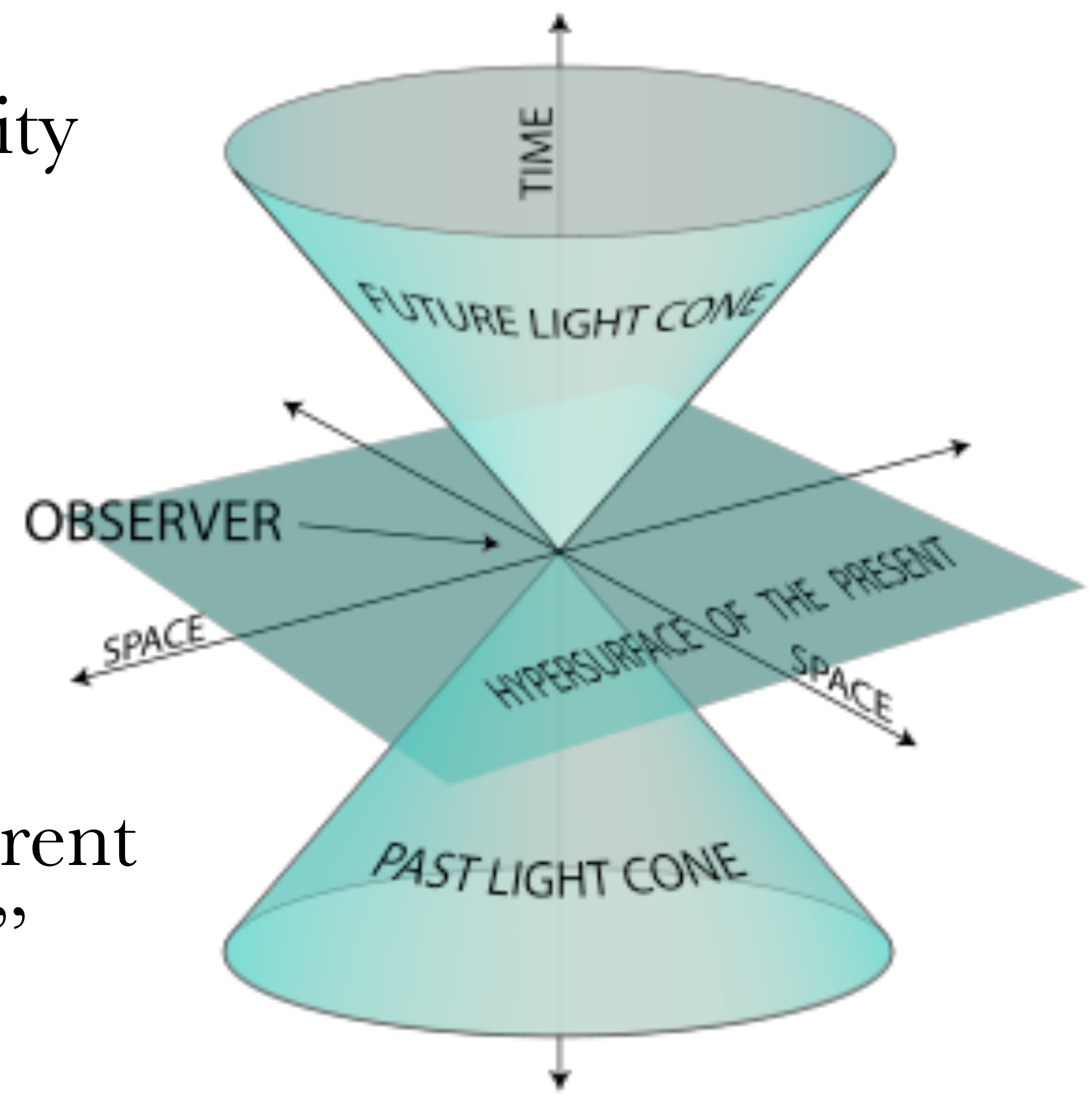
Causal structure in special relativity

- LV with linear dispersion relations

$$\omega \propto k$$

- Different modes have different speeds and different "light" cones

