Victoria University of Wellington

*Te Whare Wānanga o te Ūpoko o te Ika a Maui*

# "Prior structure" in non-Einstein theories of gravity

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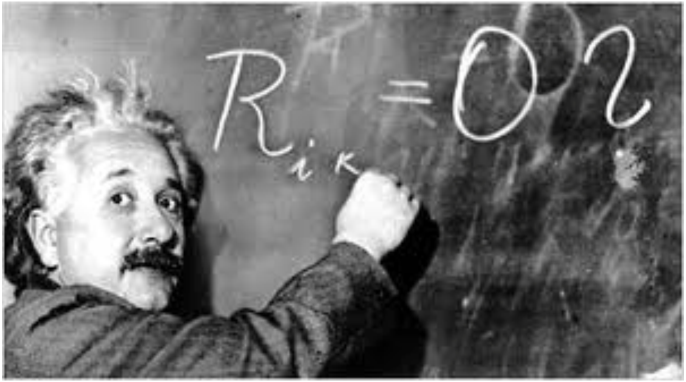
# Abstract


- "Prior structure" is an anathema in standard general relativity, but is often present to some extent in various modified theories of gravity.
- I shall discuss some of the issues and possibilities.

# Thanks to our hosts:

KAVLI
IPMU

# Prior structure:



- One of the foundations of standard GR is the complete absence of "prior structure".
- There are many reasons for maybe relaxing this condition.

Prior structure is present in:

- Preferred frame models...
  - von Ignatowski; groupoids...
  - thresholds...
- "Functionally constrained" gravity...
  - Nordström gravity... [Einstein–Fokker 1914]...
  - Unimodular gravity...
  - Hořava-inspired gravity...
  - Einstein-aether-inspired gravity...
- Massive gravity...
- Etcetera...

Prior structure can lead to useful physics...

Preferred frame (aether) models:

- Relativity principle violated...
- The inertial transformations form a groupoid, not a group...
- Spacetime transformations still linear...
- To go from inertial frame @ velocity $\vec{v}_1$ to inertial frame @ velocity $\vec{v}_2$:

$$X_2 = M(\vec{v}_2, \vec{v}_1)\, X_1$$

- Then

$$M(\vec{v}_3, \vec{v}_1) = M(\vec{v}_3, \vec{v}_2)\, M(\vec{v}_2, \vec{v}_1)$$

- BUT

$$M(\vec{v}_4, \vec{v}_3)\, M(\vec{v}_2, \vec{v}_1)$$

is physically meaningless.

- Explictly:

$$M(\vec{v}_2, \vec{v}_1) = M(\vec{v}_2, \vec{0})\; M(\vec{0}, \vec{v}_1) = M(\vec{v}_2, \vec{0})\; M(\vec{v}_1, \vec{0})^{-1}.$$

- It is this groupoid (~~group~~) structure that allows one to evade the von Ignatowski theorems...
- If the relativity principle is respected then

$$M(\vec{v})$$

  depends only on the relative velocities...
- If the relativity principle is violated, then

$$M(\vec{v}_2, \vec{v}_1)$$

  depends separately on the 2 velocities of the 2 inertial frames being compared.

- OPERA-inspired discussion, but the message to take from this is much more general...
- Inertial frames without the relativity principle.
  Valentina Baccetti, Kyle Tate, Matt Visser.
  Published in JHEP **1205** (2012) 119.
  e-Print: arXiv:1112.1466 [gr-qc]
- The groupoid *versus* group distinction is important...

# Preferred frame models...

In the presence of preferred frame physics, particle production thresholds are much more subtle than in standard SR.

- Useful to consider decay thresholds and scattering thresholds separately...
- Can now have both upper and lower thresholds...
  (Minimum energies and/or maximum energies for certain processes.)
- Thresholds could be due to "enabling" or to "saturation"...
  (Saturation: maximum energy for infinite momentum...)
- Lorentz violating kinematics: Threshold theorems.
  Valentina Baccetti, Kyle Tate, Matt Visser
  Published in JHEP **1203** (2012) 087
  e-Print: arXiv:1111.6340 [hep-ph]

Let us work in $n = d + 1$ dimensions.

