FEATURE

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Research Area : Theoretical Physics

String Theory: Genetic Code of the Cosmos

The only known consistent description of matter and gravity at the quantum level is known as string theory. Fundamentally, string theory dictates the laws of physics in a very simple way: When particles are examined at the shortest distances, they are replaced by tiny strings. Strings replace both the massive particles constituting matter -- electrons, quarks, and Higgs bosons -- and the massless particles that transmit forces between the particles -- photons, gluons, and gravitons.

In particle theory, the properties of the particle in isolation -- its mass or its angular momentum, for instance -- can be considered separately from the properties of the particle's interaction with other particles or forces. A particle in isolation is represented by a line in a Feynman diagram. The particle's interaction with a force carrier is a vertex in the Feynman diagram where the line meets other lines. The types of such vertices, and the probabilities of their appearance, are arbitrary in particle theory.

The rules in string theory are different. Force carriers and matter particles are all different configurations of the same type of string. After one specifies the basic properties of the string such as its tension, and the shape of the space it moves in, the properties of matter particles and forces are uniquely determined by those choices. Instead of lines with sharp junctures between them, the splitting and joining of strings is described by a smooth two-dimensional surface traced out in space and time. Isolated motion and interaction are merged into a single smooth event, so the interactions of particles are therefore dictated by the same dynamics as are the properties of the particles propagating in isolation.

The consistency conditions for this equivalence are stringent but well-defined, and solution to the conditions is known as a conformal field theory (CFT). A CFT setting the rules for string propagation is sometimes also called a "string vacuum". The CFT is called a vacuum because it specifies not a particular configuration of particles, but a configuration of empty space before any particles are even added.

Vacua from Calabi-Yau Threefolds

Each distinct vacuum has different particle masses and different forces and interactions between particles. When the gravitational force is very weak, the physical consistency conditions of a CFT become identical with the mathematical consistency conditions for a very special type of geometric shape called a Calabi-Yau 3-manifold, or Calabi-Yau threefold (CY3).

A Calabi-Yau threefold can be thought of as defining extra dimensions of space, curled up very small, or "compactified". Calabi-Yau compactification was discovered by Philip Candelas, Gary Horowitz,

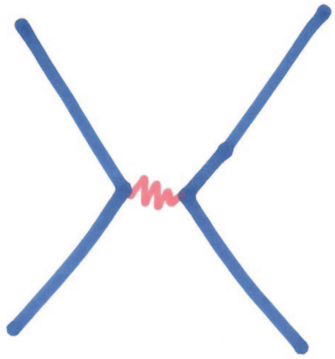

Figure 1a. A Feynman diagram describing interactions of point particles.

Figure 1b. In string theory, a Feynman diagram is replaced by a smooth string trajectory.

Andrew Strominger, and Edward Witten in 1984, a discovery that helped to bring on the first superstring revolution.

One Theory, Many Vacua

CY3 geometries occur in at least millions of different types, possibly even an infinite number. Enumerating the full set of possibilities for CY3s is an unsolved problem, of great interest to mathematicians in its own right.

Moreover, the classification and enumeration of possible consistent string theories involves CY3 geometries only as a small and simple piece of a much larger and more elaborate puzzle. In addition to the shape of the CY3 itself, a string vacuum is characterized by the distribution of certain energies and extended objects inside the CY3, known as fluxes and branes. These branes and fluxes wrap themselves around various closed shapes, called homology cycles, embedded inside the CY3. Homology cycles can be thought of as two-spheres or three-spheres in the space. The numbers of homology cycles of a given type of CY3 are called its Hodge numbers.

Because each homology cycle gives Nature independent choices for how many branes and how much flux to wrap on it, the number of possibilities for a vacuum configuration grows exponentially with the Hodge numbers. The largest theoretically possible Hodge numbers for a CY3 are not known, but some known examples already have Hodge numbers that are quite large, around 500 or so. This implies that string theory may have as many as 10^{500} vacua that can be described with various choices of branes and fluxes on homology cycles of a CY3.

Thus string theory has a vast number of different vacua. Only one string vacuum can describe dynamics of the world we inhabit, where the rules of the Standard Model of particle physics have prevailed in experiments so far. Most of the other vacua are inevitably very different from our own world. That is to say, in our own world, the charge of the electron is 1.6×10^{-19} Coulombs, and not some other value; the recently discovered Higgs boson has a mass of 125.5×10^{9} electron-Volts and is not heavier or lighter than that; et cetera. The electron charge, Higgs boson mass, and other such quantities vary wildly over the set of string vacua, distinguishing the overwhelming majority of possible Universes from our own.