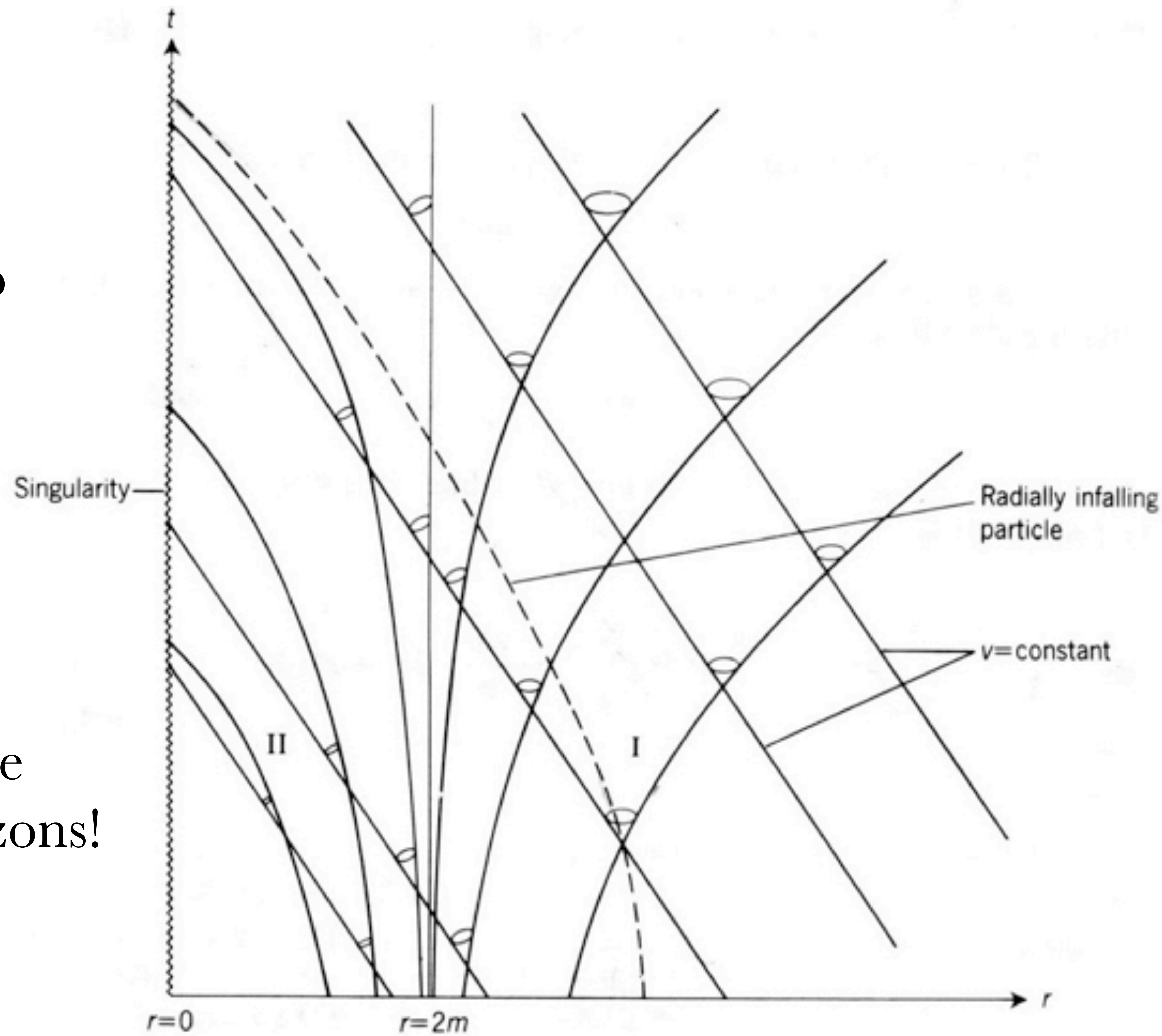
- But there are still "light" cones!



LV and black hole structure

What happens to black holes?

- They will have multiple horizons!



