Theoretical Expectations for Neutrino Parameters: Mixing Schemes, Models and Deviations

Max-Planck-Institut für Kernphysik

Werner Rodejohann
(MPIK, Heidelberg)

IPMU, 09/11/10

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

What to talk about?

- "Neutrino oscillations": `Jose`
- "Neutrino models with flavor symmetry": `Morimitsu`

$\Rightarrow$ try to cover the "middle"

Outline

- Current Status of PMNS
- Tri-bimaximal Mixing (TBM)
- Alternatives to TBM
- Importance of θ_{13}
- Deviating Neutrino Mixing Schemes: example $|U_{e3}| \simeq 0.1$ from zero
- "Expectation" for Non-Standard Neutrino Physics
- application to MINOS anomaly
- Importance of neutrino mass observables

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) Matrix

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2\theta_{23} = \tfrac{1}{2}) \quad (\sin^2\theta_{13} = 0) \quad (\sin^2\theta_{12} = \tfrac{1}{3})$$

$$= \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

`Harrison, Perkins, Scott` (2002)

Mixing close to TBM $\Rightarrow$ expand around it

$$U = R_{23}\left(-\frac{\pi}{4}\right)\, R_{23}(\epsilon_{23})\, \tilde{R}_{13}(\epsilon_{13};\delta)\, R_{12}(\epsilon_{12})\, R_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right)$$

Only one small ϵ_{ij} responsible for deviation of (and only of) θ_{ij} from $\theta_{ij}^{\mathrm{TBM}}$

$$\begin{aligned}
\sin^2\theta_{12} &= \frac{1}{3}\left(\cos\epsilon_{12} + \sqrt{2}\,\sin\epsilon_{12}\right)^2 \\
&\simeq \frac{1}{3} + \frac{2\sqrt{2}}{3}\,\epsilon_{12} + \frac{1}{3}\,\epsilon_{12}^2 \\
\sin^2\theta_{23} &= \frac{1}{2} + \sin\epsilon_{23}\,\cos\epsilon_{23} \simeq \frac{1}{2} + \epsilon_{23} \\
U_{e3} &= \sin\epsilon_{13}\, e^{-i\delta}
\end{aligned}$$

"Triminimal Parametrization"

Pakvasa, W.R., Weiler, PRL **100**, 111801 (2008)

Mass Matrix

Special case of μ–τ symmetry

$$(m_\nu)_{\rm TBM} = U_{\rm TBM}^* \, m_\nu^{\rm diag} \, U_{\rm TBM}^\dagger = \left(\begin{array}{ccc} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{array} \right)$$

where

$$A = \frac{1}{3} \left(2\, m_1 + m_2 \, e^{-2i\alpha} \right) \, , \quad B = \frac{1}{3} \left(m_2 \, e^{-2i\alpha} - m_1 \right) \, , \quad D = m_3 \, e^{-2i\beta}$$

- $m_{ee} + m_{e\mu} + m_{e\tau} = m_{\mu e} + m_{\mu\mu} + m_{\mu\tau} = m_{\tau e} + m_{\tau\mu} + m_{\tau\tau}$
- masses independent on mixing (i.e., not $V_{us} = \sqrt{m_d/m_s}$)

Correlations between mass matrix elements $\leftrightarrow$ flavor symmetries:

A_4, $\Delta(27)$, $\Sigma(81)$, T', $\mathcal{PSL}_2(7)$, $SU(3)$,...

