Two Faces of Anisotropy in Lifshitz Holography

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Focus Week on Gravity and Lorentz Violation

February 18, 2013

Based on 1211.4872, 1112.5660 with T. Griffin and P. Horava

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$$S_{\rm scalar} = \int dt\, d^D x \left(\dot{\phi}^2 - (\nabla^z \phi)^2 \right)$$

$$S_{\rm gauge} = \int dt\, d^D x\, {\rm tr} \left(\frac{1}{2e^2} \vec{E}^2 - \frac{1}{4g^2} (\nabla^{z-1} F)^2 \right)$$

Hořava-Lifshitz Gravity

- Gravity: Theory of a dynamical metric
- Anisotropy requires additional structure
- Natural candidate: manifold with foliation, symmetries = foliation-preserving diffs

$$t \mapsto f(t) \qquad \vec{x} \mapsto \vec{g}(t, \vec{x})$$

- ADM decomposition is now canonical:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

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- Symmetries allow two conditions for lapse:

$$N = N(t) \text{ “projectable”}$$
$$\text{or} \quad N = N(t, \vec{x}) \text{ “non-projectable”}$$

Projectable Theory

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- No natural way to embed holography

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$$\lambda > 1 \quad \text{or} \quad D\lambda < 1$$

Vacuum Solutions for $\Lambda<0$

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$$S_{\mathrm{IR}} = \frac{1}{2\kappa^2}\int dt\, d^Dx\, dr\, N\sqrt{g}\,\Big(K^{ij}K_{ij} - \lambda K^2 + \beta R + \frac{\alpha}{2}\left(\frac{\nabla N}{N}\right)^2 - 2\Lambda\Big)$$

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with

- Invariant under time/space translations, spatial rotations, and

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$$ds^2 = -\left(\frac{r}{\ell}\right)^{2z} dt^2 + \left(\frac{r}{\ell}\right)^2 d\vec{x}^2 + \frac{dr^2}{r^2}$$

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- Invariant under time/space translations, spatial rotations, and

$$(t, \vec{x}, r) \mapsto (\Omega^z t, \Omega\vec{x}, \Omega^{-1} r)$$

- Acts on the asymptotic region like scaling symmetry in a theory at a Lifshitz point!
- Solutions well-known: “Lifshitz Space”
- Proposed as background for holographic duals to theories at a Lifshitz point
- Naturalness: Boundary anisotropy from bulk anisotropy
- Holography using Horava-Lifshitz gravity

Basics of Holography

- Equivalence between gravitational theory on AdS(-like) spacetime and a field theory living on its asymptotic boundary
- Specifically:
  - Partition function of gravity is a functional of asymptotic boundary conditions $Z_{\rm gravity}[g_{ij}, \phi_a, \ldots]$
  - Partition function of field theory can be taken as a functional of its sources $Z_{\rm FT}[g_{ij}; J_a, \ldots]$

Gravity/Field Theory Correspondence

- Holographic correspondence:

$$Z_{\text{gravity}}[g_{ij}, J_a, \ldots] = Z_{\text{FT}}[g_{ij}; J_a, \ldots]$$

- Semi-classical limit ($\ell_{\text{Planck}} \ll R_{\text{AdS}}$):

$$Z_{\text{gravity}}[g_{ij}, \ldots] \approx Z_0 e^{iS_{\text{on-shell}}[g_{ij}, \ldots]}$$

- AdS(-like) spacetimes have infinite volume => require regularization and counterterms to obtain a finite result

- Relativistic case: asymptotically locally AdS

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \frac{dr^2}{r^2}$$

$$g_{\mu\nu}(x^\lambda, r) = r^2 g^{(0)}_{\mu\nu} + O(1)$$

- Asymptotic vector fields act non-trivially on boundary values

$$V = \omega(x^\mu) r \partial_r \qquad \delta_V g^{(0)}_{\mu\nu} = 2\omega g^{(0)}_{\mu\nu}$$

Weyl transformation.

- Formally doesn't change the on-shell action, so Weyl transformations symmetries of boundary theory.