- Suppose the spacetime metric $g_{ab}(x)$ is functionally constrained to depend on position only implicitly and algebraically through a finite number $N$ of fields $\Phi^A(x)$.
- That is

$$g_{ab} \to g_{ab}(\Phi_A(x)).$$

- (More precisely, one might wish to assert some preferred set of coordinate charts in which such functional dependence can be made explicit.)

As we shall soon see, this is only useful if

$$N < \frac{n(n+1)}{2} = \frac{(d+1)(d+2)}{2}.$$

That is, useful for $N < 10$ in 3+1 dimensions.

# “Functionally constrained” gravity...

Take the usual action:

$$S = \int \sqrt{-g}\,\{R + 8\pi\mathcal{L}_{\mathrm{matter}}\}\,\mathrm{d}^n x.$$

But emphasize

$$S(\Phi,\Psi) = \int \sqrt{-g(\Phi)}\,\{R(g(\Phi) + 8\pi\mathcal{L}_{\mathrm{matter}}(\Psi, g(\Phi))\}\,\mathrm{d}^n x.$$

Equations of motion:

$$\frac{\delta S}{\delta \Psi} = 0; \qquad \frac{\delta S}{\delta \Phi} = 0.$$

Matter EOM are standard.

But gravitational EOM must be evaluated via the chain rule...

Gravitational equations of motion:

$$\frac{\delta S}{\delta \Phi_A} = \frac{\delta S}{\delta g_{ab}} \; \frac{\delta g_{ab}}{\delta \Phi_A} = 0.$$

But if the functional dependence is taken to be algebraic

$$\frac{\delta g_{ab}}{\delta \Phi_A} \to \frac{\partial g_{ab}}{\partial \Phi_A}.$$

We also know that for the standard action

$$\frac{\delta S}{\delta g_{ab}} \to G^{ab} - 8\pi T^{ab}.$$

Then the gravitational EOM are simply

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \; \frac{\partial g_{ab}}{\partial \Phi_A} = 0.$$

There are $N$ equations for $N$ unknowns.

# "Functionally constrained" gravity...

- This is not GR...
- A priori constraint $\neq$ a posteriori constraint...
- For a posteriori [symmetry] constraints the EOM are simply

$$G^{ab}\Big|_{g\to g(\Phi)} = 8\pi \;\; T^{ab}\Big|_{g\to g(\Phi)}.$$

- Contrast a priori constraints, where the EOM are

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \frac{\partial g_{ab}}{\partial \Phi_A} = 0.$$

- Considerably weaker than the standard Einstein equations.
- Solutions of the standard Einstein equations that a posteriori satisfy the prior geometry constraint are immediately included as solutions of this weaker set of equations.

# Example: Nordström gravity...

Simply take:

$$g_{ab} = \exp(2\Phi)\ \eta_{ab}; \qquad N = 1; \qquad \frac{\partial g_{ab}}{\partial \Phi} = 2 g_{ab}.$$

The gravitational EOM are:

$$G - 8\pi T = 0,$$

or

$$R + 8\pi T = 0.$$

One gravitational EOM for the one degree of freedom of the metric.

(Nordström gravity is an interesting historical precursor to standard GR; it [famously] does not lead to the bending of starlight or gravitational lensing, but passes many other sanity checks...)

Nordström gravity [Einstein–Fokker 1914] is still a useful toy model...

# Example: Non-dynamical modulus gravity...

As is completely standard, simply take:

$$g_{ab} = \omega^2\, \Phi_{ab}; \qquad \det(\Phi_{ab}) = 1; \qquad (\omega \text{ non-dynamical}).$$

Then (small calculation required)

$$\frac{\partial g_{ab}}{\partial \Phi_{cd}} = \omega^2 \left[ \delta_{(a}{}^{(c}\, \delta_{b)}{}^{d)} - \frac{1}{n}\, g_{ab}\, g^{cd} \right].$$

This guarantees that

$$\frac{\partial \det(g)}{\partial \Phi_{cd}} = \det(g)\; g^{ab}\; \frac{\partial g_{ab}}{\partial \Phi_{cd}} = 0.$$

Now consider the gravitational EOM...

