Mini-workshop "Massive gravity and its cosmological implications" @IPMU

Vainshtein mechanism in Horndeski's general scalar-tensor theory (and in massive gravity)

Tsutomu Kobayashi 小林 努

Rikkyo University 立教大学

Based on work with

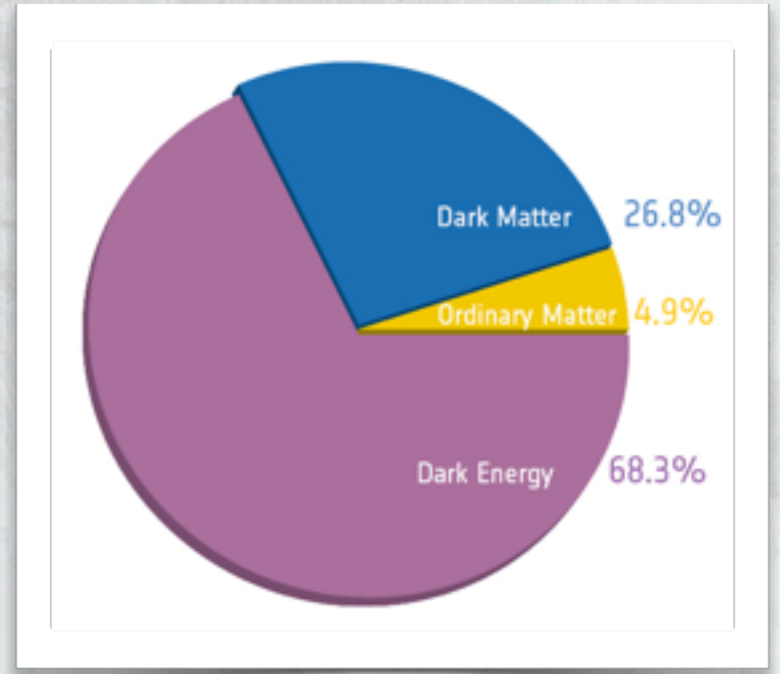
Tastuya Narikawa (Osaka), **Ryo Saito** (YITP), **Daisuke Yamauchi** (RESCEU)

arXiv:1302.2311

Talk plan

- Introduction & Motivation
- Horndeski's most general scalar-tensor theory
- Static, spherically symmetric, weak gravitational field
- Application
- Summary

Introduction



Mystery of dark energy

Modified gravity as an alternative to dark energy?

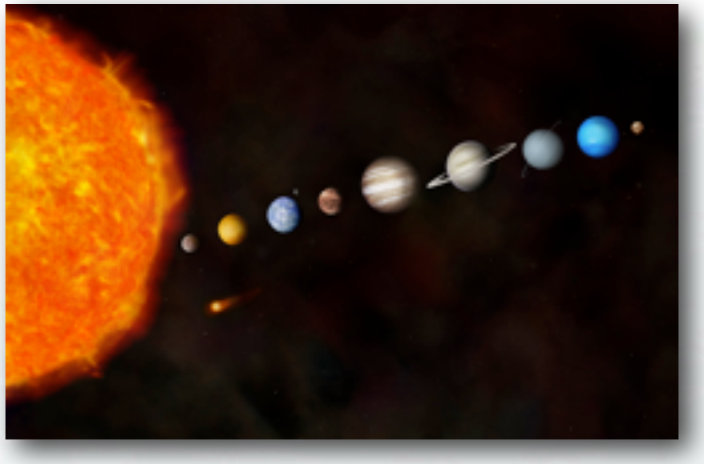
Modification would persist down to small length scales…

Need **screening mechanism** in the vicinity of matter

Basic idea

Extra d.o.f is *effectively* weakly coupled to matter

—— **Vainshtein mechanism**   Vainshtein (1972)



Example

Cubic **Galileon** non-minimally coupled to matter:

$$\mathcal{L} = \frac{1}{8\pi G}\left[-\frac{1}{2}(\partial\varphi)^2 - \frac{r_c^2}{3}(\partial\varphi)^2\Box\varphi\right] + \varphi T_\mu{}^\mu$$

(φ : dimensionless)

Key non-linearity

$r_c^2\Box\varphi$ **can be large** even if $\varphi \ll 1$

$$\varphi \sim \frac{r_g}{r} \ll 1, \quad r_c^2\Box\varphi \sim \frac{r_c^2 r_g}{r^3} \gtrsim 1 \quad \text{for} \quad r \lesssim (r_c^2 r_g)^{1/3}$$

Equation of motion

Static, spherically symmetric, non-relativistic source: $T_\mu^{\ \mu} = -\rho$

$$\Rightarrow \quad \frac{1}{r^2}\left\{(r^2\varphi')' + \frac{4r_c^2}{3}\left[r(\varphi')^2\right]'\right\} = 8\pi G\rho$$

$\Rightarrow$ Quadratic algebraic equation for φ'

$$\Rightarrow \quad \varphi' = \frac{3r}{8r_c^2}\left(-1+\sqrt{1+\frac{16}{3}\frac{r_c^2 r_g}{r^3}}\right)$$

Vainshtein radius

$$r_V = (r_c^2 r_g)^{1/3}$$

$$\varphi' \sim \frac{r_g}{r^2} \sim \Phi'$$ for $r \gg r_V$ unscreened

$$\varphi' \sim \frac{r_g}{r^2}\left(\frac{r}{r_V}\right)^{3/2} \ll \Phi'$$ for $r \ll r_V$ screened

(Φ : gravitational potential)

Suppose $r_c = 3 \text{ Gpc}$

$r_V \sim 100 \text{ pc}$ for the Sun ($M = M_\odot$)

$r_V \sim 1 \text{ Mpc}$ for a galaxy cluster ($M = 10^{14}\ M_\odot$)

Motivation

- Study the Vainshtein mechanism in the *most general* scalar-tensor theory, clarifying the conditions under which a screened solution is realized

- Offer a basic tool to test general scalar-tensor type gravity

Horndeski's most general scalar-tensor theory

Nicolis, Rattazzi,Trincherini (2009)

Galileon

$$\mathcal{L} = c_1\phi + c_2(\partial\phi)^2 + \underline{c_3(\partial\phi)^2\Box\phi} + \underline{c_4(\partial\phi)^2\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]}$$
$$\underline{+c_5(\partial\phi)^2\left[(\Box\phi)^3 - 3\Box\phi(\partial_\mu\partial_\nu\phi)^2 + 2(\partial_\mu\partial_\nu\phi)^3\right]}$$

Vainshtein mechanism operates

Burrage, Seery (2010); ....

Unique scalar-field theory in 4D flat spacetime having

- ✔ Galilean shift symmetry $\phi \to \phi + b_\mu x^\mu + c$
- ✔ 2nd-order equation of motion

Deffayet, Gao, Steer, Zahariade (2011)

Generalized Galileon

- Include gravity
- 2nd-order equation of motion both for $g_{\mu\nu}$ and ϕ
- Forget about any symmetry…

$$G_{4X} := \frac{\partial G_4}{\partial X}$$

$$\begin{aligned}\mathcal{L} \quad = \quad & K(\phi, X) - G_3(\phi, X)\Box\phi \\ & + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3\right. \\ & \qquad \left. - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

$$X = -\frac{1}{2}(\partial\phi)^2$$

Horndeski (1974); Rediscovered by Charmousis *et al.* (2011)

Horndeski's theory

The most general scalar-tensor theory with second-order field equations

$$
\begin{aligned}
\mathcal{L}_H \;=\; & \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma}+\frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \\
& +\kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma}+2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big] \\
& +\delta^{\alpha\beta}_{\mu\nu}\left[(F+2W)R_{\alpha\beta}{}^{\mu\nu}+2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi+2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right] \\
& -6\left(F_\phi+2W_\phi-X\kappa_8\right)\Box\phi+\kappa_9
\end{aligned}
$$

The generalized Galileon is equivalent to Horndeski's theory

TK, Yamaguchi, Yokoyama (2011)

Static, spherically symmetric, weak gravitational field

Narikawa, TK, Yamauchi, Saito, 1302.2311

Background

Start with the most general scalar-tensor theory

$$\begin{aligned}\mathcal{L} &= K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &\quad + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

Minkowski background

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu, \quad \phi = \phi_0 = \mathrm{const}, \quad X = 0$$

(Require $K(\phi_0, 0) = 0$, $K_\phi(\phi_0, 0) = 0$ for the theory to admit Minkowski background)

Approximations

Static, spherically symmetric perturbations produced by non-relativistic matter

$$\begin{aligned} ds^2 &= -[1+2\Phi(r)]dt^2 + [1-2\Psi(r)]d\mathbf{x}^2 \\ \phi &= \phi_0 + \varphi(r) \end{aligned}$$

$$T_t^{\ t} = -\rho(r)$$

Perturbations are *small*, but non-linear terms can be as large as linear terms

Do not neglect $(\partial\partial\epsilon)^n$ $\qquad\qquad$ $\partial\partial\epsilon$

$$\epsilon \sim \frac{r_g}{r} \quad \Longrightarrow \quad r_c^2(\partial\partial\epsilon)^2 \gtrsim \partial\partial\epsilon \quad \text{for} \quad r \lesssim (r_g r_c^2)^{1/3}$$

$(\ll 1)$

Gravitational field equations

Time-time component:

$$G_4 \frac{(r^2\Psi')'}{r^2} - G_{4\phi}\frac{(r^2\varphi')'}{2r^2} - (G_{4X} - G_{5\phi})\frac{[r(\varphi')^2]'}{2r^2} + G_{5X}\frac{[(\varphi')^3]'}{6r^2} = \frac{\rho}{4}$$

Background quantities $G_{5X} = G_{5X}(\phi_0, 0), \cdots$

Space-space component:

$$2G_4 \frac{\left[r^2\left(\Psi' - \Phi'\right)\right]'}{r^2} - 2G_{4\phi}\frac{(r^2\varphi')'}{r^2} - (G_{4X} - G_{5\phi})\frac{[r(\varphi')^2]'}{r^2} = 0$$

$$\boxed{(\text{l.h.s.}) = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(\cdots)}$$

Scalar field equation

$$
\begin{aligned}
&(K_X - 2G_{3\phi})\frac{(r^2\varphi')'}{r^2} - 2(G_{3X} - 3G_{4\phi X})\frac{[r(\varphi')^2]'}{r^2} \\
&+2G_{4\phi}\frac{[r^2(2\Psi - \Phi)']'}{r^2} + 4(G_{4X} - G_{5\phi})\frac{[r\varphi'(\Psi' - \Phi')]'}{r^2} \\
&+2\left(G_{4XX} - \frac{2}{3}G_{5\phi X}\right)\frac{[(\varphi')^3]'}{r^2} + 2G_{5X}\frac{[(\varphi')^2\Phi']'}{r^2} \\
&= \cancel{-K_{\phi\phi}\varphi}
\end{aligned}
$$