Nonetheless, it is important to understand the entire set of all vacua, rather than just the one we live in. One reason is that we theorists have not yet identified the particular vacuum describing our world, the particular Calabi-Yau manifold or CFT that produces the dynamics of the Standard Model. It may be impossible to do so without understanding string vacua more

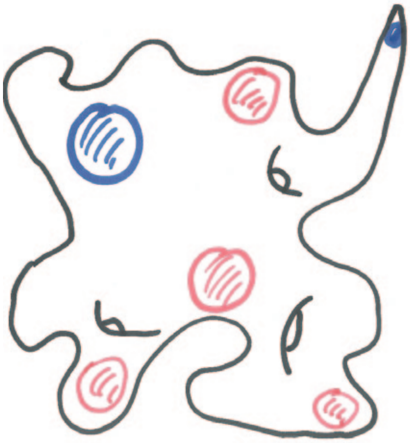

Figure 2. Calabi-Yau manifolds are partially characterized by the number of distinct closed two-spheres (blue) and three-spheres (red) in the geometry. The Calabi-Yau shown here has two two-spheres and four three-spheres.

systematically than we do at present.

Also, cosmological considerations suggest all the various vacua are probably realized in one region of space-time or another, in island-Universes connected tenuously to one another and accelerating rapidly apart. If other vacua are not only theoretical possibilities but are actually realized elsewhere in the cosmos, then some information about these other island-Universes may be accessible experimentally, possibly through patterns in the cosmic microwave background radiation.

The Holographic Bound as a Limiting Principle

Since possible types of CY3 are not classified or enumerated yet, how can we know where the complication stops? Could there be CY3s waiting to be discovered that have Hodge numbers hugely greater than 500, whose combinatorics of branes and fluxes would lead to unfathomably more complex vacua? In order to survey the range of possible vacua string theory allows, it is important to understand what principle limits the complexity of a Calabi-Yau threefold, or a string vacuum more generally.

Progress on understanding this issue has involved developments from the seemingly unrelated direction of black holes. In the 1970's, Steven Hawking discovered that black holes absorb coherent quantum information and turn it into seemingly random statistical noise by emitting thermal radiation. Since this discovery, the quantum mechanics of black holes has been an area providing important insight into the information-theoretic properties of quantum gravity.

In the early 90's, theorists led by Gerard 't Hooft and Leonard Susskind, deduced a fundamental rule known as the holographic principle. The holographic principle states that the entirety of quantum mechanical information in any gravitational system in any region of space, can always be stored on the surface of that region rather than in the interior. And furthermore, 't Hooft and Susskind postulated that the information on the surface could be packed no more tightly than one "bit" (B[inary dig]IT, a 1 or 0) per 10^{-70} square meters. This size is the square of the distance scale known as the Planck length, at which quantum gravity dominates over all other forces.

The holographic principle was originally proposed and advocated based on logical deduction using thought experiments, rather than inferred from actual experiments, or even from any calculation within a detailed model. 't Hooft and Susskind's arguments were independent of string theory. However, the holographic principle gained wide acceptance when realized in concrete form in string theory through the work of Juan Maldacena in 1997. The holographic principle is now widely accepted and believed to apply

Figure 3a. The auther enjoys an evening in 和室 (a Japanese room). Gravity holds him down on the 畳 (tatami mat), and in turn holds the 畳 down on the floor.

Figure 3b. A mathematically equivalent holographic theory simulates the events of the evening. The data of the configuration of the room at each moment in time is stored on the (two-dimensional) walls of the room.

to any consistent theory of quantum gravity, including string theory in particular.

The holographic principle is not itself merely a model or a particular theory, but a logical principle is applicable to all possible quantum theories containing gravity -- no matter what might be the content of the rest of the theory, so long as it is relativistic and quantum mechanical. For any relativistic physical theory of any type of objects with any type of interactions or laws of motion, in any number of space dimensions, the holographic principle must apply if quantum mechanics and gravity are both part of the model.

To understand the implications of the holographic principle, let us apply it (if you don't object!) to your own life. In our familiar Universe of three space dimensions, holography tells us that all physical events of the room in which you are sitting and reading this article, are simulated in real time by an equivalent quantum system living on the walls of your room: Yes, your pleasant evening of coffee and music in your lovely 和室 (*WASHITSU*, a Japanese room) was not real, but simulated by a very thin quantum computer covering the wall!