For the gravitational EOM we have

$$\left\{ G^{ab} - 8\pi\, T^{ab} \right\} \left[ \delta_{(a}{}^{(c}\, \delta_{b)}{}^{d)} - \frac{1}{n}\, g_{ab}\, g^{cd} \right] = 0.$$

That is, the traceless part of Einstein equals the traceless part of the stress-energy:

$$G^{ab} - \frac{1}{n} G\, g^{ab} = 8\pi \left\{ T^{ab} - \frac{1}{n} T\, g^{ab} \right\}.$$

Since the matter EOM are unaffected, we still have

$$T^{ab}{}_{;b} = 0,$$

but this is now being derived from the matter EOM.

# Example: Non-dynamical modulus gravity...

Consequently, in complete agreement with Anderson and Finklestein, and Unruh, and Henneaux and Teitelboim (Bunster), we have:

$$G^{ab} = 8\pi T^{ab} + \Lambda g^{ab},$$

now for a non dynamical cosmological constant.
See for instance:

- Cosmological Constant and Fundamental .
  James L. Anderson and David Finkelstein
  American Journal of Physics **39** (1971) 901–904.
- A unimodular theory of canonical quantum gravity.
  Bill Unruh
  Physical Review **D40** (1989) 1048–1052.
- The cosmological constant and general covariance.
  Marc Henneaux and Claudio Teitelboim
  Physics Letters **B222** (1989) 195–199.

With these sanity checks completed, let's try something more subtle...

Consider the "unit-lapse" metric:

$$g_{ab}(v,h) = \left[\begin{array}{c|c} -1 + h^{ij} v_i v_j & -v_j \\ \hline -v_i & h_{ij} \end{array}\right].$$

Many interesting spacetimes can be put in this form...

- Schwarzschild (Painleve–Gullstrand).
- Reissner–Nordström (Painleve–Gullstrand).
- Kerr (Doran).
- Friedmann–Lemaitre–Robertson–Walker.

This is a specialization of the "projectable" version of Hořava gravity.
(Though for now I throw away all the higher-spatial-derivative terms.)

So we take:

$$g_{ab}(v,h) = \left[\begin{array}{c|c} -1 + h^{ij} v_i v_j & -v_j \\ \hline -v_i & h_{ij} \end{array}\right].$$

Then (small calculation required):

$$N = \frac{n(n+1)}{2} - 1 = \frac{(d+1)(d+2)}{2} - 1 = \frac{d(d+3)}{2};$$

$$\frac{\partial g_{ab}}{\partial v_k} = \left[\begin{array}{c|c} 2v^k & -\delta_j{}^k \\ \hline -\delta_i{}^k & 0 \end{array}\right];$$

$$\frac{\partial g_{ab}}{\partial h_{kl}} = \left[\begin{array}{c|c} -v^k v^l & 0 \\ \hline 0 & \delta_{(i}{}^{(k}\,\delta_{j)}{}^{l)} \end{array}\right].$$

Gravitational EOM:

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \frac{\partial g_{ab}}{\partial v_k} = 0.$$

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \frac{\partial g_{ab}}{\partial h_{kl}} = 0.$$

That is:

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \left[ \begin{array}{c|c} 2v^k & -\delta_j{}^k \\ \hline -\delta_i{}^k & 0 \end{array} \right] = 0.$$

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \left[ \begin{array}{c|c} -v^k v^l & 0 \\ \hline 0 & \delta_{(i}{}^{(k}\, \delta_{j)}{}^{l)} \end{array} \right] = 0.$$

Gravitational EOM:

$$\left\{G^{tt} - 8\pi T^{tt}\right\} v^k - \left\{G^{tk} - 8\pi T^{tk}\right\} = 0.$$

$$\left\{G^{tt} - 8\pi T^{tt}\right\} v^k v^l - \left\{G^{kl} - 8\pi T^{kl}\right\} = 0.$$

These are $d + \frac{d(d+1)}{2}$ equations in $d + \frac{d(d+1)}{2}$ unknowns.

These equations should be viewed as the primary equations.

