Observing Quantum Gravity in the Sky

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See arXiv:1007.0185

IPMU String Cosmology Workshop

October 4, 2010

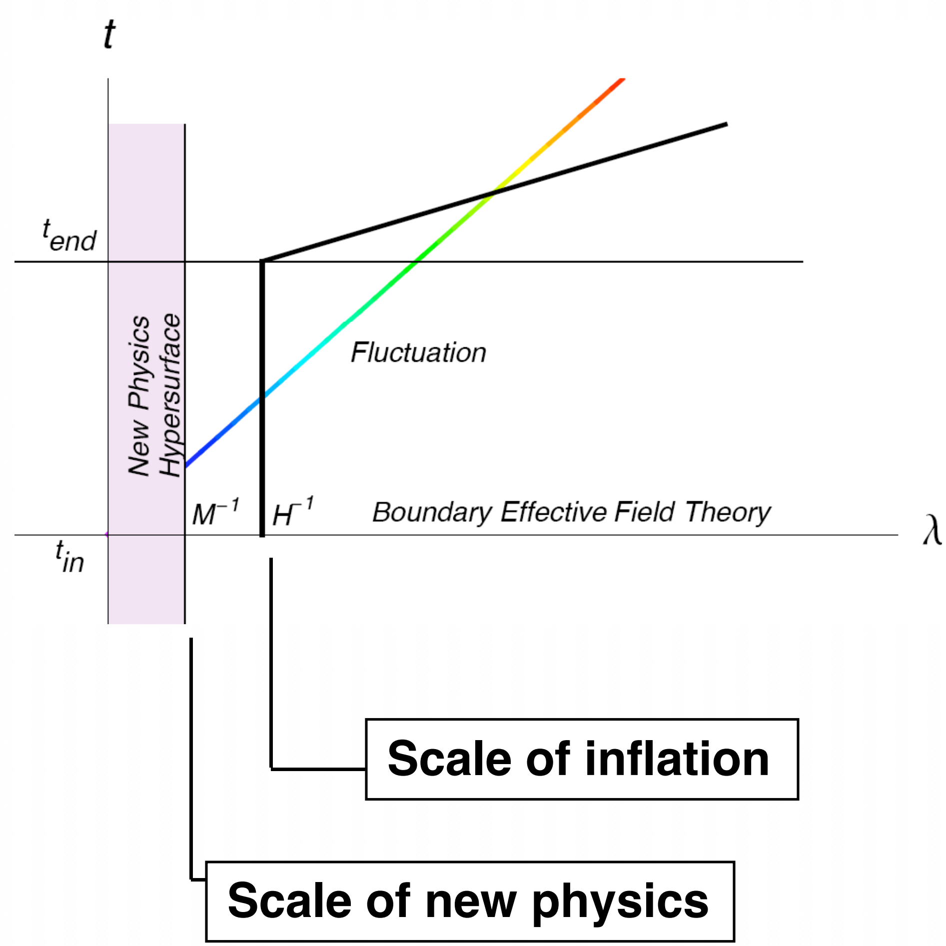
The Transplanckian Opportunity: Sensitivity to High Energies

Though the observed fluctuations currently have low energy, they were once very high:

$$p = k / a(t)$$

Thus CMB observables should be sensitive to new physics at some 'Transplanckian' scale *M*
(Brandenberger '99)

Dimensional analysis suggests that these modifications to low-energy observables must scale as $(H/M)^n$.



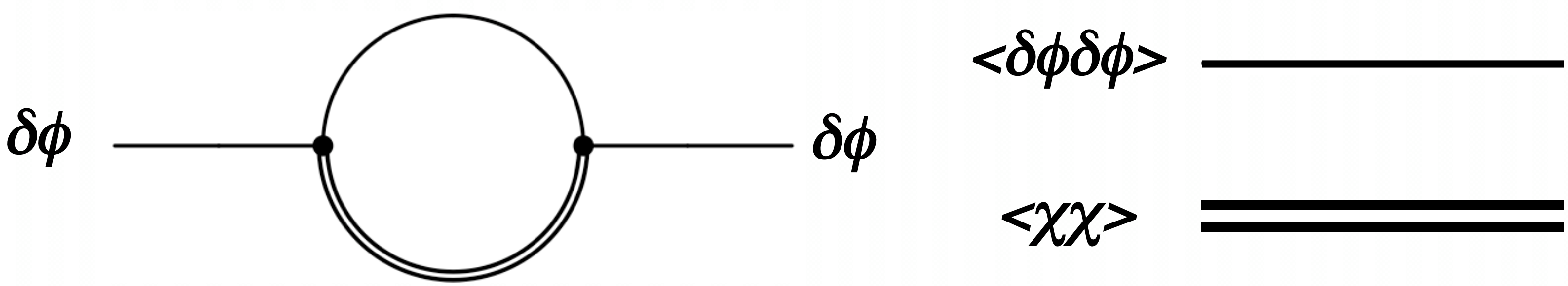
Prime Example: The Primordial Power Spectrum