Neglect "mass term"

(Scalar field is screened if it is sufficiently massive)

$$
\boxed{\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(\cdots) = 0}
$$

Three equations are integrated once to give **algebraic equations** for Φ', Ψ', φ'

Introduce two mass scales $(M_{\rm Pl}, \Lambda)$ and six dimensionless parameters

$$
\begin{aligned}
G_4 &= \frac{M_{\rm Pl}^2}{2}, \\
G_{4\phi} &= M_{\rm Pl}\xi, \\
K_X - 2G_{3\phi} &= \eta, \\
-G_{3X} + 3G_{4\phi X} &= \frac{\mu}{\Lambda^3}, \\
G_{4X} - G_{5\phi} &= \frac{M_{\rm Pl}}{\Lambda^3}\alpha, \\
G_{4XX} - \frac{2}{3}G_{5\phi X} &= \frac{\nu}{\Lambda^6}, \\
G_{5X} &= -\frac{3M_{\rm Pl}}{\Lambda^6}\beta
\end{aligned}
$$

Previous example:

$$\Lambda \to \frac{M_{\rm Pl}}{r_c^2}$$

(one is redundant)

Useful dimensionless quantities:

Enclosed mass

$$x(r) := \frac{1}{\Lambda^3}\frac{\varphi'}{r}, \quad A(r) := \frac{1}{M_{\rm Pl}\Lambda^3}\frac{M(r)}{8\pi r^3}$$

Master equations:

$$\begin{aligned}\frac{M_{\rm Pl}}{\Lambda^3}\frac{\Phi'}{r} &= -\xi x + \beta x^3 + A(r), \\ \frac{M_{\rm Pl}}{\Lambda^3}\frac{\Psi'}{r} &= \xi x + \alpha x^2 + \beta x^3 + A(r),\end{aligned}$$

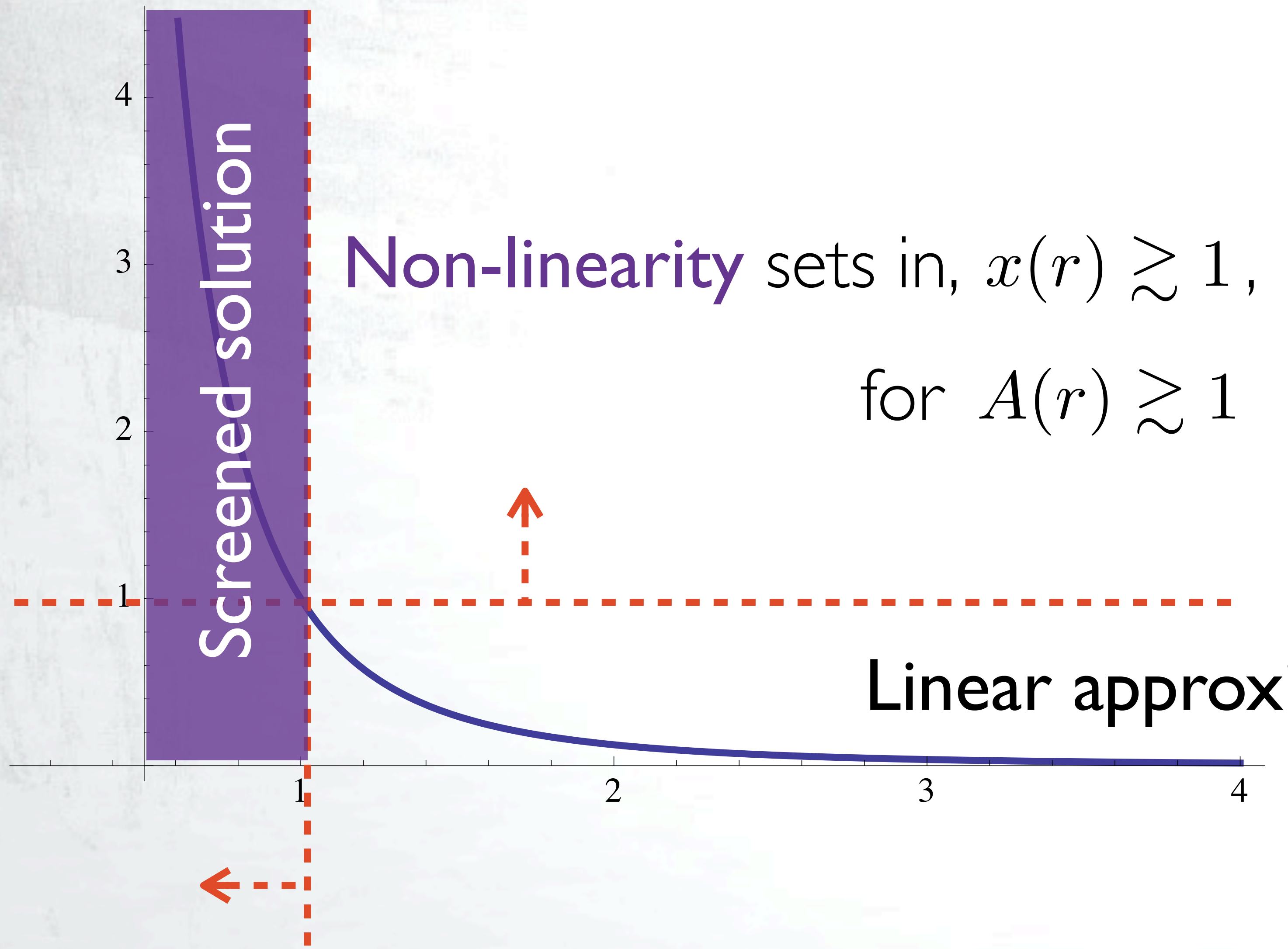
and

$$\boxed{\begin{aligned}P(x, A) &:= \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + \left[\mu + 6\alpha\xi - 3\beta A(r)\right] x^2 \\ &\quad + \left(\nu + 2\alpha^2 + 4\beta\xi\right) x^3 - 3\beta^2 x^5 \\ &= 0 \qquad \Rightarrow \qquad x(r) = x[A(r)]\end{aligned}}$$

Problem reduces to solving **quintic equation**

Vainshtein radius

$A(r)$ (Concrete form depends on density profile)



Non-linearity sets in, $x(r) \gtrsim 1$,

for $A(r) \gtrsim 1$

$A(r_V) = 1$

Linear approximation is good

r

r_V

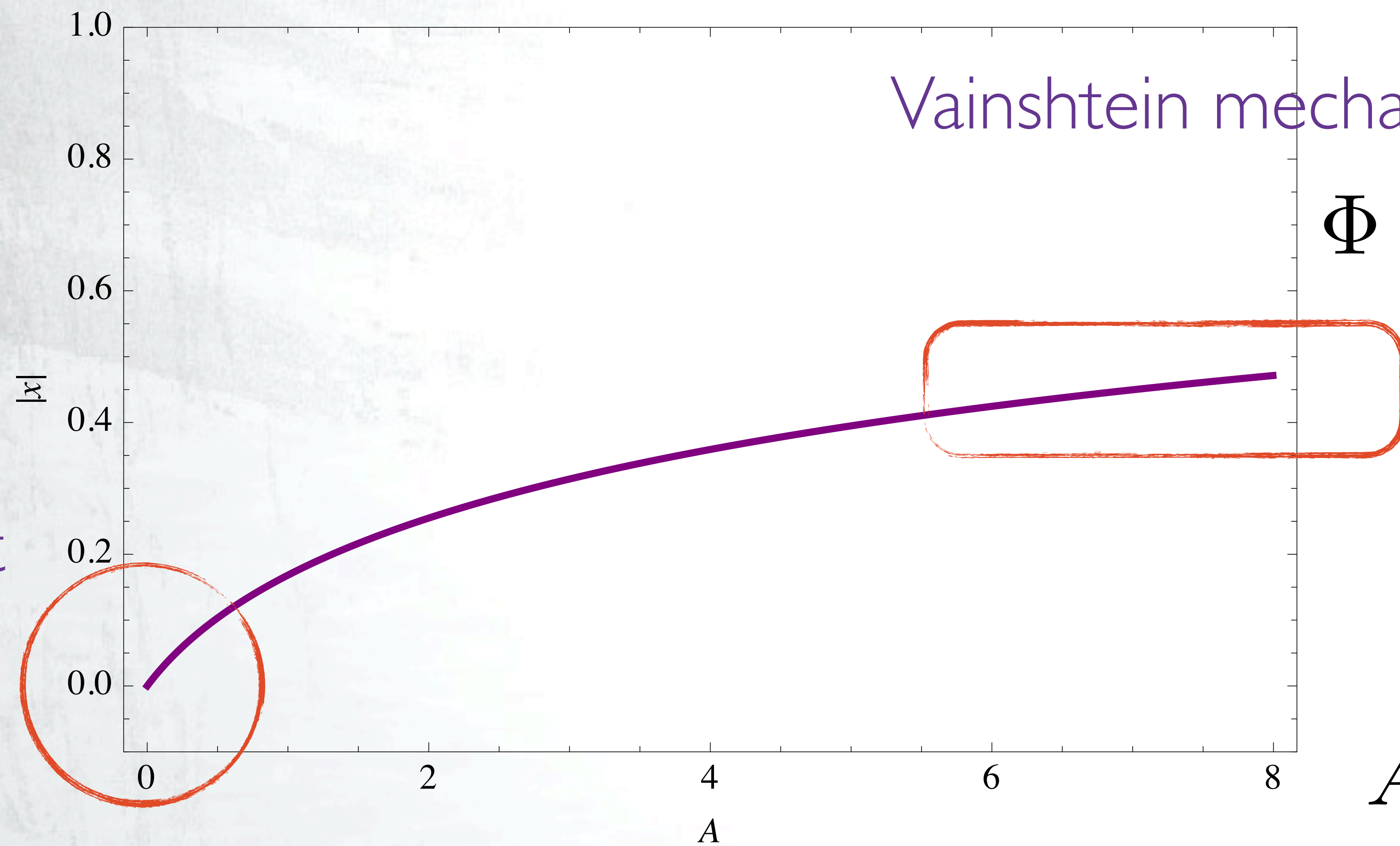
Solution we are looking for

$|x(A)|$

Inner region

Vainshtein mechanism operates

$$\Phi \simeq \Psi \simeq \Phi_{\mathrm{GR}}$$



A

Outer region

Asymptotically flat

$x \to 0$

$$A \ll 1 \Leftrightarrow r \gg r_V$$

$$A \gg 1 \Leftrightarrow r \ll r_V$$

Outer solution

Linear regime: $$P(x,A) \simeq \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x$$

$$\Leftrightarrow \quad x \approx x_{\mathrm{f}} := -\frac{2\xi A(r)}{\eta + 6\xi^2} \ll 1$$