Reassemble:

$$G^{ab} = 8\pi T^{ab} + \left\{G^{tt} - 8\pi T^{tt}\right\} \left[\begin{array}{c|c} 1 & v^j \\ \hline v^i & v^i v^j \end{array}\right].$$

# Example: Hořava-inspired gravity...

The great reinterpretation:

$$\left\{G^{tt} - 8\pi T^{tt}\right\} \to 8\pi\rho_{\rm d}.$$

Also

$$V^a = -g^{ab}\nabla_a \tau = (1; v^i) = (1; h^{ij} v_j)$$

is a unit 4-vector.
Then the gravitational EOM imply

$$G^{ab} = 8\pi\left\{T^{ab} + \rho_{\rm d}\; V^a V^b\right\}.$$

The "extra piece" is just (a simplified version of) Mukohyama's "comoving dark dust".

- Note it's not intrinsically related to Hořava gravity per se.
- It's really just the a priori enforcement of the unit lapse condition.
- The dark dust is automatically comoving with the preferred foliation.

# Example: Hořava-inspired gravity...

For analysis within a specifically Hořava gravity framework, see:

- Dark matter as integration constant in Horava-Lifshitz gravity.
  Shinji Mukohyama.
  e-Print: arXiv:0905.3563 [hep-th]
  Physical Review **D80** (2009) 064005.
- Caustic avoidance in Horava-Lifshitz gravity.
  Shinji Mukohyama.
  e-Print: arXiv:0906.5069 [hep-th]
  JCAP **0909** (2009) 2009.

See also further references therein.

*** do worry about the caustics ***

*** more on this later ***

# Example: Hořava-inspired gravity...

Warning: You cannot just blindly "solve"

$$G^{ab} = 8\pi \left\{ T^{ab} + \rho_d \; V^a V^b \right\},$$

since the physics is more adequately described by:

$$G^{ab}(v,h) = 8\pi \, T^{ab}(\Psi, v, h) + \left\{ G^{tt}(v,h) - 8\pi \, T^{tt}(\Psi, v, h) \right\} \left[ \begin{array}{c|c} 1 & v^j \\ \hline v^i & v^i v^j \end{array} \right].$$

Important to focus attention on the assumed physical degrees of freedom...

Important to focus attention on the a priori constraints imposed on the form of the metric...

Warning:

You must do the job properly...

Solve the coupled set of PDEs:

$$G^{ab} = 8\pi \left\{ T^{ab} + \rho_d \; V^a V^b \right\},$$

$$V^a = -g^{ab} \, \nabla_b \tau,$$

$$g^{ab} \, \nabla_a \tau \, \nabla_b \tau = -1.$$

This is an invariant way of enforcing the unit-lapse constraint.

Consequences:

$$\nabla_a \left( \rho_\mathrm{d} \; V^a \right) = 0; \qquad \nabla_V V = 0.$$

# Example: Hořava-inspired gravity...

In an adapted coordinate system:

$$\partial_t \left(\sqrt{h}\, \rho_{\rm d}\right) + \partial_i \left(\sqrt{h}\, \rho_{\rm d}\, v^i\right) = 0.$$

- FLRW:

$$\rho_{\rm d} = \frac{C}{\sqrt{h}} \qquad \Rightarrow \qquad \rho_{\rm d} = \frac{C}{a(t)^3}.$$

- Static spherically symmetric:

$$\rho_{\rm d} = \frac{C}{4\pi r^2 \sqrt{h_{rr}}\; v^r}.$$

In vacuum, in an adapted coordinate system:

$$G_{tt} = 8\pi \rho_{\rm d}; \quad \text{(other components vanish).}$$

$$SSS \Rightarrow \text{Schwarzschild}$$

# Dark fluid:

Combine the unit-lapse constraint with the unimodular constraint:

$$g_{ab}(v,h) = \left[ \begin{array}{c|c} -1 + \omega^{-2}\Phi^{ij} v_i v_j & -v_j \\ \hline -v_i & \omega^2 \Phi_{ij} \end{array} \right].$$

$$h_{ij} = \omega^2 \Phi_{ij}; \qquad \det(\Phi_{ij}) = 1; \qquad (\omega \text{ non-dynamical});$$

$$\Phi^{ij} = [\Phi_{ij}]^{-1}; \qquad \det(g_{ab}) = \omega^6.$$

Still general enough to accommodate:

- Schwarzschild,
- Reissner–Nordström,
- and Kerr spacetimes...
- (FLRW a little trickier...)