- **The power spectrum is simply the 2-pt correlation function of inflaton field fluctuations:**

$$P_s(k) = \lim_{t\to\infty} \frac{k^3}{2\pi^2} \langle \delta\phi_{\mathbf{k}}(t)\delta\phi_{-\mathbf{k}}(t)\rangle \quad = A_s(k_\star)\left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1}$$

$$\mathrm{WMAP7:} \quad A_s = (2.43 \pm 0.11) \times 10^{-9}, \quad n_s = 0.963 \pm 0.012$$

- **(Naively) interpreting this as a propagator, we expect that it encodes high-energy physics via virtual heavy χ-particles:**



Inflaton Field Effective Action

- **Consider the effective action for ϕ:**

$$S_{eff}[\phi] = \int d^4p\ \phi(p)\phi(-p)\{p^2/2 + H^2/2 + c_0 H^2(H^2/M^2) + c_1 p^2(H^2/M^2)\ + \ldots\}.$$

- **The freezeout scale is *p=H*, thus the 2-pt function is**

$$\langle\phi(p)\phi(-p)\rangle|_{p=H} = H^2 + c_0 H^2(H^2/M^2) + c_1 H^2(H^2/M^2)$$

- **Only even powers of *p* are allowed in S_{eff}, so we have an expansion in $(H/M)^2$.**

  Which is disastrous, since $H/M \sim 0.01$

  (Brandenberger, Burgess, Cline, Danielsson, Easther, Greene, Lemieux, Kaloper, Kinney, Kleban, Lawrence, Martin, Schalm, Shenker, Shiu, v.d. Schaar)

A Possible Solution: Vacuum State Modification

- **Fortunately, there appears to be a loophole (Easther, Greene, Kinney, v.d. Schaar, Schalm, Shiu).**
- **Note that time-localized ('boundary') terms are one energy-dimension lower, and thus would scale only as *H/M*:**

$$S_{\text{boundary}} = \int d^4x \sqrt{g}\ m\phi^2 \delta(t - t_c).$$

- **A simple calculation shows that such boundary terms modify the inflaton vacuum state.**

- **Previous analysis assumed a Bunch-Davies vacuum,**

$$a_{\mathbf{k}}|0\rangle = 0,$$

whereas De Sitter space allows for more general vacua:

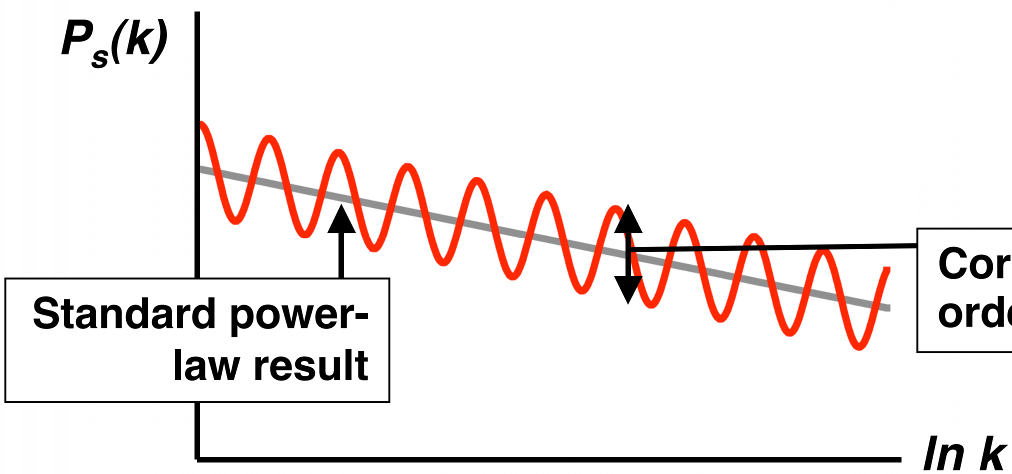
$$\left(a_{\mathbf{k}} + \beta_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}}\right)|\beta_{\mathbf{k}}\rangle = 0, \qquad |\beta_{\mathbf{k}}\rangle = \mathcal{N} \exp\left[-\beta_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}} a^{\dagger}_{\mathbf{k}}\right]|0\rangle.$$