Stable if $\eta + 6\xi^2 > 0$ (Kinetic term for small fluctuations has right sign)

Other solutions (if they exist) do not correspond to asymptotically flat spacetime

Inner solution

$$\begin{aligned} P(x, A) &:= \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + \left[\mu + 6\alpha\xi \underline{- 3\beta A(r)}\right] x^2 \\ &\quad + \left(\nu + 2\alpha^2 + 4\beta\xi\right) x^3 \underline{- 3\beta^2 x^5} \\ &= 0 \end{aligned}$$

Structure for $A \gg 1$ is different depending on whether $\beta = 0$ or $\beta \neq 0$

- $\beta = 0$

  $P(x, A)$ is cubic — consider separately

- $\beta \neq 0$

$$P(x, A) \approx \xi A - 3\beta A x^2 - 3\beta^2 x^5 \quad \text{for} \quad A \gg 1$$

Inner solution for $\beta \neq 0$

$$P(x, A) \approx \xi A - 3\beta A x^2 - 3\beta^2 x^5$$

- $\xi\beta < 0$

$$x^3 \approx -\frac{A}{\beta} \Rightarrow \frac{\Psi'}{r} \sim A^{2/3}, \quad \frac{\Phi'}{r} \sim A^{1/3}$$

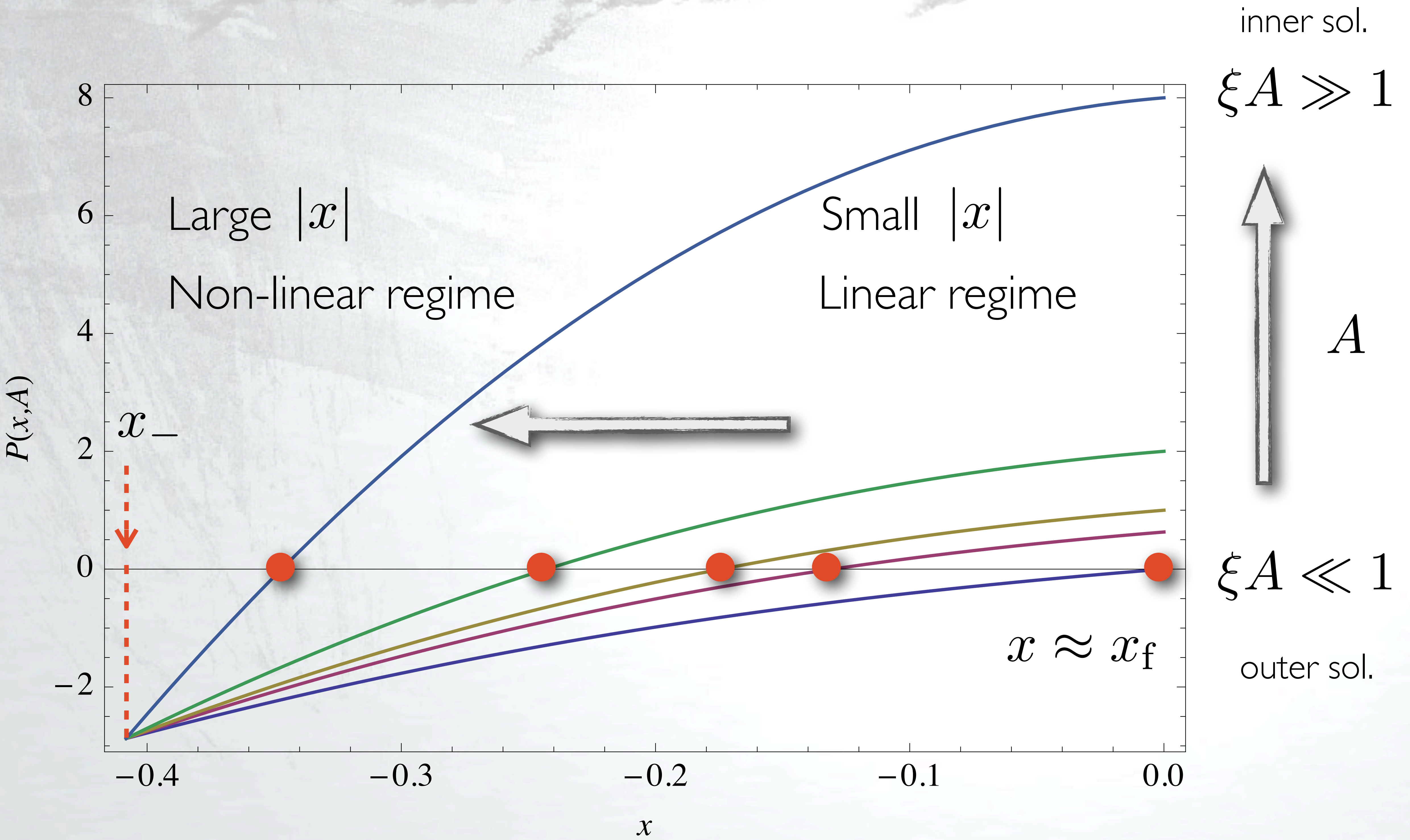
Not GR

- $\xi\beta > 0$

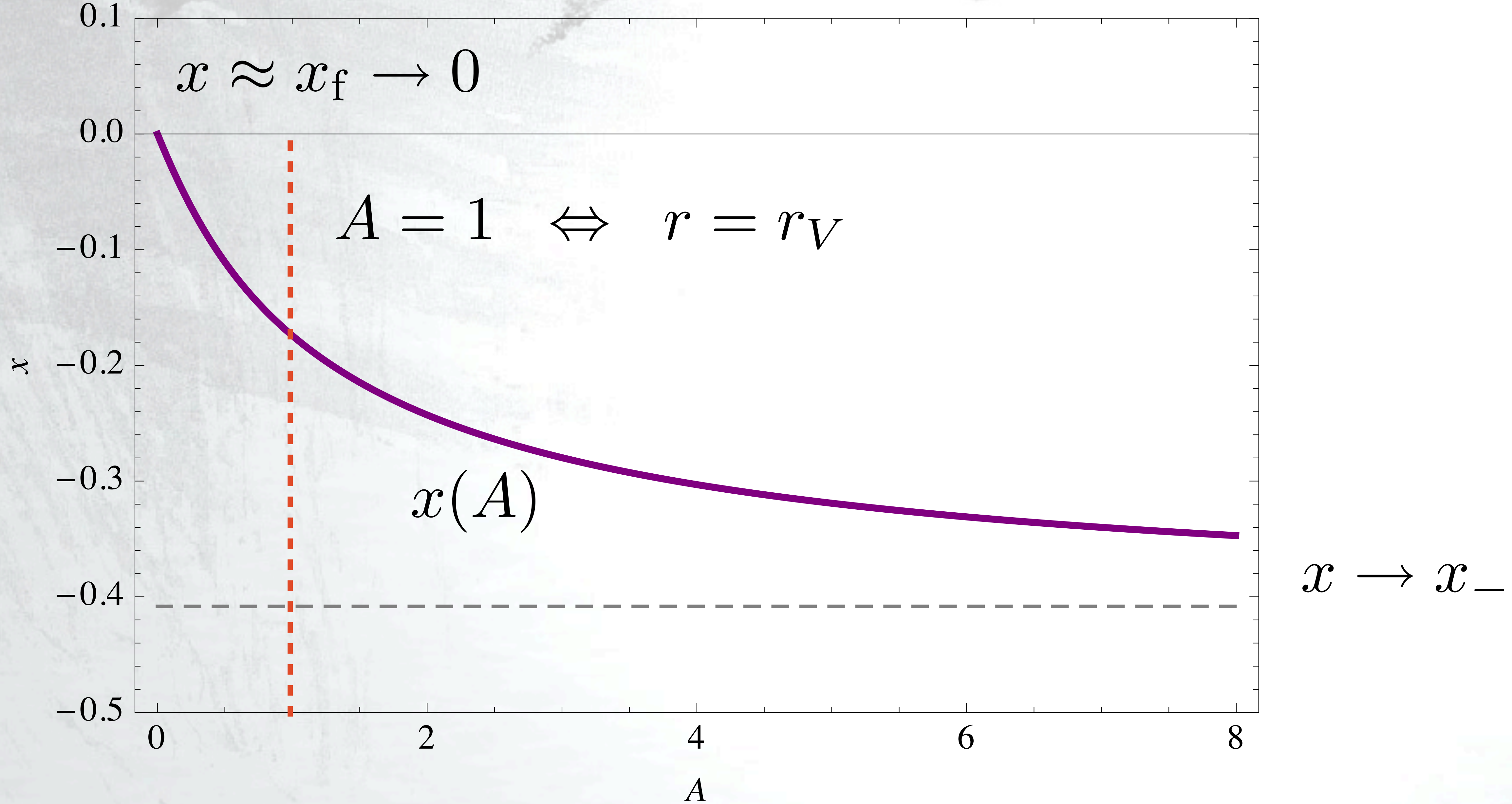
$x^3 \approx -\dfrac{A}{\beta}$ and $x \approx x_\pm := \pm\sqrt{\dfrac{\xi}{3\beta}} \Rightarrow \Phi \simeq \Psi \simeq \Phi_{\rm GR}$

(Consider for simplicity the case $\xi > 0$)