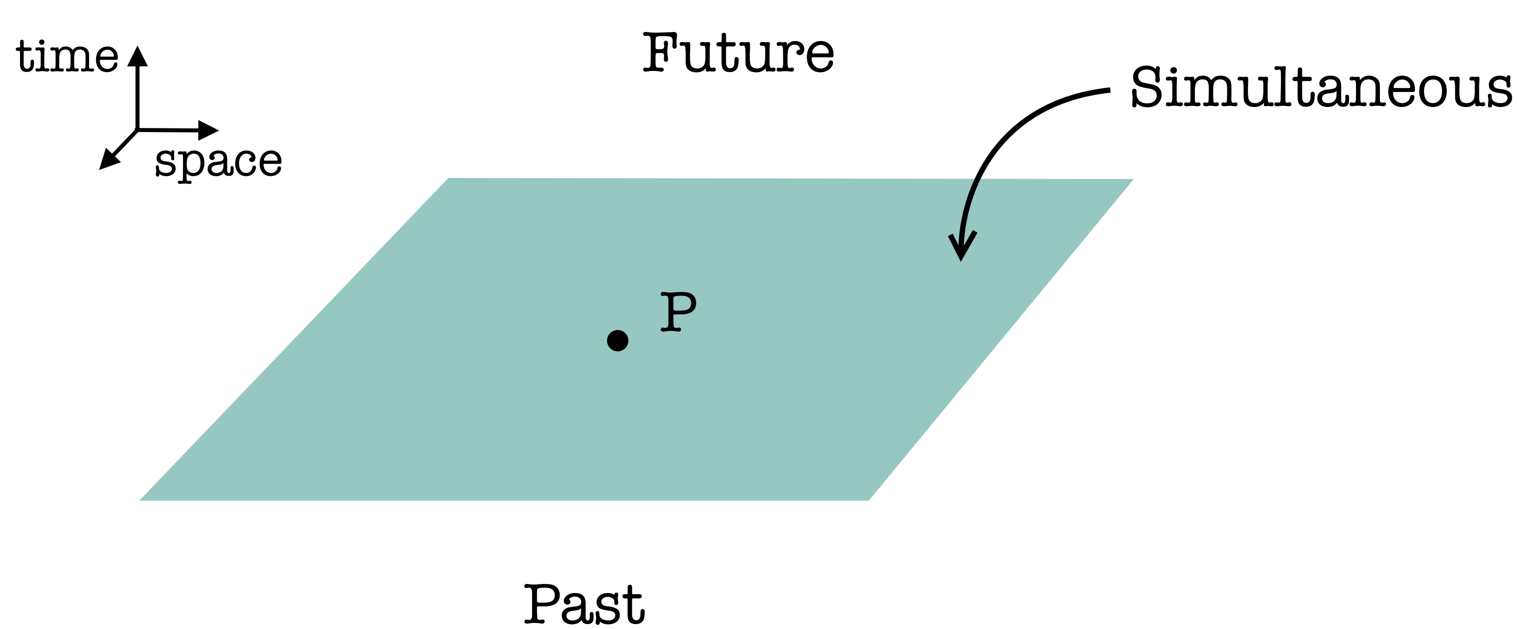
LV and black hole structure

- LV with non-linear dispersion relations

$$\omega^2 \propto k^2 + ak^4 + \ldots$$

- No light cones!

Causal structure without relativity



No black holes at all??

Our goal

We are interested in vacuum black hole solutions which are

- spherically symmetric (so, also h.o. aether)
- static
- asymptotically flat
- and can (in principle) form from gravitational collapse

Finding such solutions analytically seems unfeasible, so we find them numerically

- There is a one-parameter family of such solutions
- I suppress all the (complicated and challenging) details about how to find these solutions

E. Barausse, T. Jacobson and T. P. S., Phys. Rev. D 83, 124043 (2011)

Equations and constraints

The field equations are

$$E^{\mu\nu} \equiv G^{\mu\nu} - T^{\mu\nu}_{æ} = 0 \qquad\qquad Æ^{\mu} = 0$$

We use Eddington-Finkelstein(-like) coordinates

$$ds^2 = F(r)dv^2 - 2B(r)dvdr - r^2 d\Omega^2$$

$$u^\alpha \partial_\alpha = A(r)\partial_v - \frac{1 - F(r)A^2(r)}{2B(r)A(r)}\,\partial_r$$

and the non-redundant or non-trivial components are

$$E^{vv} = E^{vr} = E^{rr} = E^{\theta\theta} = Æ^{v} = 0$$

Equations and constraints

However, the combinations

$$C^\mu \equiv E^{r\mu} - u^r \text{Æ}^\mu = 0$$

are actually constraint equations, so we will use instead the system

$$E^{vv} = E^{\theta\theta} = \text{Æ}^v = 0, \quad C^v = C^r = 0$$

The three dynamical equations can be recast in the form

$$\begin{aligned} F'' &= F''(A, A', B, F, F') \\ A'' &= A''(A, A', F, F') \\ B' &= B'(A, A', B, F, F') \end{aligned}$$

so we need 5 pieces of initial data.