Effect of Vacuum Choice on Power Spectrum

- The excited vacuum will add an oscillating term to the power spectrum,

$$\begin{aligned}
P_\varphi^\beta(k) &= \frac{k^3}{2\pi^2}\langle\beta_{\mathbf{k}}|\varphi_{\mathbf{k}}(0)\varphi_{-\mathbf{k}}(0)|\beta_{\mathbf{k}}\rangle \\
&= \frac{k^3}{2\pi^2}\langle\beta_{\mathbf{k}}|\left[U_{\mathbf{k}}(0)a_{\mathbf{k}} + U^*_{\mathbf{k}}(0)a^\dagger_{\mathbf{k}}\right]\left[U_{-\mathbf{k}}(0)a_{-\mathbf{k}} + U^*_{-\mathbf{k}}(0)a^\dagger_{-\mathbf{k}}\right]|\beta_{\mathbf{k}}\rangle \\
&\approx P_\varphi^{\mathrm{BD}}(k)\left(1+\beta_{\mathbf{k}}+\beta^*_{\mathbf{k}}\right) \\
&= P_\varphi^{\mathrm{BD}}(k)\left(1+2|\beta_{\mathbf{k}}|\sin\theta_{\mathbf{k}}\right), \qquad \beta_{\mathbf{k}} = |\beta_{\mathbf{k}}|e^{i\theta_{\mathbf{k}}}
\end{aligned}$$

- These ‘wiggles’ are expected to be a generic, model-independent feature of quantum gravity, with all new physics encoded in β.



But what is β?

Easther, Greene, Kinney, Shiu ‘01

Effective Action Construction Procedure

- We recently developed **the procedure to construct the effective description to represent high-energy physics.**
- Begin with inflating system,

$$S_{\rm inf}[\phi] = -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

and add (for example) Yukawa interactions to a heavy field χ :

$$S_{\rm new}[\varphi,\chi] \;=\; -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\chi)^2 + \frac{1}{2}M^2\chi^2 + \frac{g}{2}\varphi^2\chi\right]$$

- The power spectrum can then be computed using the in-in formalism:

$$P_\varphi(k) \;=\; \lim_{t\to\infty}\frac{k^3}{2\pi^2}\,\langle \mathbf{0}(t_0)|e^{i\int_{t_0}^t dt'\mathcal{H}(t')}|\varphi_{\mathbf{k}}(t)|^2 e^{-i\int_{t_0}^t dt''\mathcal{H}(t'')}|\mathbf{0}(t_0)\rangle$$

- Note that this can be interpreted as an in-out correlation using

$$\mathcal{S} \equiv S[\varphi_+,\chi_+] - S[\varphi_-,\chi_-]$$

(MGJ, Schalm '10)

Vacuum Construction Procedure

- **This suggests we should transform into a new field basis given by**

$$\bar{\varphi} \equiv (\varphi_+ + \varphi_-)/2, \qquad \Phi \equiv \varphi_+ - \varphi_-,$$
$$\bar{\chi} \equiv (\chi_+ + \chi_-)/2, \qquad \mathrm{X} \equiv \chi_+ - \chi_-$$

- **In this basis the action is now**

$$\mathcal{S}[\bar{\varphi}, \Phi, \bar{\chi}, \mathrm{X}] = -\int d^4x\sqrt{g}\left[\partial\bar{\varphi}\partial\Phi + \partial\bar{\chi}\partial\mathrm{X} + M^2\bar{\chi}\mathrm{X} \;+ g\bar{\chi}\bar{\varphi}\Phi + \frac{g}{2}\mathrm{X}\left(\bar{\varphi}^2 + \frac{\Phi^2}{4}\right)\right].$$