The most important implication is that the quantum mechanical information contained in the room can never be too large -- it cannot exceed the fundamental "holographic bound". This sets a fundamental limitation on the complexity of the activities you can perform this evening. If you are cooking dinner, the holographic principle limits the number of recipes that can be packed into a cookbook in your kitchen. If you are listening to your MP3 player, then your playlist cannot have more songs than 10^{70} times the area of the walls of your room, in units of meters squared. This is because your MP3 player and your cookbook are not real, but are themselves information imprinted on a hard drive on the walls, with only one bit of storage capacity per Planck length.

This limitation is fundamental. Even if you get bored later in the evening, you cannot go to the Apple Store and get an upgrade to double the memory on the invisible hard drive on your wall: The holographic principle states that you would need to double the size of your wall instead.

Since the logic of the holographic principle is based on universally applicable arguments, holography may be used to limit the Hodge numbers in any possible realization of string theory. The argument is simple: The fluxes in Calabi-Yau manifolds carry information, namely information about which two-cycle or three-cycle they are threaded through. Thus a single particle of flux necessarily carries information, and the larger the number of spheres in the Calabi-Yau, the more choices the flux has in how to orient itself: Hodge numbers determine the amount of quantum

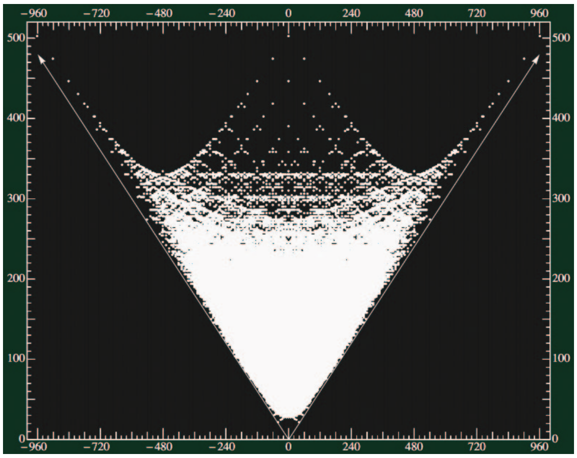

Figure 4. A plot of the properties of Calabi-Yau manifolds, showing the sum of Hodge numbers on the vertical axis. (The horizontal axis shows the difference of the two Hodge numbers.) Some yet-unknown principle of mathematics seems to limit this combined number to about 500. Image credit: P. Candelas et al., http://arxiv.org/abs/1207.4792

mechanical information carried by a single flux particle.

Hypothetically, if some inventive geometer were to find Calabi-Yau threefolds with Hodge numbers that were to be too large, eventually the quantum mechanical information carried by a single flux particle in your room would exceed the amount of information that the walls of your room could store holographically. This would violate the holographic principle, and therefore it cannot happen. We can conclude that the Hodge numbers of a Calabi-Yau should not be too large.

Recent work by myself and others has made this idea of a holographic bound more quantitative. A paper of my own in 2009 used elementary calculus and algebra to analyze a general consistency condition of CFT known as modular invariance, and to interpret that consistency condition using Maldacena's holographic correspondence. Translated using Maldacena’s rules, the consistency condition becomes a universal limitation on masses of particles in a string vacuum. Then in a joint paper with Cornelius Schmidt-Colinet (Kavli IPMU), we extended this technique to derive a limitation on the number of types of massless particle (such as the flux particles that are counted by Hodge numbers of a CY3). Using additional information following from the assumption of supersymmetry, the bound on massless particle species was improved dramatically in 2012 by Christoph Keller (Caltech) and Hirosi Ooguri (Caltech and Kavli IPMU).

Under certain technical assumptions, the theoretical holographic bound on the total Hodge numbers of a CY3 comes to about $e^{2\pi} = 535.4\cdots$, with small corrections to that value that do not change the order of magnitude. Although no rigorous universal bound is known to mathematicians, this value is strikingly close to the largest total Hodge numbers produced in any concrete construction of a Calabi-Yau threefold, which is around 500. It is striking that physical reasoning brings us so close to explaining data from pure mathematics that mathematicians themselves cannot yet explain.

Conclusion

The holographic principle is the genetic code of quantum gravity, and a complex jungle of theories realizes that underlying genetic code in myriad ways. We are now beginning to understand how the holographic DNA of string theory shapes the family resemblances among the flora and fauna of this vast ecosystem.