Comment on Model Zoo

Example: 58 models based on A_4 leading to tri-bimaximal mixing:

Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	-	[1–11] [12]#
A2				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[13, 14]
A3				$\underline{1}, \underline{3}$	[15]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[4, 16–21]# [22, 23]* [24–35]
B2				$\underline{1}, \underline{3}$	[36]#
C1	$\underline{3}$	$\underline{3}$	-	-	[2]
C2				$\underline{1}$	[37, 38] [39]#
C3				$\underline{1}, \underline{3}$	[40]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[41]
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	-	[42, 43]* [44, 45]
D2				$\underline{1}$	[46] [47]*
D3				$\underline{1}'$	[48]*
D4				$\underline{1}', \underline{3}$	[49]*
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[50, 51]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[52]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[53]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[54]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[55]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[56, 57]
K	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	$\underline{1}$	[58]*
L	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}$	-	[59]*

Barry, W.R., PRD **81**, 093002 (2010)

	Bari	GM-I	GM-II	STV	TBM
$\sin\theta_{13}$	$0.126^{+0.053}_{-0.049}$	$0.097^{+0.053}_{-0.047}$	$0.089^{+0.051}_{-0.057}$	$0.114^{+0.047}_{-0.063}$	0
$\sin^2\theta_{23}$	$0.466^{+0.073}_{-0.058}$	$0.462^{+0.082}_{-0.050}$	$0.462^{+0.082}_{-0.050}$	$0.50^{+0.07}_{-0.06}$	0.5
$\sin^2\theta_{12}$	$0.312^{+0.019}_{-0.018}$	$0.319^{+0.016}_{-0.016}$	$0.321^{+0.016}_{-0.016}$	$0.318^{+0.019}_{-0.016}$	0.333

all groups find deviations from one or more TBM values

Taking Bari results as example:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.731 & -0.683 \\ 0 & -0.683 & 0.731 \end{pmatrix} \begin{pmatrix} 0.992 & 0 & 0.126\, e^{-i\delta} \\ 0 & 1 & 0 \\ -0.126\, e^{i\delta} & 0 & 0.992 \end{pmatrix} \begin{pmatrix} 0.830 & 0.559 & 0 \\ -0.559 & 0.830 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

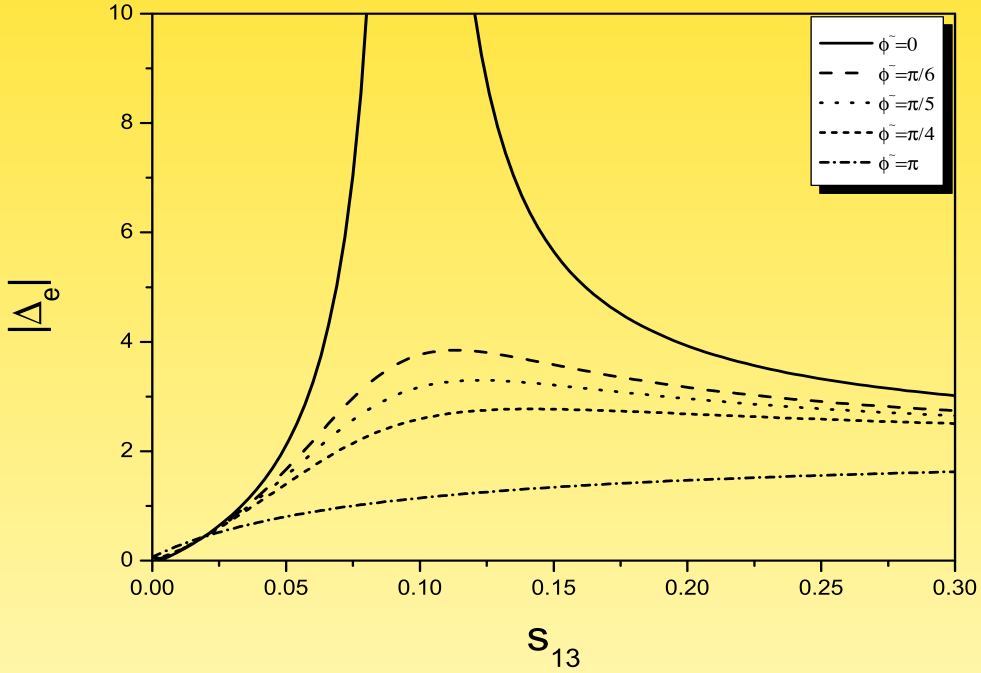
$$= \begin{pmatrix} 0.823 & 0.554 & 0.126\, e^{-i\delta} \\ -0.408 - 0.072\, e^{i\delta} & 0.606 - 0.048\, e^{i\delta} & 0.677 \\ 0.381 - 0.077\, e^{i\delta} & -0.566 - 0.052\, e^{i\delta} & 0.731 \end{pmatrix}$$

`Abbas, Smirnov, PRD` **`82`**`, 013008 (2010)`:

deviations from $m_\nu^{\rm TBM}$ possible: "TBM accidental?"