- **The correlations can now be evaluated using Green's and Wightman's functions for these fields:**

$$\begin{aligned} F_{\mathbf{k}}(\tau_1,\tau_2) &\equiv \langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\bar{\varphi}_{-\mathbf{k}}(\tau_2)\rangle \\ &= \mathrm{Re}\left[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)\right], \\ G^R_{\mathbf{k}}(\tau_1,\tau_2) &\equiv i\langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\Phi_{-\mathbf{k}}(\tau_2)\rangle \\ &= -2\theta(\tau_1-\tau_2)\mathrm{Im}\left[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)\right], \\ G^A_{\mathbf{k}}(\tau_1,\tau_2) &\equiv G^R_{\mathbf{k}}(\tau_2,\tau_1), \\ 0 &= \langle\Phi_{\mathbf{k}}(\tau_1)\Phi_{-\mathbf{k}}(\tau_2)\rangle \end{aligned}$$

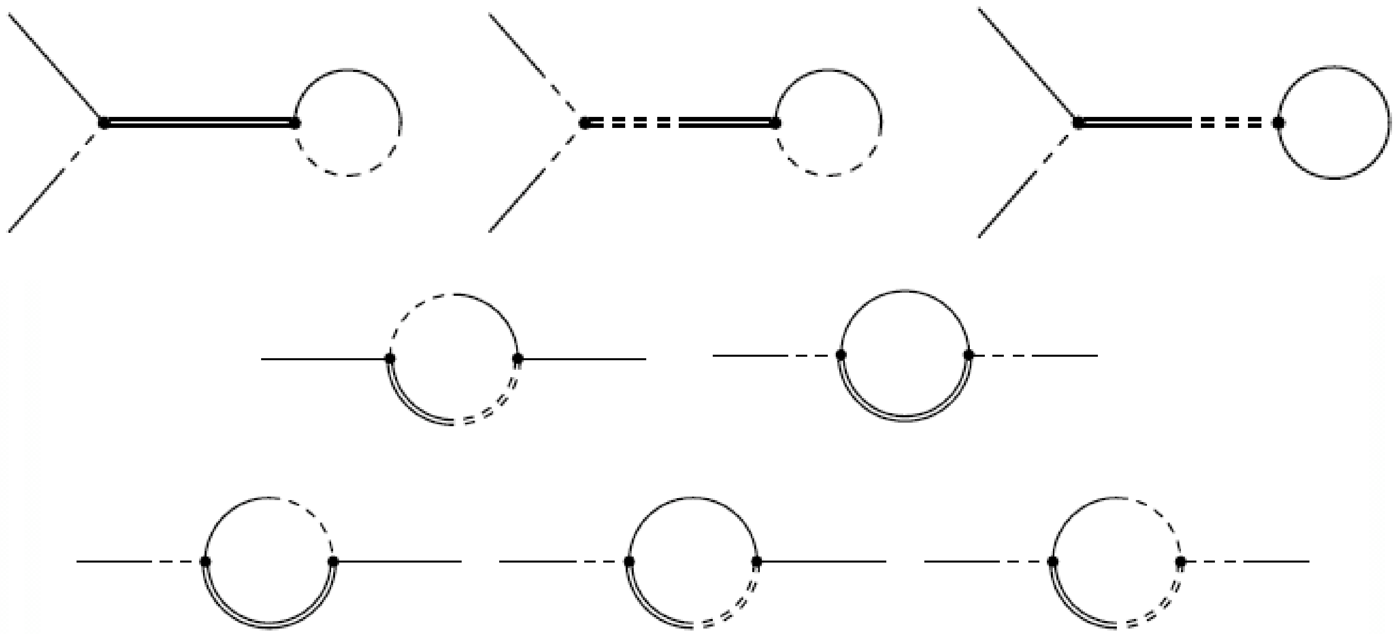
$$U_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}(1-ik\tau)\,e^{-ik\tau}$$

$$V_{\mathbf{k}}(\tau) \approx -\frac{H\tau\exp\left[-i\int^\tau d\tau'\sqrt{k^2+\frac{M^2}{H^2\tau'^2}}\right]}{\sqrt{2}\left(k^2+\frac{M^2}{H^2\tau^2}\right)^{1/4}}$$

(MGJ, Schalm '10)

Power Spectrum Corrections

- **2-pt correlation can then be computed using normal methods, producing Feynman diagrams:**



- **Which are significant? We need some approximations!**

(MGJ, Schalm ‘10)

