Vainshtein mechanism in quasi-dilaton massive gravity

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Mini-workshop "Massive gravity and its cosmological implications"
04/09/2013

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In progress

dRGT massive gravity

de Rham, Gabadadze, Tolley (2011)

✓ The action

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu},\psi]$$

where the potential is given by

$$\mathcal{U}_2 = 2\varepsilon_{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta\rho\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta = 4\left([\mathcal{K}^2] - [\mathcal{K}]^2\right)$$
$$\mathcal{U}_3 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\rho}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta = -[\mathcal{K}]^3 + 3[\mathcal{K}][\mathcal{K}^2] - 2[\mathcal{K}^3]$$
$$\mathcal{U}_4 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta\mathcal{K}^\rho{}_\sigma = -[\mathcal{K}]^4 + 6[\mathcal{K}]^2[\mathcal{K}^2] - 3[\mathcal{K}^2]^2 - 8[\mathcal{K}][\mathcal{K}^3] + 6[\mathcal{K}^4]$$

and

$$\mathcal{K}^\mu{}_\nu = \delta^\mu_\nu - \sqrt{\eta_{ab}g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b}$$

(BD) *Ghost free* massive gravity

Hassan and Rosen (2011)

Quasi-dilaton theory

D'Amico, Gabadadze, Hui, Pirtskhalava (2012)

✓ Symmetry

$$\sigma \to \sigma - \alpha M_{\rm Pl}, \qquad \phi^a \to e^{\alpha}\phi^a$$

✓ Define the new tensor

$$\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - e^{\sigma/M_{\rm Pl}}\sqrt{\eta_{ab} g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b}$$

K satisfies the symmetry

✓ The action

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{\omega}{M_{\rm Pl}^2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu},\psi]$$

massive graviton + scalar (DOF=6)

Decoupling limit in QMG

D'Amico, Gabadadze, Hui, Pirtskhalava (2012)

✓ Decoupling limit

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \phi^a = \delta^a_\mu x^\mu - \eta^{a\mu}\partial_\mu \pi / M_{\rm Pl} m^2$$

$$M_{\rm Pl} \to \infty, \qquad m \to 0, \qquad \Lambda = (M_{\rm Pl} m^2)^{1/3} = \text{fixed}, \qquad \frac{T_{\mu\nu}}{M_{\rm Pl}} = \text{fixed}$$

✓ The action

$$\mathcal{L}_{\rm DL} = -\frac{1}{4} h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{\omega}{2}\partial^\mu\sigma\partial_\mu\sigma - h^{\mu\nu}\left[\frac{1}{4}\varepsilon_\mu\varepsilon_\nu\Pi - \frac{\alpha}{4\Lambda^3}\varepsilon_\mu\varepsilon_\nu\Pi\Pi - \frac{\beta}{2\Lambda^6}\varepsilon_\mu\varepsilon_\nu\Pi\Pi\Pi\right]$$
$$+\sigma\left[\gamma_0\varepsilon\varepsilon\Pi + \frac{\gamma_1}{\Lambda^3}\varepsilon\varepsilon\Pi\Pi + \frac{\gamma_2}{\Lambda^6}\varepsilon\varepsilon\Pi\Pi\Pi + \frac{\gamma_3}{\Lambda^9}\varepsilon\varepsilon\Pi\Pi\Pi\Pi\right] + \frac{1}{M_{\rm Pl}} h^{\mu\nu} T_{\mu\nu}$$

$$\varepsilon\varepsilon\Pi \equiv \varepsilon^{\mu\alpha\beta\gamma}\varepsilon^{\nu}{}_{\alpha\beta\gamma}\partial_\mu\partial_\nu\pi$$
$$\varepsilon_\mu\varepsilon_\nu\Pi \equiv \varepsilon_\mu{}^{\alpha\gamma\delta}\varepsilon_\nu{}^\beta{}_{\gamma\delta}\partial_\alpha\partial_\beta\pi$$

$$\alpha,\ \beta,\ \text{and}\ \gamma_i = \text{functions of}\ \alpha_3,\ \text{and}\ \alpha_4$$

$$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$$

Decoupling limit in QMG

✓ Transformation

$$h_{\mu\nu} \to h_{\mu\nu} + \pi\eta_{\mu\nu} + \frac{\alpha}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