Matching inner and outer solutions



Profile of x



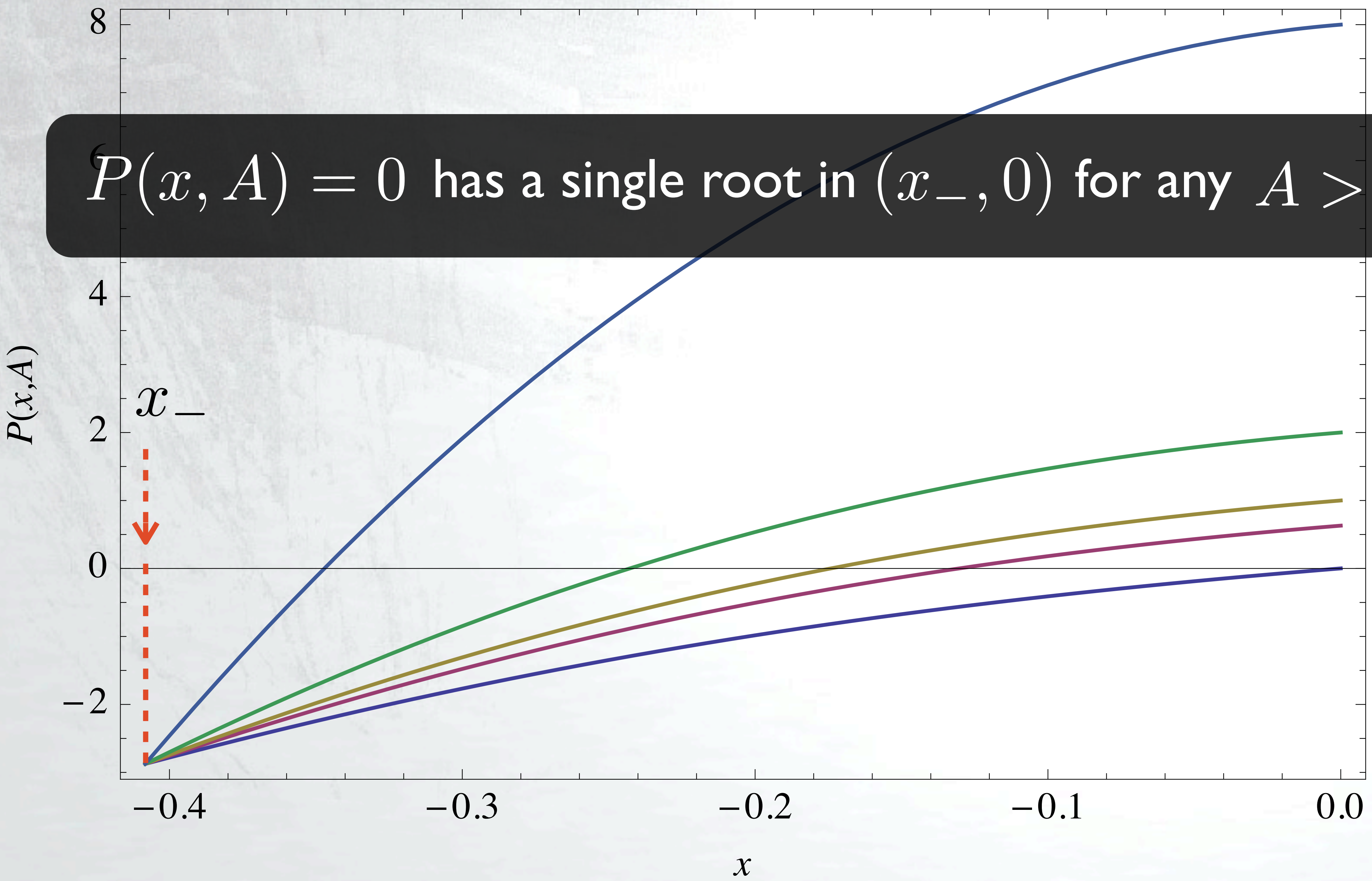
$A \ll 1$

$A \gg 1$

Outer region

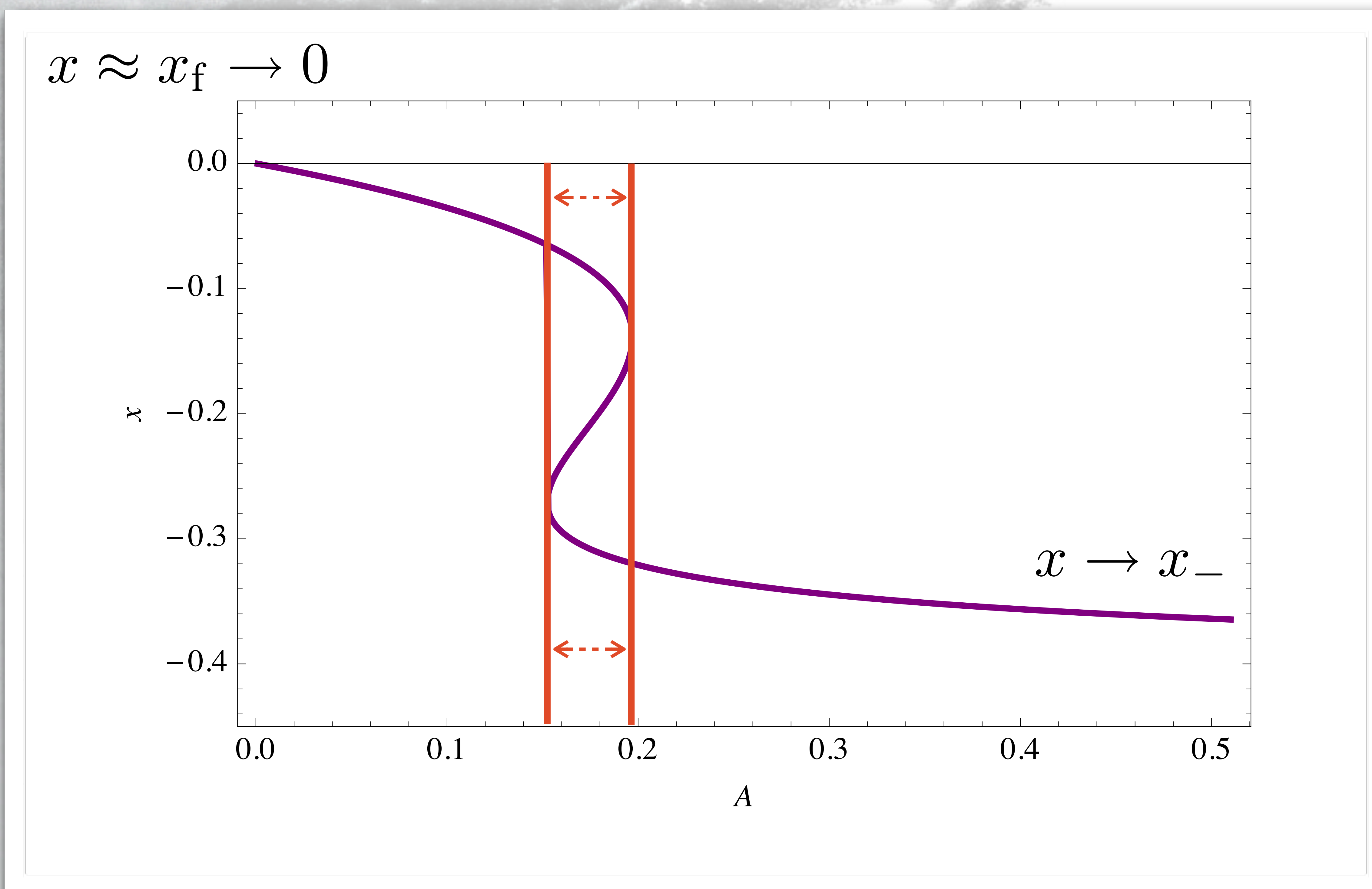
Inner region

Conditions for smooth matching



$P(x, A) = 0$ has a single root in $(x_-, 0)$ for any $A > 0$

Otherwise...