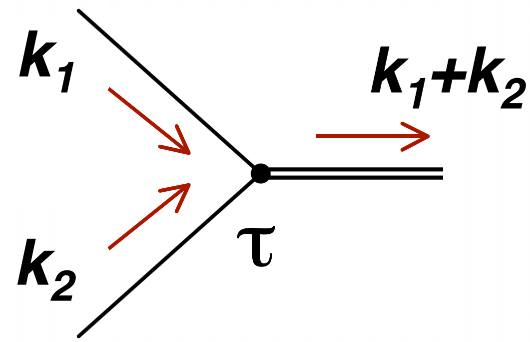
Power Spectrum Corrections

- **Each vertex is an integral over the time of interaction, and has the following form:**

$$\mathcal{A}_1(\mathbf{k}_1, \mathbf{k}_2) \equiv \int_{\tau_0}^{0} d\tau\ a^4(\tau) U_{\mathbf{k}_1}(\tau) U_{\mathbf{k}_2}(\tau) V^*_{-(\mathbf{k}_1+\mathbf{k}_2)}(\tau)$$

$$\approx -\frac{1}{2\sqrt{2k_1^3 k_2^3} H} \int_{\tau_0}^{0} \frac{d\tau}{\tau^3} \frac{(1 - ik_1\tau)(1 - ik_2\tau)}{\left(|\mathbf{k}_1 + \mathbf{k}_2|^2 + \frac{M^2}{H^2\tau^2}\right)^{1/4}}$$

$$\times \exp\left[-i(k_1 + k_2)\tau + i \int^{\tau} d\tau' \sqrt{|\mathbf{k}_1 + \mathbf{k}_2|^2 + \frac{M^2}{H^2\tau'^2}}\right].$$



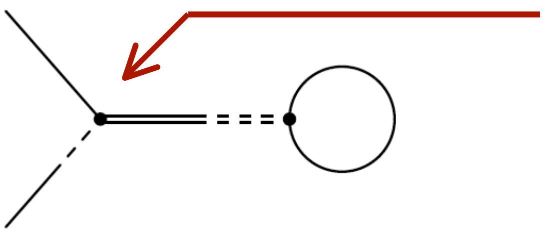
- **This admits a stationary phase approximation near the moment of energy-conservation,**

$$\tau_*^{-1} = -\frac{H}{M}\sqrt{2k_1 k_2(1 - \cos\theta)}, \qquad \cos\theta = \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2}.$$

- **The vertex (to leading order in *H/M*) is then simply**

$$\mathcal{A}_1(\mathbf{k}_1, \mathbf{k}_2) \approx -\frac{\sqrt{\pi i}}{2\sqrt{k_1 k_2}\,[2k_1 k_2(1 - \cos\theta)]^{1/4}\, H}\sqrt{\frac{H}{M}}\left[\frac{2M}{H}\left(k_1 + k_2 + \sqrt{2k_1 k_2(1 - \cos\theta)}\right)\right]^{-i\frac{M}{H}}$$

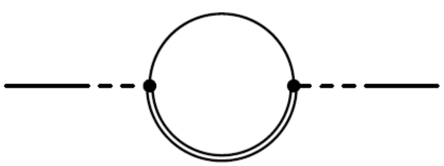
Power Spectrum Corrections



Boundary correction generated by stationary phase at $\tau = -M/2Hk$, when pair production occurs

$$\Delta P_\varphi^{(1)}(k) = \left(\frac{gH}{M}\right)^2 \frac{\sqrt{\pi}}{4(2\pi)} \left(|k\tau_0|^{-3/2} \ln\frac{\Lambda}{\mu} + \frac{1}{2}|k\tau_0|^{1/2}\frac{\Lambda^2}{k^2}\right) \sin\left[\frac{M}{H}\ln 2(k|\tau_0|) + \zeta\right]$$

This is exactly the form we expected (and desired)!