✓ The action with β=0

Galileon terms

$$\begin{aligned}\mathcal{L}_{\rm DL} = & -\frac{1}{4}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{\omega}{2}\partial^\mu\sigma\partial_\mu\sigma \boxed{- \frac{1}{8}\pi\left[\varepsilon\varepsilon\Pi - \frac{2\alpha}{\Lambda^3}\varepsilon\varepsilon\Pi\Pi + \frac{\alpha^2}{\Lambda^6}\varepsilon\varepsilon\Pi\Pi\Pi\right]} \\ & + \boxed{\sigma\left[\gamma_0\varepsilon\varepsilon\Pi + \frac{\gamma_1}{\Lambda^3}\varepsilon\varepsilon\Pi\Pi + \frac{\gamma_2}{\Lambda^6}\varepsilon\varepsilon\Pi\Pi\Pi + \frac{\gamma_3}{\Lambda^9}\varepsilon\varepsilon\Pi\Pi\Pi\Pi\right]} \\ & + \frac{1}{M_{\rm Pl}}h^{\mu\nu}T_{\mu\nu} + \frac{1}{M_{\rm Pl}}\pi T + \frac{\alpha}{M_{\rm Pl}\Lambda^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}\end{aligned}$$

Bi-galileon interaction term

where

$$\alpha = -\frac{3}{4}\alpha_3 - 1, \qquad \beta = -\frac{1}{8}\alpha_3 - \frac{1}{2}\alpha_4, \qquad \gamma_0 = \frac{1}{2} - \frac{2}{3}\alpha_5,$$

$$\gamma_1 = \frac{3}{8}\alpha_3 - \frac{1}{2} - \alpha_5, \qquad \gamma_2 = \frac{1}{2}\alpha_4 - \frac{3}{8}\alpha_3 - \frac{2}{3}\alpha_5, \qquad \gamma_3 = -\frac{1}{2}\alpha_4 - \frac{1}{6}\alpha_5.$$

Spherically symmetric case

✓ Master equation for λ

$$\frac{3}{2}\left(1-\frac{6}{\omega}\right)\lambda-\left(3\alpha+\frac{36\gamma_1}{\omega}\right)\lambda^2+\left(\alpha^2-\frac{32\gamma_1^2+24\gamma_2}{\omega}\right)\lambda^3-40\frac{\gamma_1\gamma_2}{\omega}\lambda^4-\frac{12\gamma_2^2}{\omega}\lambda^5=\left(\frac{r_*}{r}\right)^3$$

$$\lambda_\sigma=\frac{1}{\omega}(3\lambda+4\gamma_1\lambda^2+2\gamma_2\lambda^3)$$

where

$$\lambda\equiv\frac{\pi'}{\Lambda^3 r},\qquad \lambda_\sigma\equiv\frac{\sigma'}{\Lambda^3 r},\qquad r_*\equiv\left(\frac{M}{M_{\rm Pl}^2 m^2}\right)^{1/3}$$

✓ Solution **inside** Vainshtein radius

$$\pi'\propto r^{2/5}$$

$$\lambda\simeq-\left(\frac{3\omega}{16(1+\alpha)^2}\right)^{1/5}\left(\frac{r_*}{r}\right)^{3/5}$$

Negative λ (repulsive force)

π is suppressed by usual gravity → Newton gravity

Spherically symmetric case

✓ Solution **outside** Vainshtein radius

$$\frac{3}{2}\left(1-\frac{6}{\omega}\right)\lambda-\left(3\alpha+\frac{36\gamma_1}{\omega}\right)\lambda^2+\left(\alpha^2-\frac{32\gamma_1^2+24\gamma_2}{\omega}\right)\lambda^3-40\frac{\gamma_1\gamma_2}{\omega}\lambda^4-\frac{12\gamma_2^2}{\omega}\lambda^5=\cancelto{0}{\left(\frac{r_*}{r}\right)^3}$$

$$\lambda = 0 \text{ and } 4 \text{ constants}$$

✓ λ=0 solution (Asymptotically flat solution)

$$\lambda \simeq \frac{2\omega}{3(\omega-6)}\left(\frac{r_*}{r}\right)^3 \qquad \pi' \propto \frac{1}{r^2}$$

vDVZ solution

✓ λ=constant solutions (Asymptotic cosmological solutions)

$$\lambda \simeq \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$$

$$\pi' \propto r$$

$$h_{00} \propto h_{ij} \propto r^2$$

Ghost ??