Types of solutions

We also have 2 constraints though: a 3-parameter family of solutions. Additionally,

- asymptotic flatness
- regularity of the spin-0 horizon

will lead to a 1-parameter family.

- Fixing the horizon radius one has a unique solution

Without regularity of the spin-0 horizon

- A two parameter “hairy” solution!

Parameter space

We impose the following viability constraints

- Classical and quantum-mechanical stability
- Avoidance of vacuum Cherenkov radiation by matter
- Exact agreement with Solar system experiments (vanishing preferred frame parameters)

The last constraint is more restrictive than actually required. However,

- It reduces the dimensions of the parameters space to 2
- It provided an important simplification with little given away

Characteristic quantities

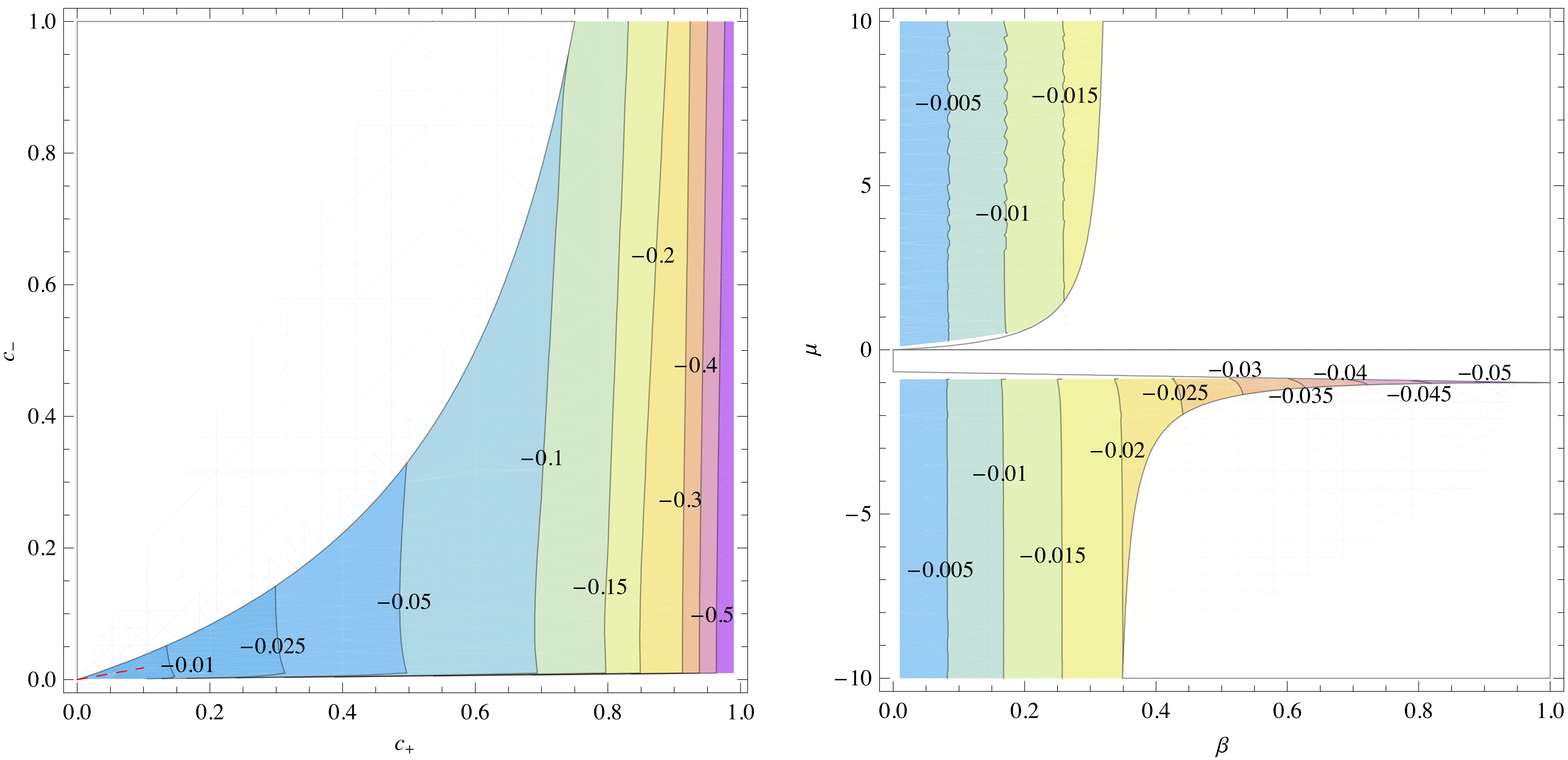
$\omega_{\mathrm{ISCO}}\, r_g$: orbital frequency at the ISCO times gravitational radius. Can be measured using X-ray spectra from accretion or gravitational waves from EMRIs