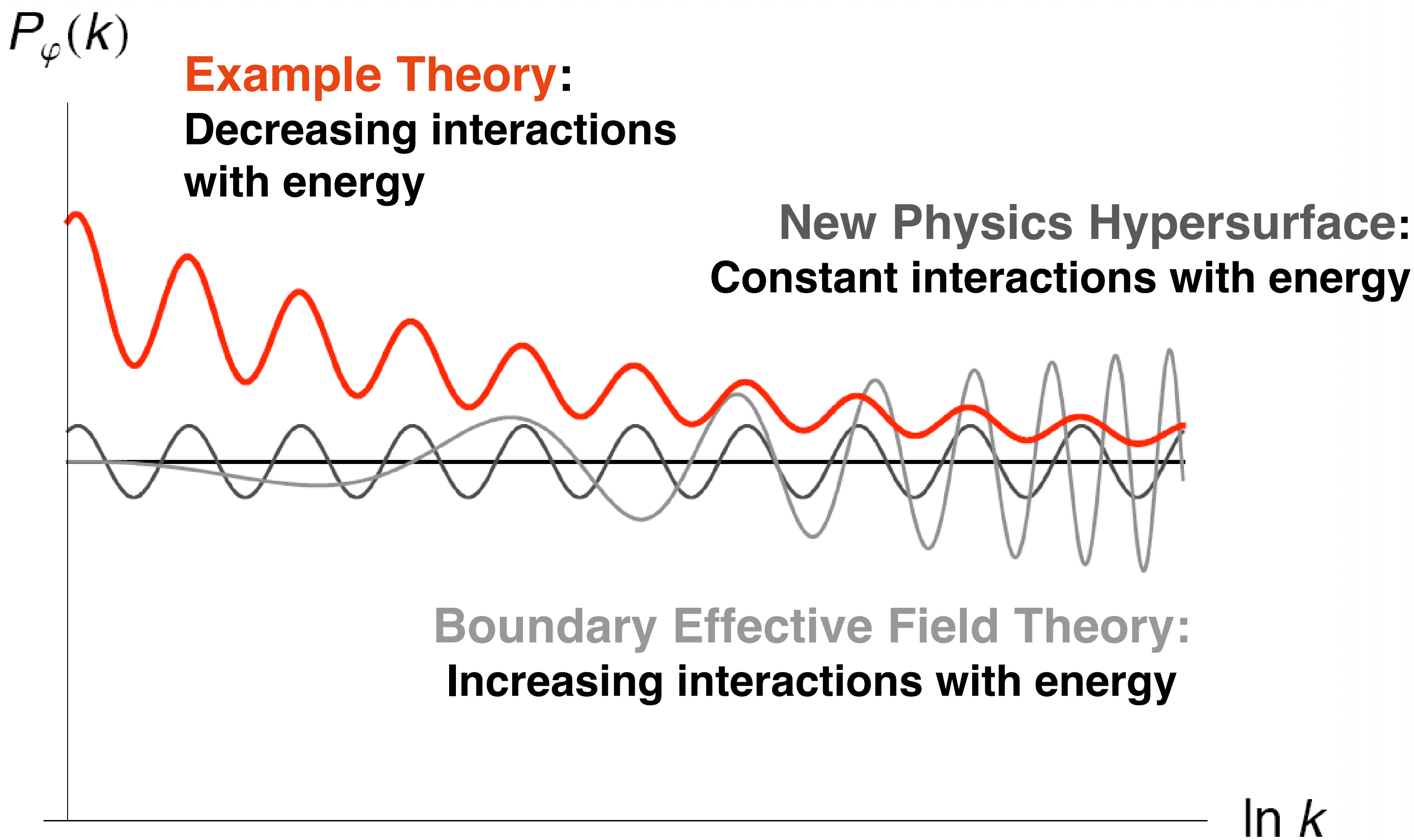
Bulk correction generated by virtual χ-exchange

$$\Delta P_\varphi^{(2)}(k) = \frac{g^2 H}{M} \frac{1}{12(2\pi)^3} \left(\frac{\Lambda}{k}\right)^{3/2}$$

(MGJ, Schalm '10)

This is a non-oscillating shift in the power spectrum

Power Spectrum Corrections



(MGJ, Schalm '10)

Low-Energy Effective Interactions

- **Most importantly, these corrections can be derived from inflaton-only interactions:**

$$\mathcal{S}_{\rm int}[\bar\varphi,\Phi] = -\int a^4(\tau_1)d\tau_1 a^4(\tau_2)d\tau_2 \prod_i \frac{d^3\mathbf{q}_i}{(2\pi)^3}(2\pi)^3\delta^3(\sum_i \mathbf{q}_i)$$