$$\pi \to \Phi(r) + \phi(t,x)$$
$$\sigma \to \Psi(r) + \psi(t,x)$$

✓ Quadratic Lagrangian

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm DL} &= \mathcal{A}_1(\partial_t\phi)^2 - \mathcal{A}_2(\partial_r\phi)^2 - \mathcal{A}_3(\partial_\Omega\phi)^2 + \mathcal{B}_1(\partial_t\psi)^2 - \mathcal{B}_2(\partial_r\psi)^2 - \mathcal{B}_3(\partial_\Omega\psi)^2 \\ &\quad +\mathcal{C}_1\partial_t\phi\,\partial_t\psi - \mathcal{C}_2\partial_r\phi\,\partial_r\psi - \mathcal{C}_3\partial_\Omega\phi\,\partial_\Omega\psi\end{aligned}$$

where

$$\begin{aligned}
\mathcal{A}_1 &= \frac{3}{4} - \frac{1}{\Lambda^3}\left[\frac{3}{2}\alpha\left(\Phi'' + 2\frac{\Phi'}{r}\right) + 2\gamma_1\left(\Psi'' + 2\frac{\Psi'}{r}\right)\right] \\
&\quad + \frac{1}{\Lambda^6}\left[\frac{3}{2}\alpha^2\left(\frac{\Phi'^2}{r^2} + 2\frac{\Phi'\Phi''}{r}\right) - 6\gamma_2\left(\frac{\Psi'\Phi'}{r^2} + \frac{\Psi''\Phi'}{r} + \frac{\Psi'\Phi''}{r}\right)\right] \\
&\quad - \frac{12\gamma_3}{\Lambda^9}\left(\frac{\Psi''\Phi'^2}{r^2} + 2\frac{\Psi'\Phi'\Phi''}{r^2}\right) \\
\mathcal{A}_2 &= \frac{3}{4} + \frac{3}{2}\alpha a + 2\gamma_1 b - \frac{1}{\Lambda^3}\left[3(\alpha + \alpha^2 a - 2\gamma_2 b)\frac{\Phi'}{r} + 2(2\gamma_1 - 3\gamma_2 a)\frac{\Psi'}{r}\right] \\
&\quad + \frac{1}{\Lambda^6}\left[\frac{3}{2}\left(\alpha^2 + 8\gamma_3 b\right)\frac{\Phi'^2}{r^2} - 6(\gamma_2 - 4\gamma_3 a)\frac{\Psi'\Phi'}{r^2}\right] \\
\mathcal{A}_3 &= \frac{3}{4} + \frac{3}{2}\alpha a + 2\gamma_1 b - \frac{1}{\Lambda^3}\left[\frac{3}{2}(\alpha + \alpha^2 a - 2\gamma_2 b)\left(\Phi'' + \frac{\Phi'}{r}\right) + (2\gamma_1 - 3\gamma_2 a)\left(\Psi'' + \frac{\Psi'}{r}\right)\right] \\
&\quad + \frac{1}{\Lambda^6}\left[\frac{3}{2}\left(\alpha^2 + 8\gamma_3 b\right)\frac{\Phi'\Phi''}{r} - 3(\gamma_2 - 4\gamma_3 a)\frac{\Phi'\Psi'' + \Psi'\Phi''}{r}\right] \\
\mathcal{B}_1 &= \frac{\omega}{2} \\
\mathcal{B}_2 &= \frac{\omega}{2} \\
\mathcal{B}_3 &= \frac{\omega}{2} \\
\mathcal{C}_1 &= -6\gamma_0 - \frac{4\gamma_1}{\Lambda^3}\left(\Phi'' + 2\frac{\Phi'}{r}\right) - 6\frac{\gamma_2}{\Lambda^6}\left(\frac{\Phi'^2}{r^2} + 2\frac{\Phi'\Phi''}{r}\right) - \frac{24\gamma_3}{\Lambda^9}\frac{\Phi'^2\Phi''}{r^2} \\
\mathcal{C}_2 &= -6\gamma_0 + 4\gamma_1 b - \frac{8\gamma_1 - 6\gamma_2(a+b)}{\Lambda^3}\frac{\Phi'}{r} - \frac{6\gamma_2 - 8\gamma_3(2a+b)}{\Lambda^6}\frac{\Phi'^2}{r^2} \\
\mathcal{C}_3 &= -6\gamma_0 + 4\gamma_1 b - \frac{4\gamma_1 - 3\gamma_2(a+b)}{\Lambda^3}\left(\Phi'' + \frac{\Phi'}{r}\right) - \frac{6\gamma_2 - 8\gamma_3(2a+b)}{\Lambda^6}\frac{\Phi'\Phi''}{r}
\end{aligned}$$