- Regularization requires cutting off action integral in the bulk.
- Regularization breaks Weyl invariance.
- Only divergent terms can spoil Weyl invariance
- Divergent terms are local and can be removed by counterterm
- => Weyl invariance violated at most by a local integral, the Weyl anomaly

- Familiar relativistic example: 2D CFT

- Partition function almost invariant,

$$Z[e^{2\omega}g_{\mu\nu}] = Z[g_{\mu\nu}]\exp\left(iC\int d^2x\sqrt{g}(\frac{1}{2}(\partial\omega)^2 + \omega R)\right)$$

  C proportional to central charge.

- More generally, a CFT is a theory invariant under Weyl transformations, up to an anomaly term: a local functional of the metric and the Weyl parameter.

- Generalize to locally asymptotically Lifshitz space:

$$N(t,\vec{x},r) = r^z N^{(0)}(t,\vec{x}) + \cdots$$

$$N_i(t,\vec{x},r) = r^2 N_i^{(0)}(t,\vec{x}) + \cdots$$

$$g_{ij}(t,\vec{x},r) = r^2 g_{ij}^{(0)}(t,\vec{x}) + \cdots$$

- Then under the vector V

$$\delta N^{(0)} = z\omega N^{(0)} \quad \delta N_i^{(0)} = 2\omega N_i^{(0)} \quad \delta g_{ij}^{(0)} = 2\omega g_{ij}^{(0)}$$

- Precisely what we call "anisotropic Weyl transformations"

Anisotropic CFTs

- Anisotropic Conformal Field Theory: partition function transforms as a local integral under anisotropic Weyl transformations

$$\delta Z[N, N_i, g_{ij}] = i\left(\int dt d^D x N\sqrt{g}\omega\mathcal{A}\right) Z[N, N_i, g_{ij}]$$

$$\delta N = z\omega N \quad \delta N_i = 2\omega N_i \quad \delta g_{ij} = 2\omega g_{ij}$$

- A is the (infinitesimal) anomaly density

- Dual field theory to HL gravity on Lifshitz space is an anisotropic conformal theory

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$$A_{\rm ct}[N, N_i, g_{ij}; \sigma] = Q S_{\rm ct}$$

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- Precisely the possible action terms in z=D=2 conformal HL gravity

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- Anomaly provides a simple, universal quantity to compare proposed dual theories

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$$\mathcal{L}_{\rm div} = \sum_{\Delta > 0} \mathcal{L}^{(\Delta)} r^{\Delta} + \tilde{\mathcal{L}}^{(0)} \log r$$

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$$c_1 = \frac{1}{2\kappa^2} \qquad c_2 = 0$$

Anomaly from HL Gravity

- Same procedure works for HL gravity
- Using IR limit of non-projectable theory,

$$c_1 = \frac{1}{2\kappa^2} \qquad c_2 = \frac{\beta}{48\kappa^2}$$

- Second central charge is non-vanishing!

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- Has both anomaly terms non-vanishing.

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- Obtain condition on λ for good spectrum:

$$\lambda \geq 1 \quad \text{or} \quad \lambda \leq \frac{(z-D)^2 + 4D}{(D+z)^2}$$

- For case z=D, this is $D\lambda \leq 1$
- D+2 dimensions => in flat space $(D+1)\lambda \leq 1$
- => Like with tachyonic masses in AdS, unitarity bounds in Lifshitz space allow a wider range of parameters than in flat space
- For $1/(D+1) \leq \lambda \leq 1/D$ the scalar graviton seems to allow two boundary conditions => effect on boundary theory?

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  - Black hole solutions

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- Strongly coupled dynamics in theories without Lorentz invariance
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- May provide diagnostics of HL gravity as a sensible quantum theory

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- Computations in Horava-Lifshitz gravity on give sensible holographic renormalization

Conclusions

- Holography in Lifshitz space gives rise to anisotropic conformal field theories
- Lifshitz space a vacuum solution of HL gravity => most natural embedding of anisotropy in holographic context
- Computations in Horava-Lifshitz gravity on give sensible holographic renormalization
- Possible applications and various interesting questions

Thank you!