$z_{\max}$: $(= \nu_{\mathrm{emitted}}/\nu_{\mathrm{measured}} - 1)$ the maximum redshift for a photon emitted at the ISCO. Can be measured using iron-Kα line

b_{ph}/r_g : impact parameter for circular photon orbit/grav. radius. Can be measured by gravitational lensing or in the future via black hole quasinormal modes

r_g/r_H : grav. radius/horizon radius. Not measurable but gives info about near horizon region

Results: exterior



$$\omega_{\mathrm{ISCO}}\, r_g$$

Interior solution

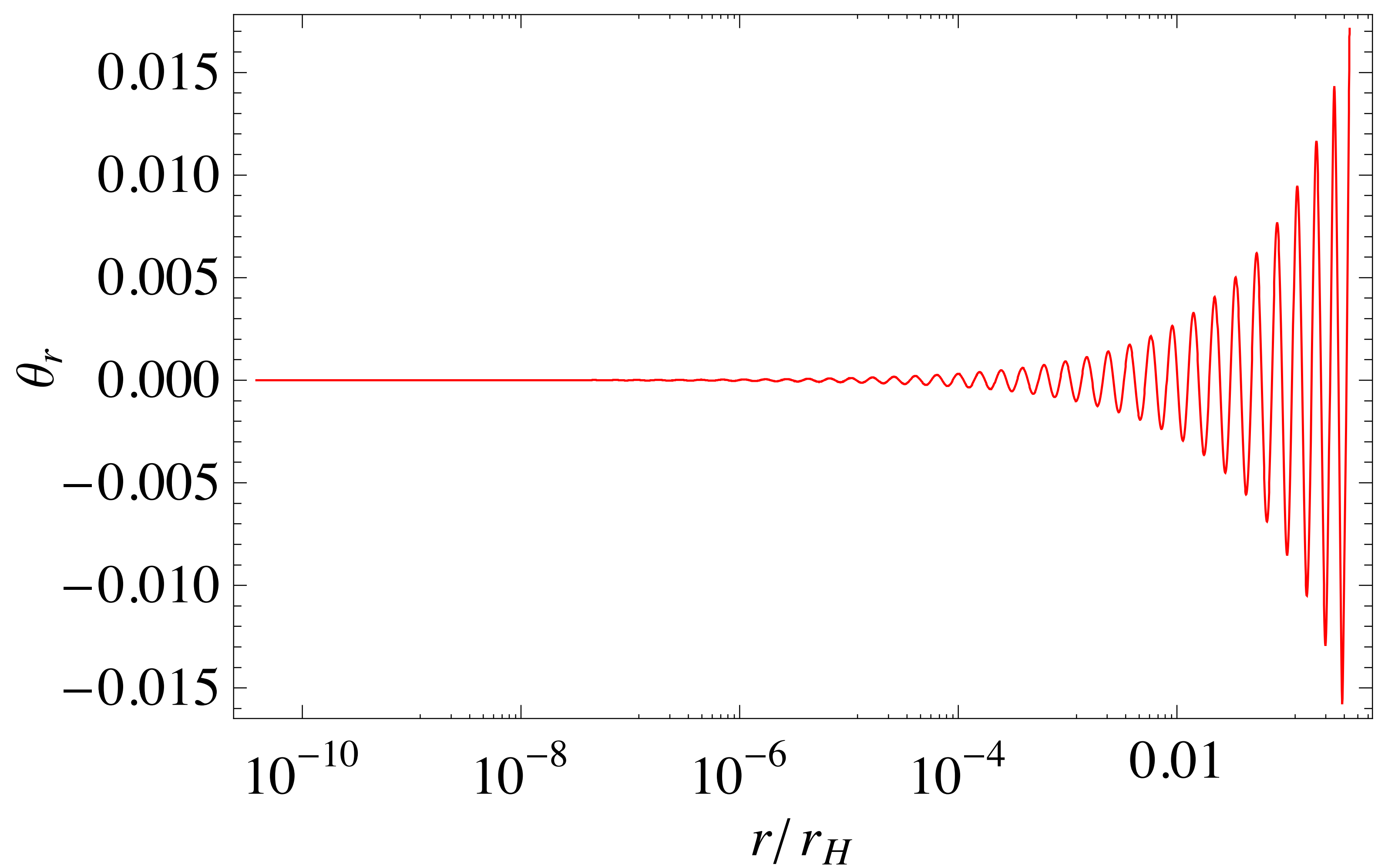
- Curvature singularity at the centre

Lorentz factor of the aether as measured by the future directed observer orthogonal to $r = $ const. hypersurfaces