Ghost ??

$$\pi \to \Phi(r) + \phi(t,x)$$
$$\sigma \to \Psi(r) + \psi(t,x)$$

✓ The static clump of dust of constant density $\quad T_{\mu\nu} = \rho\delta^0_\mu\delta^0_\nu\theta(R-r)$

✓ Quadratic Lagrangian (Inside the source + Vainshtein radius)

$$\mathcal{L}^{(2)}_{DL} \supset \left[\boxed{\alpha\frac{\rho}{M_{\rm Pl}\Lambda^3}} - \boxed{\frac{3^{4/5}}{10\times 2^{1/5}}\frac{(1+\alpha)(10+7\alpha)}{\omega}\left(\frac{\omega}{(1+\alpha)^2}\right)^{4/5}\left(\frac{r_*}{R}\right)^{12/5}}\right](\partial_t\phi)^2$$

$$\frac{\alpha}{M_{\rm Pl}\Lambda^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu} = \alpha\left(\frac{r_*}{R}\right)^3$$

from galileon and bi-galileon

For $\quad R \ll \xi^{-5/3} r_* \qquad \alpha > 0 \qquad\qquad \xi \equiv \frac{(1+\alpha)(10+7\alpha)}{\alpha\,\omega}\left(\frac{\omega}{(1+\alpha)^2}\right)^{4/5}$

✓ Quadratic Lagrangian (Outside the source, but inside Vainshtein radius)

$$\mathcal{L}^{(2)}_{DL} \supset -\frac{3^{4/5}}{10\times 2^{1/5}}\frac{(1+\alpha)(10+7\alpha)}{\omega}\left(\frac{\omega}{(1+\alpha)^2}\right)^{4/5}\left(\frac{r_*}{r}\right)^{12/5}(\partial_t\phi)^2$$

Negative sign of the kinetic term

Ghost ??

$$\pi \to \Phi(r) + \phi(t,x)$$
$$\sigma \to \Psi(r) + \psi(t,x)$$

✓ Quadratic action for π and σ

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm DL} &\supset \mathcal{A}_1(\partial_t\phi)^2 + \mathcal{B}_1(\partial_t\psi)^2 + \mathcal{C}_1(\partial_t\phi)(\partial_t\psi) \\ &= \mathcal{A}_1\left(\partial_t\phi + \frac{\mathcal{C}_1}{2\mathcal{A}_1}\partial_t\psi\right)^2 + \left(\mathcal{B}_1 - \frac{\mathcal{C}_1^2}{4\mathcal{A}_1}\right)(\partial_t\psi)^2\end{aligned}$$

✓ Inside Vainshtein radius,

$$\mathcal{B}_1 = \omega/2$$
$$\frac{\mathcal{C}_1^2}{4\mathcal{A}_1} \propto \left(\frac{r_*}{r}\right)^{12/5}$$

$$\mathcal{L}^{(2)}_{DL} \simeq \mathcal{A}_1(\partial_t\chi)^2 - \frac{\mathcal{C}_1^2}{4\mathcal{A}_1}(\partial_t\psi)^2$$

One of the scalar field has always ghost !!!

Summary

- Quasi-dilaton massive gravity with $\beta=0$ (Restricted bi-galileon)
  - does not have "healthy" asymptotically flat solution (vDVZ solution) as well as cosmological solution
  - It seems that there is some "healthy" time-dependent solutions (in progress)