Strings, matrices, gauge theories, plane waves (and cosmology)

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(with 2 disclaimers)

Waves

$$ds^2 = -2dX^+dX^- - F(X^+)\sum_{i=1}^{d}(X^i)^2(dX^+)^2 + \sum_{i=1}^{d}(dX^i)^2.$$

- Strong gravitational wave
- Arbitrary profile
- Lack of interaction between “gravitons” following parallel courses
- The metric (Brinkmann form) can be made independent of the transverse coordinates (Rosen form)
- Manifest propagating wave-front (but Rosen coordinates tend to suffer from singularities)
- Dilaton (or other matter) fields to compensate for the curvature

Waves

$$ds^2 = -2dX^+dX^- - F(X^+)\sum_{i=1}^{d}(X^i)^2(dX^+)^2 + \sum_{i=1}^{d}(dX^i)^2.$$

$$F(X^+) \sim \frac{1}{(X^+)^2}$$

- Why bother? $\rightarrow$ tractable time-dependent singularities in quantum gravity
- Light-like singularities (pity!)
- Arise as Penrose limits of Friedmann-type cosmologies, etc
- String theory action quadratic in the light-cone gauge
- Supergravity equations of motion remain uncorrected (essential!)
- Matrix theory description can be given

Strings

- Background consistency requirements in string theory and related approaches to quantum gravity (cf. Papadopoulos-Russo-Tseytlin)
- Unclear *a priori* what conditions to impose at $X^+ = 0$
- Resolving the singularity makes background consistency automatically satisfied
- Many ways to resolve!
- Demand **NO** additional dimensionful scales introduced
- $\frac{1}{(X^+)^2} \to \frac{1}{\epsilon^2}\Omega(X^+/\epsilon)$ with $\Omega(\eta) = \frac{k}{\eta^2} + O\left(\frac{1}{\eta^b}\right)$
- Does the $\to 0$ limit exist?

Strings

Worldsheet Hamiltonian from bosonic part of the action

$$I = -\frac{1}{4\pi\alpha'}\int \mathrm{d}t \int_0^{2\pi} \mathrm{d}\sigma \sqrt{-g}\left(g^{ab} G_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu} - \frac{1}{2}\alpha' R^{(2)}\Phi\right) + \ldots$$

Choose lightcone gauge ($X^{+} = 2\alpha' p^{+} t$)
and Fourier transform the σ coordinate

$\Rightarrow$ Decoupled set of Hamiltonians (for each mode n)

$$H_{in} = \frac{(P_{in})^2}{2} + \left(n^2 + \frac{\lambda}{\epsilon^2}\Omega(t/\epsilon)\right)\frac{\left(X_n^i\right)^2}{2}$$

Strings

WKB exact:

$$\phi(t; X_n^i) \sim \prod_n \frac{1}{\sqrt{\mathcal{C}(t_1, t_2)}} \times$$

$$\exp\left(-\frac{i}{2\mathcal{C}} \sum_{i=1}^{d} \partial_{t_1}\mathcal{C}(X_{1n}^i)^2 - \partial_t \mathcal{C}(X_n^i)^2 + 2X_{1n}^i X_n^i\right)$$

Coordinate i, mode n, initial conditions X_{1n}^i
"Classical equation of motion" for n-dependent Hamiltonians

$$\partial_t^2 \mathcal{C}(t_1, t) + \left(n^2 + \frac{\lambda}{\epsilon^2}\Omega(t/\epsilon)\right)\mathcal{C}(t_1, t) = 0$$

Specified initial conditions

$$\mathcal{C}(t_1, t)|_{t=t_1} = 0, \quad \partial_t \mathcal{C}(t_1, t)|_{t=t_1} = 1$$

Strings

- The singular limit can be analyzed, even though the equations of motion in the resolved space-time do not admit analytic solutions (an optimistic lesson)

Singular plane waves coming from Friedmann-like geometires:

- Individual modes propagate consistently whenever the center-of-mass propagates consistently
- However, if no extra scales introduced to the scale-invariant space-time, total excitation energy of the string diverges after the singularity crossing
- Dimensionful scales "buried" at the singularity needed
- Open for speculation on physical origin of such scales...