$P(x,A)$

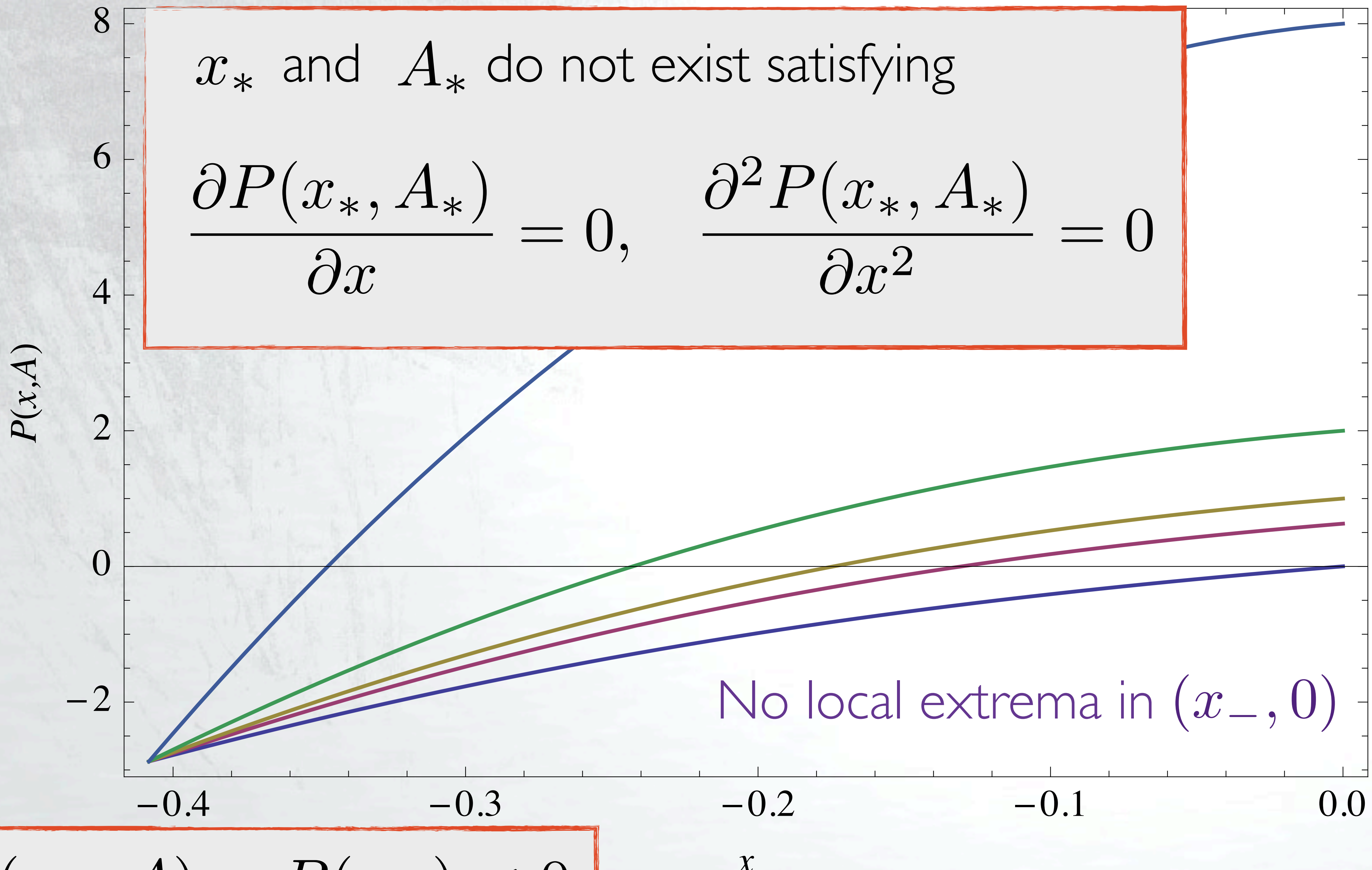
$x \longrightarrow$

$\approx x_{\mathrm{f}}$

Case I

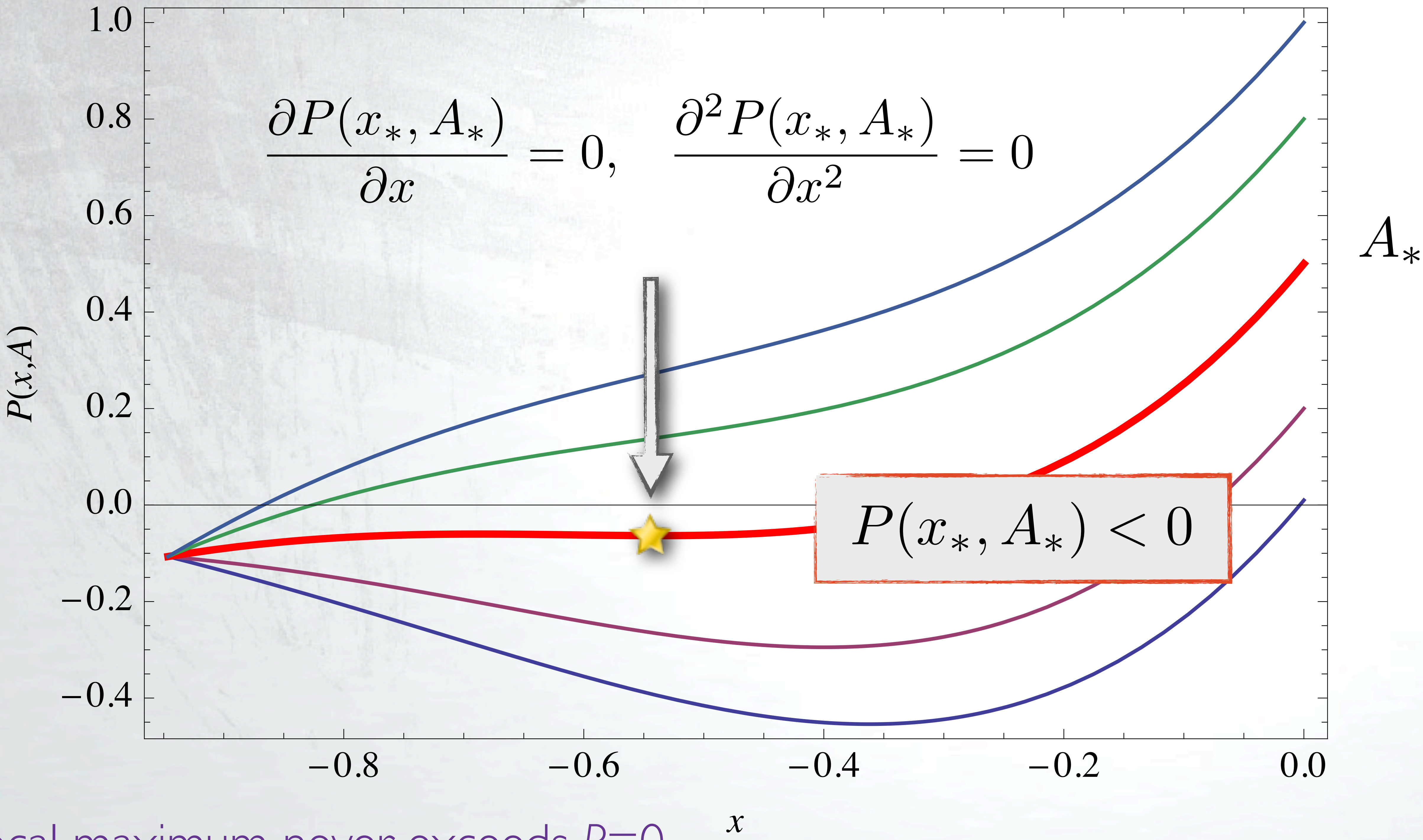
x_* and A_* do not exist satisfying

$$\frac{\partial P(x_*, A_*)}{\partial x} = 0, \quad \frac{\partial^2 P(x_*, A_*)}{\partial x^2} = 0$$



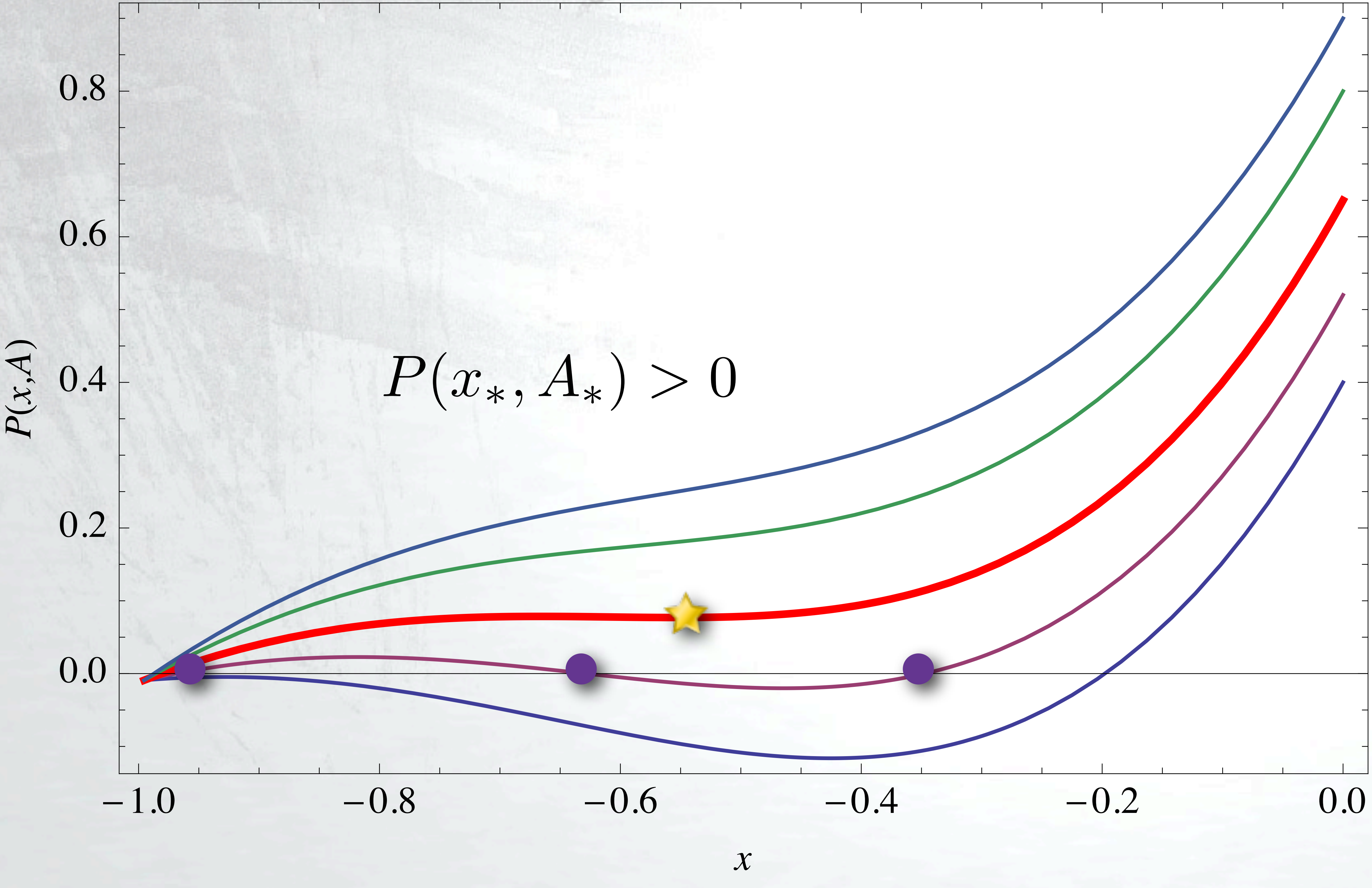
$$P(x_-, A) = P(x_-) < 0$$

Case II



Local maximum never exceeds P=0

Otherwise...



Inner solution for $\beta = 0$

$$P(x, A) \to \xi A + \left(\frac{\eta}{2} + 3\xi^2\right) x + (\mu + 6\alpha\xi)\, x^2 + \left(\nu + 2\alpha^2\right) x^3$$

Solution for $A \gg 1$

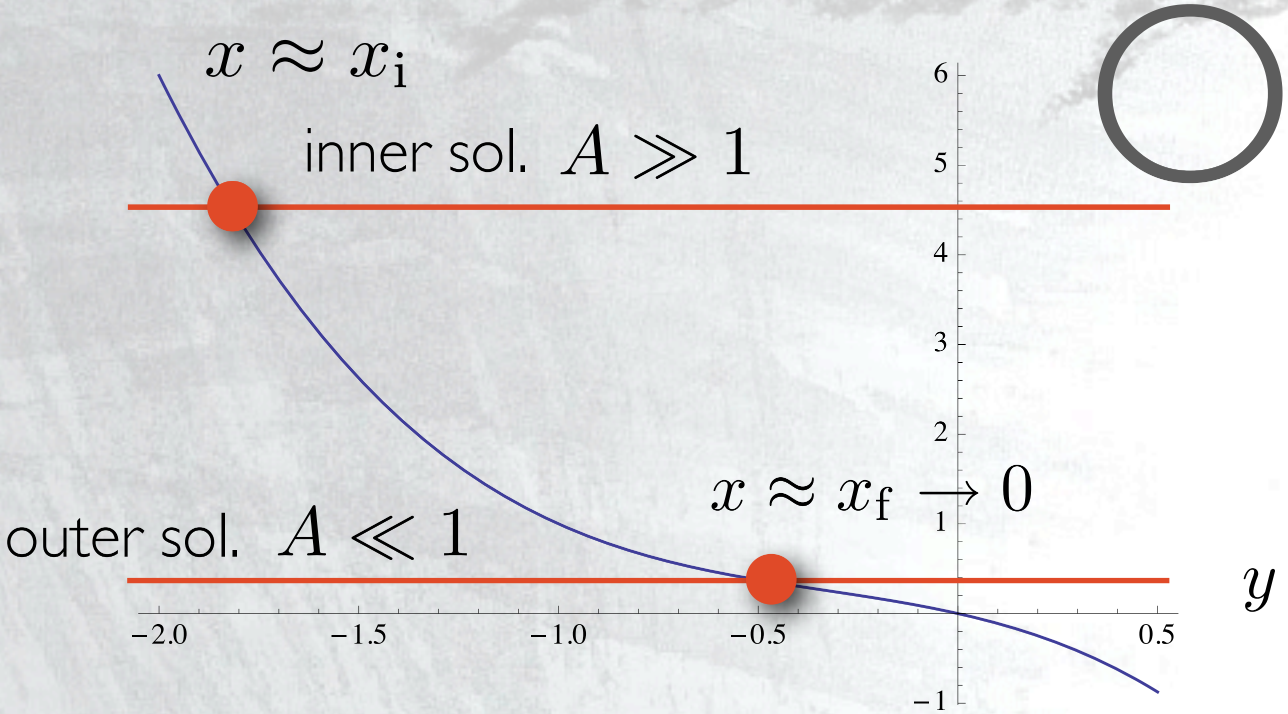
$$x^3 \approx x_{\mathrm{i}}^3 := -\frac{\xi A}{\nu + 2\alpha^2} \quad (< 0)$$