$$\gamma_r \equiv u^{\alpha}_{\text{obs}} u_{\alpha}$$

and the boost angle

$$\theta_r = \text{arccosh}\gamma_r$$



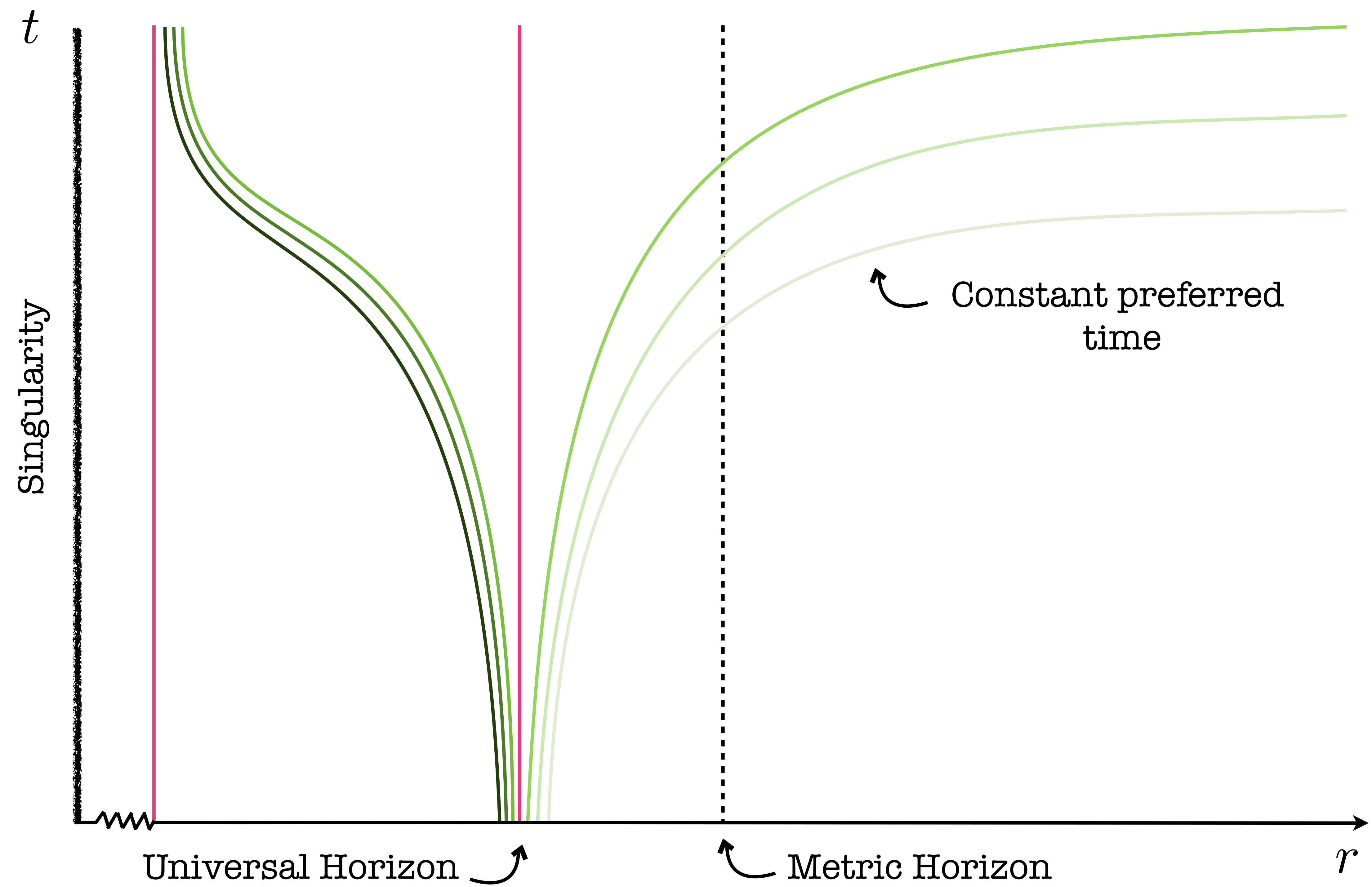
Interior solution

- Signals cannot travel backwards in time
- Future and past direction are locally defined by the aether
- The aether is orthogonal to constant time hypersurfaces in the preferred foliation