$$\Big[\frac{1}{2!}\boxed{\lambda_1(\mathbf{q}_i)}\bar\varphi^-_{\mathbf{q}_1}(\tau_1)\Phi^-_{\mathbf{q}_2}(\tau_1)\left(\bar\varphi^+_{\mathbf{q}_3}(\tau_2)\bar\varphi^+_{\mathbf{q}_4}(\tau_2)+\frac{1}{4}\Phi^+_{\mathbf{q}_3}(\tau_2)\Phi^+_{\mathbf{q}_4}(\tau_2)\right)$$

$$\times\delta(\tau_1-\tau_*^{\mathbf{q}_1,\mathbf{q}_2})\delta(\tau_2-\tau_*^{\mathbf{q}_3,\mathbf{q}_4})\theta(\tau_*^{\mathbf{q}_1,\mathbf{q}_2}-\tau_*^{\mathbf{q}_3,\mathbf{q}_4})-\text{c.c.}$$

These are determined by high-energy model

$$+\frac{1}{2!}\boxed{\lambda_2(\mathbf{q}_i,\tau_2)}\bar\varphi^-_{\mathbf{q}_1}(\tau_1)\Phi^-_{\mathbf{q}_2}(\tau_1)\bar\varphi^+_{\mathbf{q}_3}(\tau_2)\bar\varphi^-_{\mathbf{q}_4}(\tau_2)$$

$$\times\delta(\tau_1-\tau_*^{\mathbf{q}_1,\mathbf{q}_2})\theta(\tau_*^{\mathbf{q}_1,\mathbf{q}_2}-\tau_2)-\text{c.c.}$$

$$-i\boxed{\Gamma(\mathbf{q}_i)}\bar\varphi^-_{\mathbf{q}_1}(\tau_1)\Phi^-_{\mathbf{q}_2}(\tau_1)\bar\varphi^+_{\mathbf{q}_3}(\tau_2)\Phi^+_{\mathbf{q}_4}(\tau_2)$$

$$\times\delta(\tau_1-\tau_*^{\mathbf{q}_1,\mathbf{q}_2})\delta(\tau_2-\tau_*^{\mathbf{q}_3,\mathbf{q}_4})\theta(\tau_*^{\mathbf{q}_1,\mathbf{q}_2}-\tau_*^{\mathbf{q}_3,\mathbf{q}_4})\Big]$$

where

$$\bar\varphi^\pm_{\mathbf{k}} \equiv \frac{1}{2}\left(\bar\varphi_{\mathbf{k}} \pm \frac{i}{k}\dot{\bar\varphi}_{\mathbf{k}}\right), \qquad \Phi^\pm_{\mathbf{k}} \equiv \frac{1}{2}\left(\Phi_{\mathbf{k}} \pm \frac{i}{k}\dot\Phi_{\mathbf{k}}\right)$$

Observability?

- **We see that integrating out high energy physics produces low energy interactions, but an expanding background induces boundary terms**
- **These represent a modified vacuum, appearing in the power spectrum as oscillations**
- **But is this observable?**
- **We can see about four decades of comoving *k* in the CMB,**

$$k_{\rm min} \leq k_{\rm obs} \leq 10^4 \; k_{\rm min}$$

If $H/M_{\rm string} \sim 10^{-2}$ then we should see about 10^2 oscillations, just at the threshold of *Planck*'s sensitivity.

Scale of Inflation

- **The scale of inflation is directly proportional to *r*,**

$$V^{1/4} = 1.06 \times 10^{16}\,\mathrm{GeV}\left(\frac{r_\star}{0.01}\right)^{1/4}, \qquad r \equiv \frac{P_t}{P_s} = \frac{\left.\frac{2}{3\pi^2}\frac{V}{M_{\rm pl}^4}\right|_{k=aH}}{\left.\frac{1}{24\pi^2 M_{\rm pl}^4}\frac{V}{\epsilon}\right|_{k=aH}}$$

  Sensitivity to *r* ~ 0.01 is the goal of *CMBPol*
- **A detectably large tensor amplitude would demonstrate that inflation occurred at a very high energy scale, comparable to GUTs (Lyth '96; Baumann and McAllister '07)**
- **Since**

$$P_t \sim (H/M_{\rm pl})^2$$

  this also implies we should see Transplanckian effects in the power spectrum

Conclusion

- We now possess the theoretical tools to transform models of fundamental physics into low-energy interactions in an expanding background
- These will produce specific corrections to the primordial power spectrum which could soon be experimentally detected
- Such transplanckian corrections would also imply the presence of a primordial tensor background, detectable by e.g. *CMBPol*