# Dark fluid:

Metric:

$$g_{ab}(v,h) = \left[\begin{array}{c|c} -1+\omega^{-2}\Phi^{ij}v_i v_j & -v_j \\ \hline -v_i & \omega^2\Phi_{ij} \end{array}\right].$$

Then (small calculation required):

$$N = \frac{n(n+1)}{2} - 2 = \frac{(d+1)(d+2)}{2} - 2 = \frac{d^2+3d-2}{2};$$

$$\frac{\partial g_{ab}}{\partial v_k} = \left[\begin{array}{c|c} 2v^k & -\delta_j{}^k \\ \hline -\delta_i{}^k & 0 \end{array}\right];$$

$$\frac{\partial g_{ab}}{\partial \Phi_{kl}} = \left[\begin{array}{c|c} -v^k v^l + \frac{1}{d}(h^{ij}v_i v_j)h^{kl} & 0 \\ \hline 0 & \omega^2\left\{\delta_{(i}{}^{(k}\,\delta_{j)}{}^{l)} - \frac{1}{d}h_{ij}h^{kl}\right\} \end{array}\right].$$

Gravitational EOM:

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \frac{\partial g_{ab}}{\partial v_k} = 0.$$

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \frac{\partial g_{ab}}{\partial \Phi_{kl}} = 0.$$

That is:

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \left[ \begin{array}{c|c} 2v^k & -\delta_j{}^k \\ \hline -\delta_i{}^k & 0 \end{array} \right] = 0.$$

$$\left\{ G^{ab} - 8\pi T^{ab} \right\} \left[ \begin{array}{c|c} -v^k v^l + \frac{1}{d}(h^{ij} v_i v_j) h^{kl} & 0 \\ \hline 0 & \left\{ \delta_{(i}{}^{(k} \delta_{j)}{}^{l)} - \frac{1}{d} h_{ij} h^{kl} \right\} \end{array} \right] = 0.$$

# Dark fluid:

Gravitational EOM:

$$\left\{G^{tt} - 8\pi T^{tt}\right\} v^k - \left\{G^{tk} - 8\pi T^{tk}\right\} = 0,$$

and

$$\left\{G^{tt} - 8\pi T^{tt}\right\} \left\{v^k v^l - \frac{v^2 h^{kl}}{d}\right\} - \left\{G^{kl} - 8\pi T^{kl}\right\}$$

$$+\frac{1}{d}\left\{G^{ij} h_{ij} - 8\pi T^{ij} h_{ij}\right\} h^{kl} = 0.$$

These are $d + \frac{(d+2)(d-1)}{2}$ equations in $d + \frac{(d+2)(d-1)}{2}$ unknowns.

So they generate a complete set of PDEs for the metric coefficients.

The great reinterpretation:

Define:

$$\left\{G^{tt} - 8\pi T^{tt}\right\} \to 8\pi\rho_{\rm d}.$$

$$\frac{1}{d}\left\{G^{ij}h_{ij} - 8\pi T^{ij}h_{ij}\right\} - \frac{v^2}{d}\left\{G^{tt} - 8\pi T^{tt}\right\} \to 8\pi p_{\rm d}.$$

Then the gravitational EOM are:

$$8\pi\rho_{\rm d} v^k - \left\{G^{tk} - 8\pi T^{tk}\right\} = 0;$$

$$8\pi\rho_{\rm d} v^k v^l - \left\{G^{kl} - 8\pi T^{kl}\right\} + 8\pi p_d h^{kl} = 0.$$