- When the boost angle vanishes the aether is orthogonal to constant r hypersurfaces as well!
- Ultimate causal boundary for all signals!

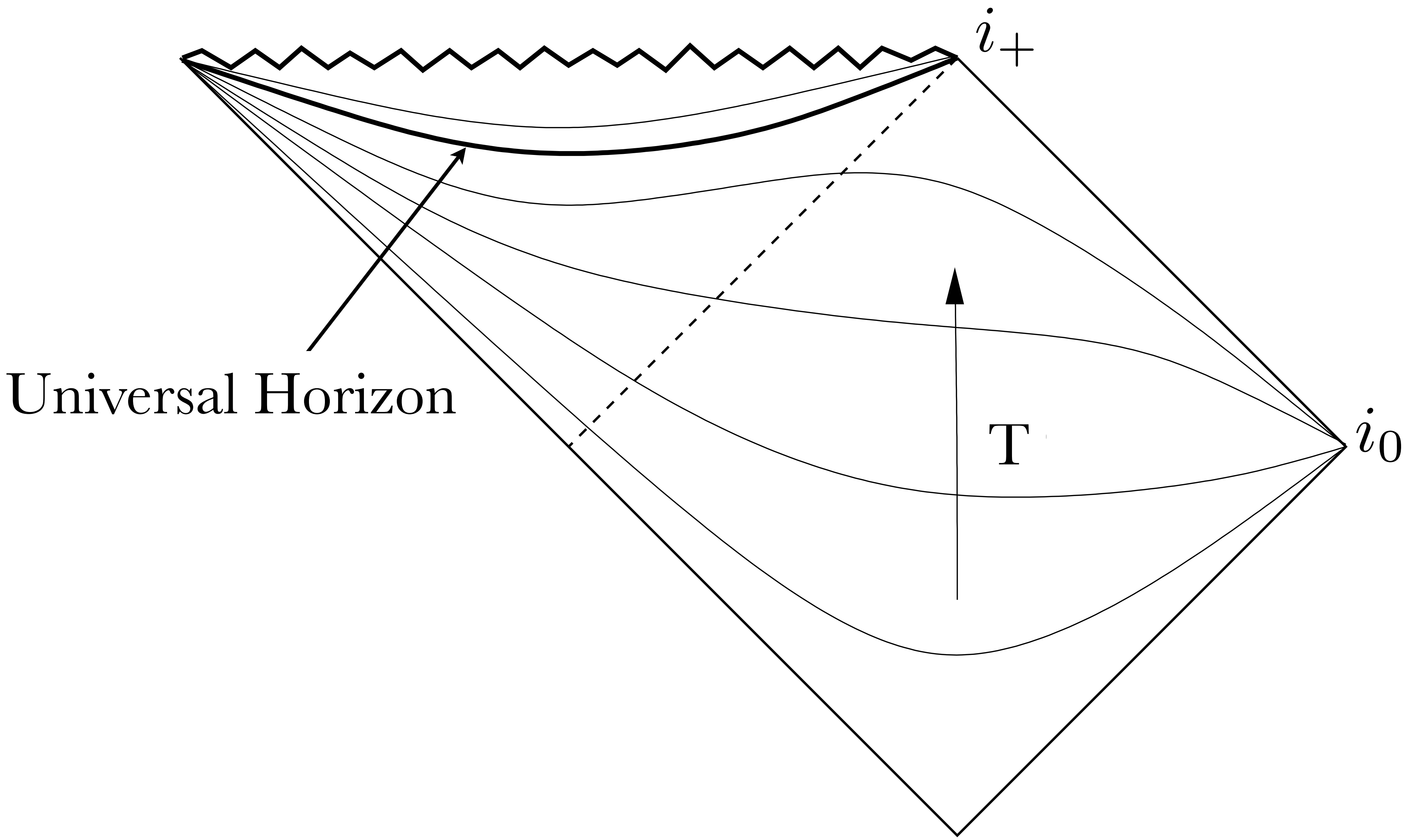
The same result found in Horava gravity, in the decoupling limit. However, this horizon seems to be unstable.