Matrices

- The background enjoys a light-like isometry (X^--translations)
- A variation of Seiberg's argument

(DLCQ$\rightarrow$IIA/M-theory-duality) gives a matrix theory description

- 10D version $\rightarrow$ matrix string theory (Craps; Blau, O'Laughlin, etc)

$$S_{BC} = \int d\tau d\sigma \,\mathrm{Tr}\left(-\frac{1}{4} g_{YM}^{-2} F_{\tau\sigma}^2 - \frac{1}{2}\left(D_\tau Z^a D_\tau Z^a - D_\sigma Z^a D_\sigma Z^a\right)\right.$$

$$\left. + \frac{1}{4} g_{YM}^2 [Z^a, Z^b][Z^a, Z^b] + \frac{1}{2} A_{ab}(\tau) Z^a Z^b\right)$$

$$g_{YM} = \frac{\exp[-\phi(y^+(\tau))]}{2\pi l_s g_s} = \frac{\tau^{-3b/2(b+1)}}{2\pi l_s g_s}.$$

Matrices

- Basically, a SYM-theory with a time-dependent coupling
- 11D-version: BFSS quantum mechanics with time-dependent prefactors inserted in the Lagrangian

- Early times: superficially, zero-coupling regime, but subtleties possible; singularity transition is currently analyzed (Blau, O'Laughlin)
- Late times: almost classical space-time (with low curvatures should emerge), related to adiabaticity of the time dependences in the Lagrangian (current work with Ben Craps, to appear very soon)

AdS/CFT

p-brane in a strong dilaton-gravity plane wave:

$$ds^2 \equiv e^{\phi/2} ds_E^2 = \left(1+e^{f(u)}\frac{R^{7-p}}{r^{7-p}}\right)^{1/2} dx_a^2$$
$$+\left(1+e^{f(u)}\frac{R^{7-p}}{r^{7-p}}\right)^{-1/2}\left[-2dudv+\ddot{f}(u)\,r^2\left(\frac{2}{9-p}-\frac{e^{f(u)}}{5-p}\frac{R^{7-p}}{r^{7-p}}\right)du^2+dy_\alpha^2\right],$$

$$\phi = f(u)+\frac{3-p}{4}\ln\left(1+e^{f(u)}\frac{R^{7-p}}{r^{7-p}}\right),$$

$$F_{uv\alpha_1\cdots\alpha_{p-1}a} = \frac{x^a}{r}\,e^{-f(u)}\,\frac{\partial}{\partial r}\left(1+e^{f(u)}\frac{R^{7-p}}{r^{7-p}}\right)^{-1}\epsilon_{\alpha_1\cdots\alpha_{p-1}}\left[\frac{1}{\sqrt{2}}\right]_{p=3} \qquad (p\leq 3),$$

$$F_{a_1\cdots a_{8-p}} = \frac{x^a}{r}\,e^{-f(u)}\,\frac{\partial}{\partial r}\left(1+e^{f(u)}\frac{R^{7-p}}{r^{7-p}}\right)\epsilon_{a_1\cdots a_{8-p}a}\left[\frac{1}{\sqrt{2}}\right]_{p=3} \qquad (p\geq 3).$$

(first solution with an arbitrary profile; previous work by Maeda, Ohta, Tanabe, Wakebe, etc)

AdS/CFT

- The near-horizon region is a time-dependent (possibly singular) space-time that should have a dual description in terms of Dp-branes in the same plane wave $\rightarrow$ time-dependent SYM theories
- Similar considerations in works by Das, Michelson, Narayan, Trivedi, but for a restricted class of plane-waves (for which the supergravity solution factorizes)
- Generalization more difficult, but potentially instructive...
- Deserves further investigation......