Rearrange:

$$G^{tk} = 8\pi\{T^{tk} + 8\pi\rho_{\rm d} v^k\};$$

$$G^{kl} = 8\pi\{T^{kl} + \rho_{\rm d} v^k v^l + p_d h^{kl}\}.$$

The great reinterpretation:

$$G^{tt} = 8\pi\{T^{tt} + 8\pi\rho_\mathrm{d}\};$$
$$G^{tk} = 8\pi\{T^{tk} + 8\pi\rho_\mathrm{d} v^k\};$$
$$G^{kl} = 8\pi\{T^{kl} + \rho_\mathrm{d} v^k v^l + p_d h^{kl}\}.$$

Then:

$$G^{ab} = 8\pi\left\{T^{ab} + \rho_\mathrm{d}\left[\begin{array}{c|c} 1 & v^j \\ \hline v^i & v^i v^j \end{array}\right] + p_\mathrm{d}\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & h^{ij} \end{array}\right]\right\}.$$

Whence (small calculation using ADM decomposition required):

$$G^{ab} = 8\pi\left\{T^{ab} + \rho_\mathrm{d} V^a V^b + p_\mathrm{d}\left[g^{ab} + V^a V^b\right]\right\}.$$

# Dark fluid:

The great reinterpretation:

Finally:

$$G^{ab} = 8\pi \left\{ T^{ab} + (\rho_\mathrm{d} + p_\mathrm{d}) V^a V^b + p_\mathrm{d}\, g^{ab} \right\}.$$

- This is now a "fake dark fluid".
- Prior structure, (specifically, an a priori unit-lapse + unimodular constraint), has lead to modified gravitational EOM.
- Pressure:

$$\nabla_V V = 0 \qquad \Rightarrow \qquad (g^{ab} + V^a V^b) \nabla_b p_\mathrm{d} = 0$$

$$\Rightarrow \qquad \partial_i p_\mathrm{d} = 0 \qquad \Rightarrow \qquad p_\mathrm{d} = p_\mathrm{d}(t).$$

- Density more subtle:

$$\nabla_a(\rho_\mathrm{d}\; V^a) + p_\mathrm{d}(\nabla_a V^a) = 0.$$

# Dark fluid in FLRW:

Combine unit-lapse, unimodular constraints, with a scale factor $a(t)$:

$$g_{ab}(a,v,h) = a(t)^2 \left[ \begin{array}{c|c} -1+\omega^{-2}\Phi^{ij} v_i v_j & -v_j \\ \hline -v_i & \omega^2 \Phi_{ij} \end{array} \right].$$

Now general enough to accommodate FLRW (in quasi-conformal form).
One new function $a(t)$...
One new constraint (an integral over spatial slices):

$$\frac{\delta g_{ab}}{\delta a(t)} \to \frac{2}{a(t)} \int \mathrm{d}^d x \; g_{ab}$$

Implies

$$\int \mathrm{d}^d x \; g_{ab} \, X^{ab} = 0$$

# Dark fluid in FLRW:

Implies

$$\int \mathrm{d}^d x \; \rho_\mathrm{d} = 3 \int \mathrm{d}^d x \; p_\mathrm{d}$$

But we already know $p_\mathrm{d}$ is spatially constant, so

$$\rho_\mathrm{d} = \bar{\rho}_d + \delta\rho_\mathrm{d}; \qquad p_\mathrm{d} = \frac{1}{3}\,\bar{\rho}_\mathrm{d}$$

Then

$$(\rho_\mathrm{d} + p_\mathrm{d}) V^a V^b + p_\mathrm{d}\; g^{ab} = (\delta\rho_\mathrm{d} V^a V^b) + p_\mathrm{d}\; (4 V^a V^b + g^{ab})$$

- Inhomogeneous dark dust plus homogeneous dark radiation.
- More complicated model building possible by introducing an $a(t)$-dependent lapse.

# Einstein-aether:

Ted Jacobson's suggestion:
Suppose the aether field $u^a$ is non-dynamical...