(required from stability)

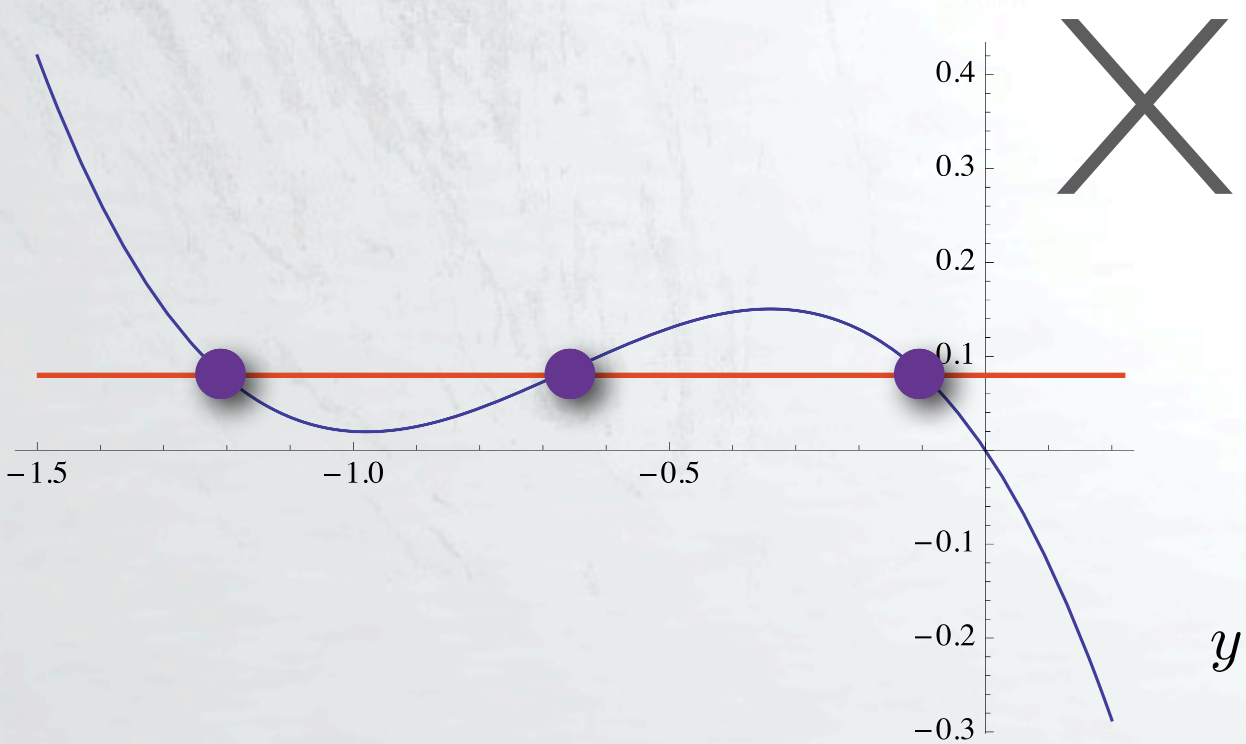
For this inner solution Vainshtein mechanism operates

$$\Phi \simeq \Psi \simeq \Phi_{\mathrm{GR}}$$

Matching inner and outer solutions



$y = \xi A$



Condition for smooth matching:

no local extrema in *x*<0

Local extrema are in *x*>0 if

$$\mu + 6\alpha\xi < 0$$

No local extrema if

$$\left(\nu + 2\alpha^2\right)\left(\eta + 6\xi^2\right) \geq \frac{2}{3}\left(\mu + 6\alpha\xi\right)^2$$

$$\mu + 6\alpha\xi \geq 0$$

$$y = -\left(\frac{\eta}{2} + 3\xi^2\right) x - \left(\mu + 6\alpha\xi\right) x^2 - \left(\nu + 2\alpha^2\right) x^3$$

Example

$$M_{\rm Pl} \to \infty, \; m \to 0$$
$$\Lambda^3 = M_{\rm Pl} m^2 \;\; \text{fixed}$$

Decoupling limit of **massive gravity** de Rham, Gabadadze, Tolley (2011)

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu}\left(X^{(1)}_{\mu\nu} + \frac{a_2}{\Lambda^3} X^{(2)}_{\mu\nu} + \frac{a_3}{\Lambda^6} X^{(3)}_{\mu\nu}\right) + \frac{1}{2M_{\rm Pl}} h^{\mu\nu} T_{\mu\nu}$$

Helicity-2 mode Interactions with **helicity-0 mode**

$$X^{(1)}_{\mu\nu} := \nabla_\mu \nabla_\nu \phi - \Box\phi \, g_{\mu\nu}$$

$$X^{(2)}_{\mu\nu} := \cdots$$

$$K = 0 = G_3$$

$$G_4 = \frac{M_{\rm Pl}^2}{2} + M_{\rm Pl}\phi + \frac{M_{\rm Pl}}{\Lambda^3} \alpha X$$

$$G_5 = -3 \frac{M_{\rm Pl}}{\Lambda^6} \beta X$$

$$(\alpha = a_2, \; \beta = a_3)$$

“Covariantization” de Rham, Heisenberg (2011)

$$h^{\mu\nu} X^{(1)}_{\mu\nu} \longleftrightarrow \phi R \longleftrightarrow G_4 \sim M_{\rm Pl}\phi$$

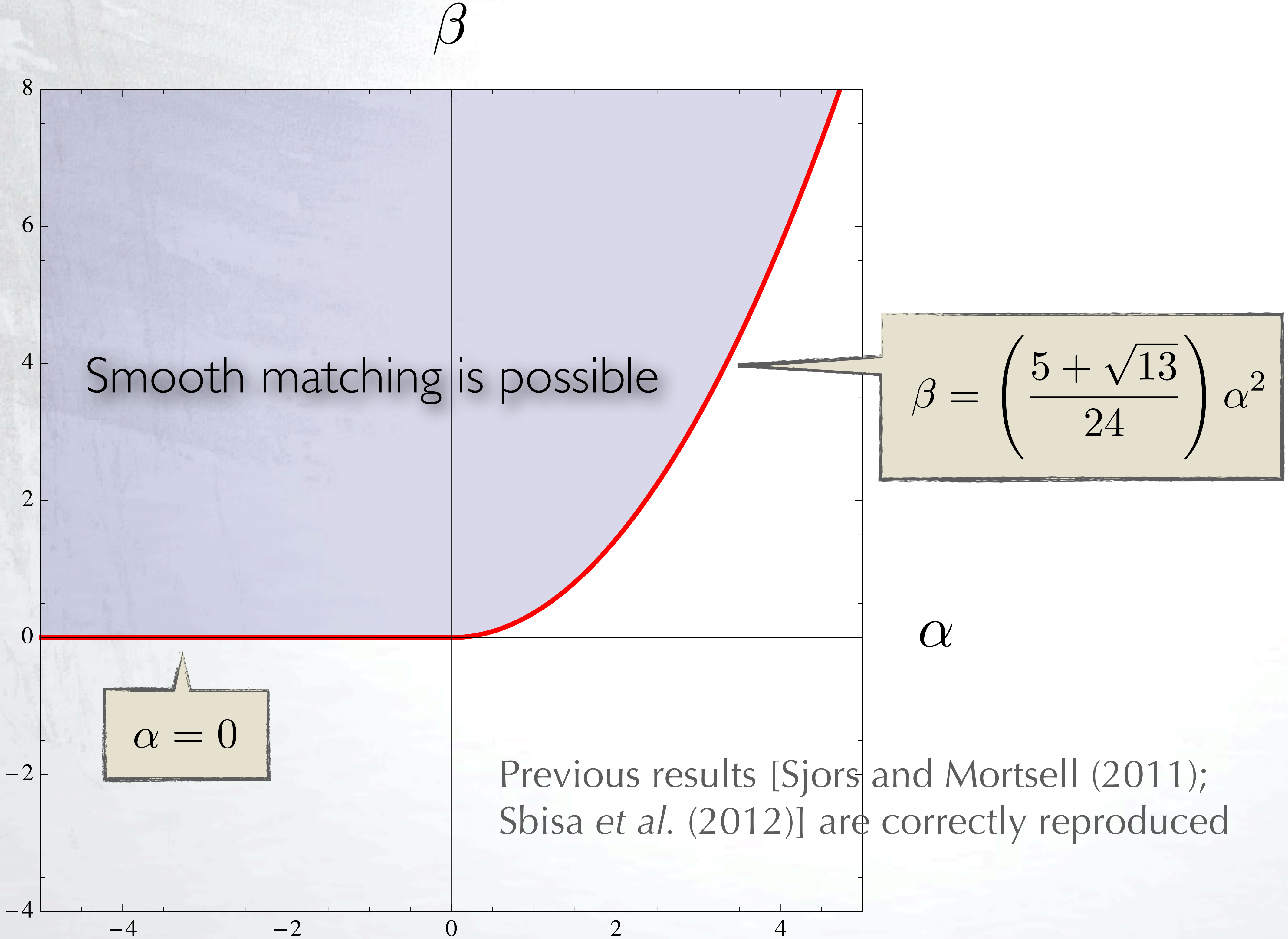
$$h^{\mu\nu} X^{(2)}_{\mu\nu} \longleftrightarrow -G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \longleftrightarrow G_4 \sim X$$

$$h^{\mu\nu} X^{(3)}_{\mu\nu} \longleftrightarrow \left(2R^{\mu\alpha\nu\beta} + \cdots\right)\partial_\mu\phi\partial_\nu\phi\partial_\alpha\partial_\beta\phi \longleftrightarrow G_5 \sim -3X$$

in Horndeski’s language

Decoupling limit of massive gravity = 2-parameter subclass of Horndeski's theory

Smooth matching of asymptotically flat and Vainshtein solutions is possible for:



Previous results [Sjors and Mortsell (2011); Sbisa *et al.* (2012)] are correctly reproduced

Application

Application: Gravitational lensing

Lensing convergence can be computed for any density profile and for any scalar-tensor theory from

$$\Delta(\Phi+\Psi) = \frac{\Lambda^3}{M_{\mathrm{Pl}}}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2(\alpha x^2 + 2\beta x^3 + 2A)\right]$$

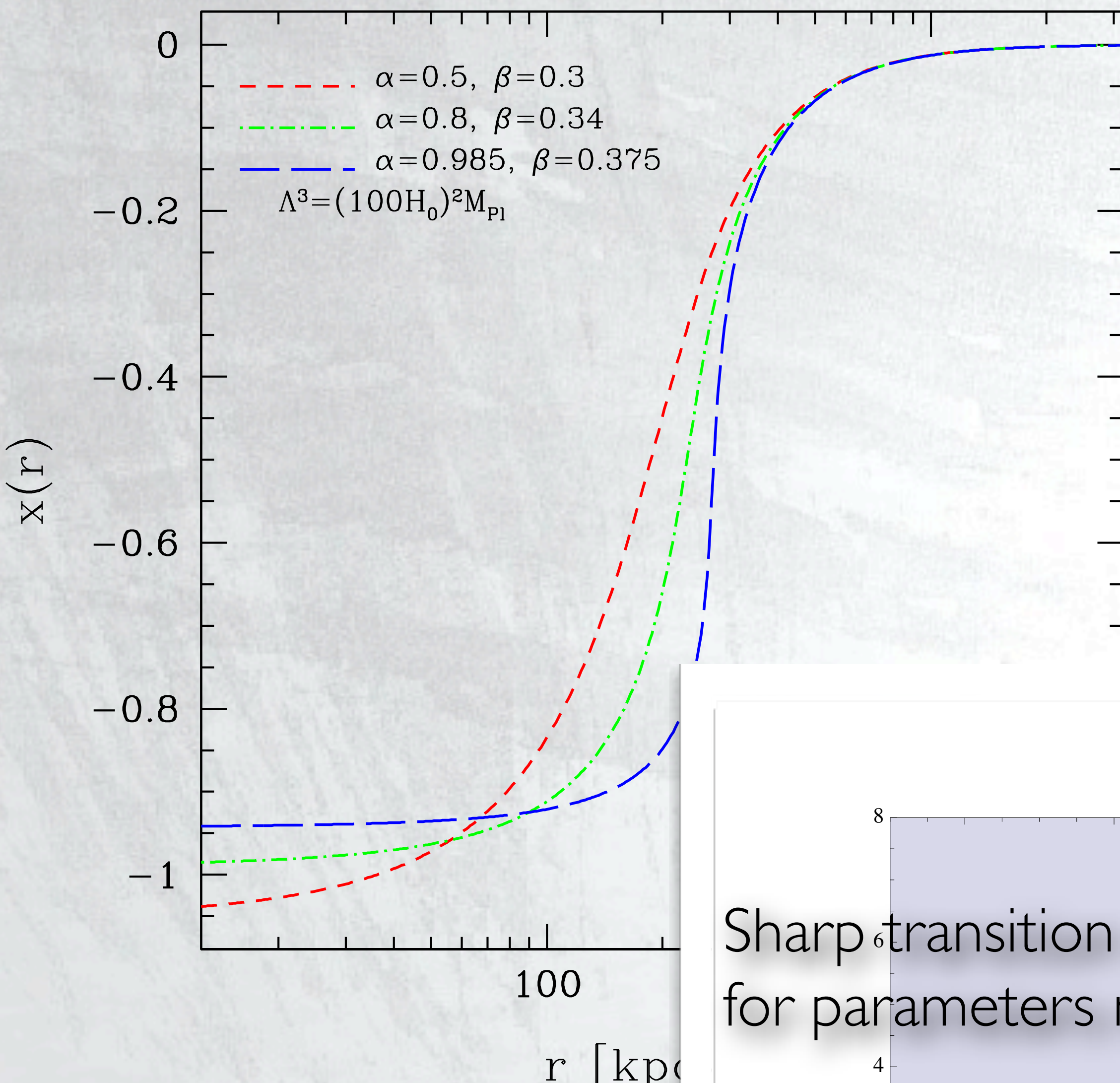
$$\supset x'(r)$$

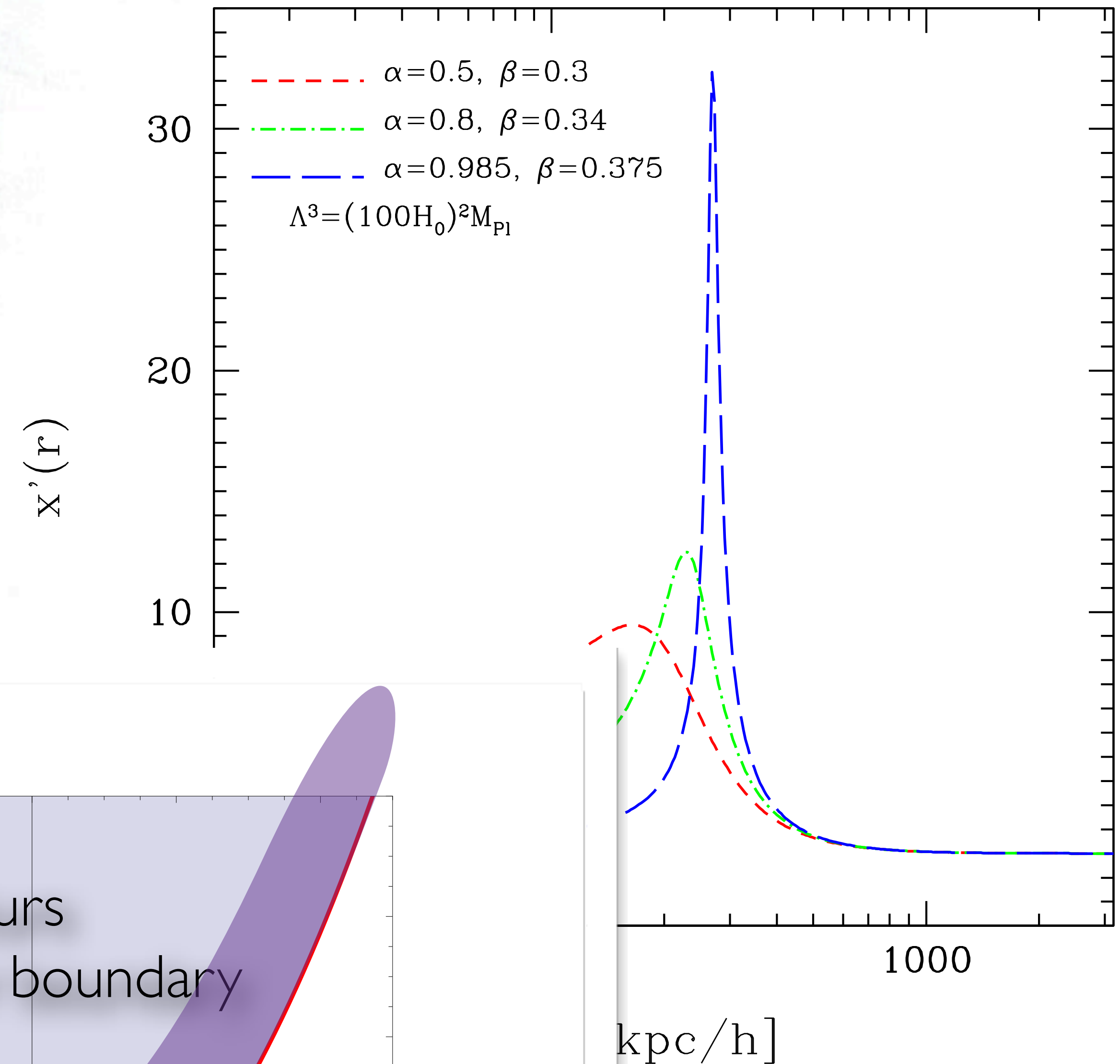
Convergence:

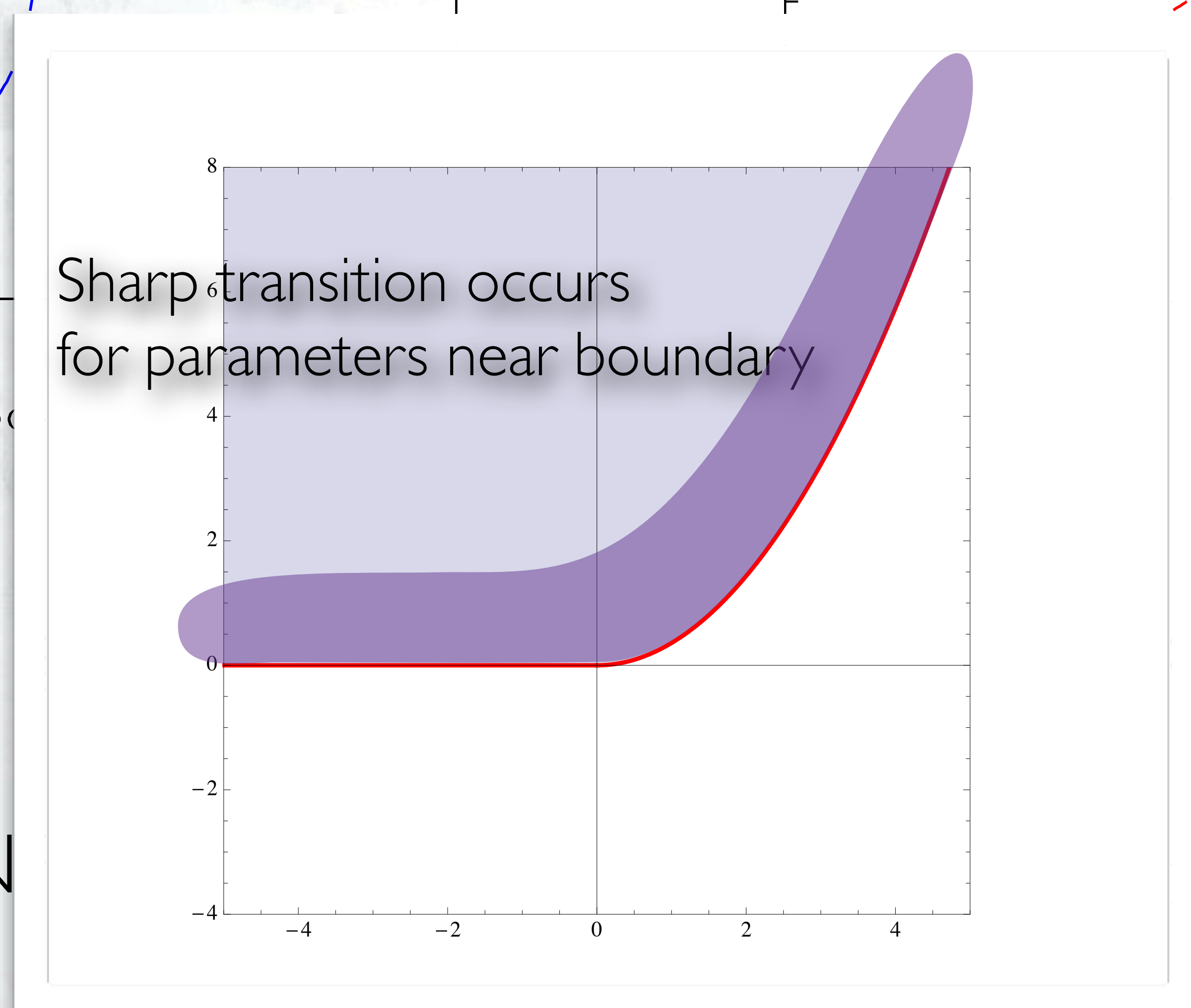
$$\kappa(\theta) = \frac{(\chi_{\mathrm{S}}-\chi_{\mathrm{L}})\chi_{\mathrm{L}}}{\chi_{\mathrm{S}}}\int_0^\infty \mathrm{d}Z\,\frac{\Delta}{a_{\mathrm{L}}^2}(\Phi+\Psi)$$

Interesting signature in cluster lensing?

$x'(r)$ can be large at transition from screened to unscreened regions





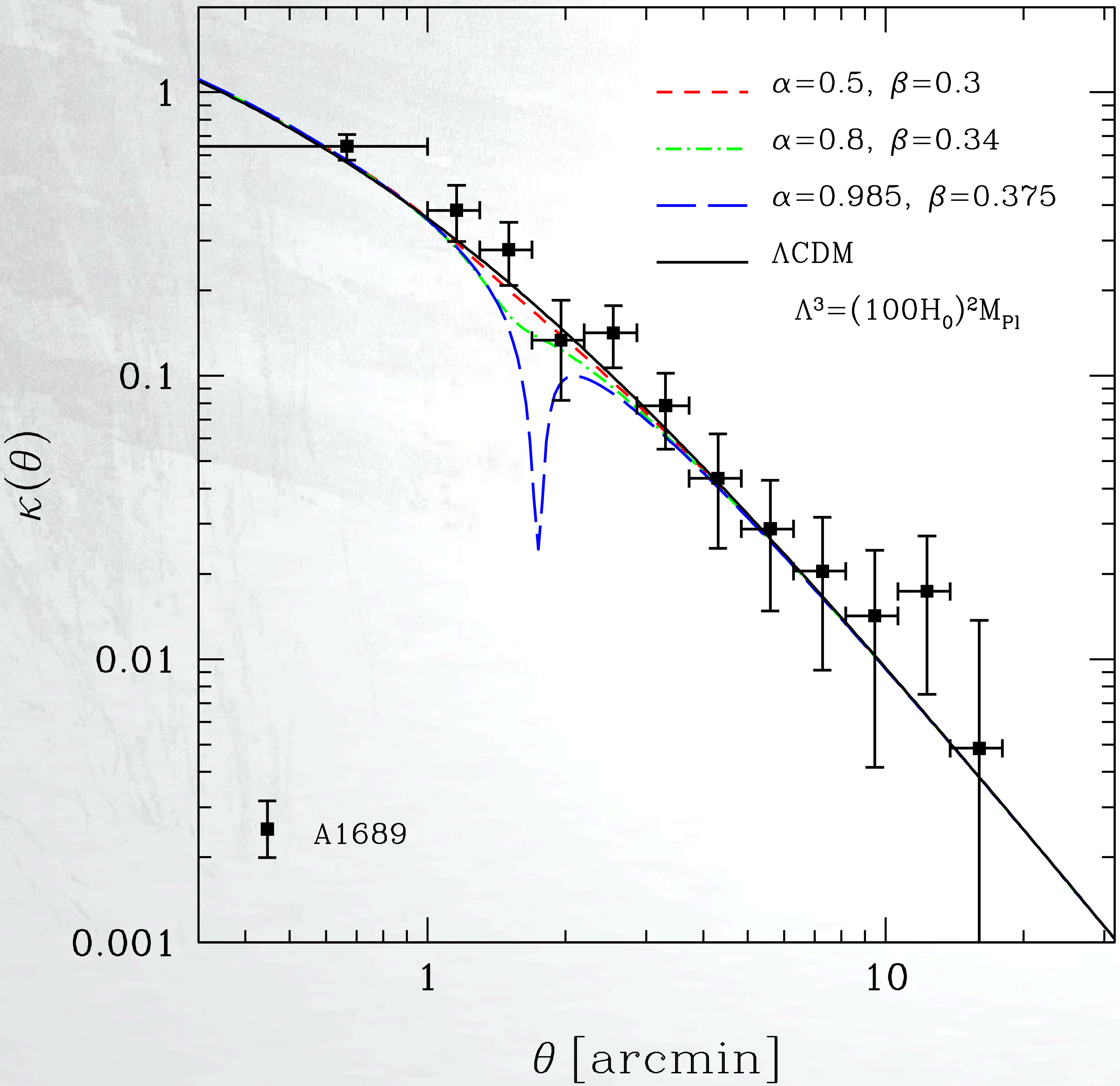


Massive … rndeski)

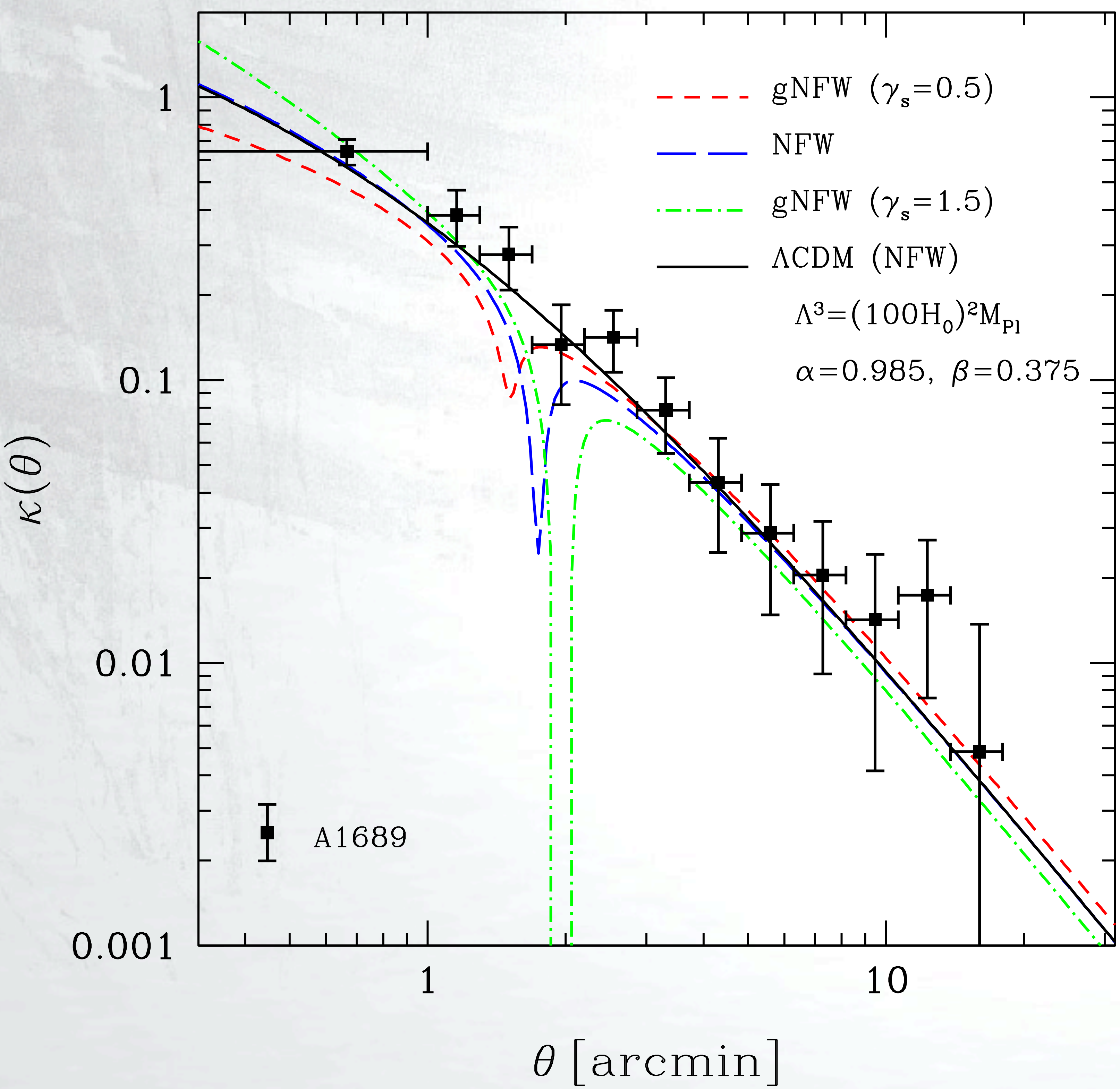
N … $)^2$

Dip in convergence power spectrum

(if we are lucky enough…?)



Dip is not a consequence of the specific choice of the density profile



Summary

- Static, spherically symmetric, weak gravitational field sourced by non-relativistic matter in **Horndeski's most general scalar-tensor theory**

- The problem reduces to solving a **quintic algebraic equation**

- Conditions under which a screened solution is realized are clarified

- Interesting applications such as testing gravity with cluster lensing

- ¿ Cosmological background? —— Sixth-order algebraic equation with time-dependent coefficients… [Kimura, TK, Yamamoto (2012)]

- ¿ Application to other cosmological probes?

Thank you!