D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

Spacetime diagram



Penrose diagram



Taken from D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

Slowly rotating BHs

- What about rotating black holes?
- Difficult to find them, easier to focus on slow rotation

Most general slowly rotating, stationary, axisymmetric metric

$$\begin{aligned} ds^2 &= f(r)dt^2 - \frac{B(r)^2}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \\ &\quad + \epsilon r^2 \sin^2\theta\, \Omega(r,\theta) dt d\varphi + \mathcal{O}(\epsilon^2) \end{aligned}$$

- $f(r)$ and $B(r)$ are the "seed" solutions, so they are known
- $\Omega(r,\theta)$ is to be determined at the next order in ϵ

Slowly rotating BHs in GR

For the Schwarzschild solution as a "seed", for which

$$f(r) = 1 - \frac{2M}{r} \qquad\qquad B(r) = 1$$

at $\mathcal{O}(\epsilon)$ one has only one equation

$$-(r-2M)\left[4\partial_r\Omega + r\,\partial_r^2\Omega\right] = \partial_\theta^2\Omega + 3\cot\theta\partial_\theta\Omega$$

with the known solutions

- $\Omega(r,\theta) = \Omega_H(2M/r)^3$, the slowly rotation Kerr BH
- $\Omega(r,\theta) = \Omega_0 =$ constant, which is the "seed" solution after the transformation $\varphi \to \varphi + \Omega_0\, t$

Slow rotation and the aether

Symmetries require that

$$\partial_t u_\mu = \partial_\phi u_\mu = 0$$

<u>If</u> the aether is hypersurface orthogonal then

$$\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma = 0$$

and then one obtains

$$u_\phi = 0$$

So, one has for the aether at

$$\boldsymbol{u} = \frac{1 + fA^2}{2A} dt + \frac{B}{2A}\left(\frac{1}{f} - A^2\right) dr + \mathcal{O}(\epsilon)^2$$

i.e. nothing to determine at $\mathcal{O}(\epsilon)$!

Solutions

If one plugs the ansaetze in the equations of Einstein-aether theory one gets

$$\Omega(r,\theta) = \text{constant}$$

i.e. no h.o., slowly-rotating solution. But one can have $u_\phi \neq 0$

If one plugs the ansaetze in the equations of HL gravity one gets

$$\Omega(r,\theta) = \Omega(r) = -12J \int_{r_\mathrm{H}}^{r} \frac{B(\rho)}{\rho^4} d\rho + \Omega_0$$

E. Barausse and T.P.S., Phys. Rev. Lett. 109, 181101 (2012);
E. Barausse and T.P.S., arXiv: 1212,1334 [gr-qc]

Solutions

- The two theories share the spherical solutions, but not the slowly rotating one!
- There are no slowly rotating solutions with a preferred foliation in Einstein-aether theory. Can there be a universal horizon then?
- In HL gravity the foliation remain the same in slow rotation

Conclusions and Perspectives

- Black holes are of particular interest in Lorentz-violating theories.
- There seems to be a new kind of black hole in such theories! ("universal horizon")
- The exterior can be similar to GR black holes. What about rotating black holes?
- Is this horizon stable?
- Will this horizon (and black holes) exist if one has less symmetry?