That is, the metric is algebraically constrained by $g_{ab}(\Phi_A)\; u^a u^b = -1$.
Then there are $N = \frac{n(n+1)}{2} - 1$ degrees of freedom...
In an adapted coordinate system

$$u^a = (1; \vec{0}); \qquad g_{ab} = \left[\begin{array}{c|c} -1 & v_j \\ \hline v_i & h_{ij} \end{array}\right].$$

Not quite unit lapse...
Not quite ADM...
Then

$$\frac{\partial g_{ab}}{\partial v_k} = \left[\begin{array}{c|c} 0 & \delta_j{}^k \\ \hline \delta_i^k & 0 \end{array}\right]; \qquad \frac{\partial g_{ab}}{\partial h_{kl}} = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \delta_{(i}{}^{(k}\; \delta_{j)}{}^{l)} \end{array}\right].$$

Ted Jacobson's suggestion:
Suppose the aether field $u^a$ is non-dynamical...

The gravitational EOM are:

$$G^{tk} - 8\pi T^{tk} = 0;$$

$$G^{kl} - 8\pi T^{kl} = 0.$$

Then

$$G^{ab} - 8\pi T^{ab} \propto \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array}\right].$$

That is:

$$G^{ab} = 8\pi\{T^{ab} + \rho_{\mathrm{d}}\, u^a u^b\}.$$

Dark dust comoving with the aether...
(Could now add unimodular constraint... $\Rightarrow$ Dark fluid...)

# Fake matter:

For any of the "functionally constrained" models you can write

$$G^{ab} = 8\pi\{T^{ab} + X^{ab}\},$$

with the "fake matter" $X^{ab}$ satisfying

$$X^{ab}\,\frac{\partial g_{ab}(\Phi)}{\partial \Phi^A} = 0.$$

And with various implicit constraints coming from $g_{ab} = g_{ab}(\Phi^A)$.

- Choose your specific model...
- Calculate...
- Easy to get cosmological constant, dark dust, dark fluid, etcetera...
- Have we suddenly killed off 96% of the mass of the universe?

# Fake matter:

Before we get too enthusiastic...

- Really should develop full-fledged cosmological perturbation theory...
  - Can you fit structure formation?
  - Can you fit galactic dynamics?
- These are very tricky questions with as yet very unclear answers...
- Considerable room for interesting developments...

# History:

- I first saw something (vaguely) along these lines in various discussions of Wilson's "conformal flatness condition" for the spatial slices of neutron stars, equivalent to taking

$$h_{ij} = e^{2\Phi}\, \delta_{ij}.$$

  This is an approximation to GR, useful in neutron star physics...

- See also:

  Waveless Approximation Theories of Gravity
  James A. Isenberg (1978!)
  International Journal of Modern Physics **D17** (2008) 265–273.
  arXiv:gr-qc/0702113

  Note 30 year hiatus...

  (Bad referee!)

Metric (isomorphic to non-relativistic acoustic metric):

$$g_{ab}(c,v,\Phi) = \left[\begin{array}{c|c} -c^2 + e^{-2\Phi}\delta^{ij} v_i v_j & -v_j \\ \hline -v_i & e^{2\Phi}\delta_{ij} \end{array}\right].$$

Then (small calculation required):

$$N = 2 + d; \qquad \frac{\partial g_{ab}}{\partial c} = \left[\begin{array}{c|c} -c & 0 \\ \hline 0 & 0 \end{array}\right]; \qquad \frac{\partial g_{ab}}{\partial v_k} = \left[\begin{array}{c|c} 2e^{-2\Phi}\delta^{ki} v_i & -\delta_j{}^k \\ \hline -\delta_i{}^k & 0 \end{array}\right];$$

$$\frac{\partial g_{ab}}{\partial \Phi} = 2\left[\begin{array}{c|c} -e^{-\Phi}\delta^{ij} v_i v_j & 0 \\ \hline 0 & e^{2\Phi}\delta_{ij} \end{array}\right].$$

Fake matter:

$$X^{tt} = 0; \qquad X^{ti} = 0; \qquad \delta_{ij}\, X^{ij} = 0.$$

Dark TT matter:

$$X^{ab} = \sigma^{ab}; \qquad \sigma^{ab}\,\nabla_a \tau = 0; \qquad g_{ab}\,\sigma^{ab} = 0; \qquad Cotton(h) = 0.$$

Transverse (to the preferred foliation), and traceless — dark TT matter.

# Prior structure is good...

# ...or at the very least, interesting...

Thank you.