Define

$$\Delta_e = \frac{(m_\nu)_{e\mu} - (m_\nu)_{e\tau}}{(m_\nu)_{e\mu}}\ , \quad \Delta_\mu = \frac{(m_\nu)_{\mu\mu} - (m_\nu)_{\tau\tau}}{(m_\nu)_{\tau\tau}}, \quad \Delta_\Sigma$$



Abbas, Smirnov, PRD **82**, 013008 (2010)

Alternatives to TBM

- μ–τ symmetry $(Z_2, D_4, \ldots)$:

$$m_\nu = \begin{pmatrix} a & b & b \\ \cdot & d & e \\ \cdot & \cdot & d \end{pmatrix} \Rightarrow U_{e3} = 0,\ \theta_{23} = \pi/4$$

solar neutrino mixing unconstrained $(\theta_{12} = \mathcal{O}(1))$

Alternatives to TBM

- Golden Ratio φ_1 (A_5)

$$\boxed{\cot\theta_{12} = \varphi} \quad \Rightarrow \sin^2\theta_{12} = \frac{1}{1+\varphi^2} = \frac{2}{5+\sqrt{5}} \quad \simeq 0.276$$

(Datta, Ling, Ramond; Kajiyama, Raidal, Strumia; Everett, Stuart)

- Golden Ratio φ_2 (D_5)

$$\boxed{\cos\theta_{12} = \frac{\varphi}{2}} \quad \Rightarrow \sin^2\theta_{12} = \frac{1}{4}(3-\varphi) = \frac{5-\sqrt{5}}{8} \quad \simeq 0.345$$

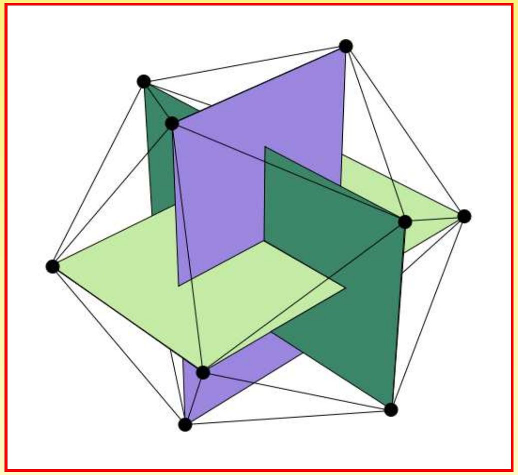
(W.R.; Adulpravitchai, Blum, W.R.)

Golden Ratio Prediction φ_1

$\boxed{\cot\theta_{12} = \varphi}$ or: $\tan 2\theta_{12} = 2$

can be generated by $m_\nu = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow Z_2 : \; S = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$

Model based on A_5 (isomorphic to rotational icosahedral symmetry group)?



Cartesian coordinates of its 12 vertices:

$(0, \pm 1, \pm\varphi)$

$(\pm 1, \pm\varphi, 0)$

$(\pm\varphi, 0, \pm 1)$

Golden Ratio Prediction φ_1

A_5 has irreps $\mathbf{1}$, $\mathbf{3}$, $\mathbf{3}'$, $\mathbf{4}$, $\mathbf{5}$

e.g., generators for triplet representation $\mathbf{3}$

$$S_3 = \frac{1}{2}\begin{pmatrix} -1 & \varphi & 1/\varphi \\ \varphi & 1/\varphi & 1 \\ 1/\varphi & 1 & -\varphi \end{pmatrix} \text{ and } T_3 = \frac{1}{2}\begin{pmatrix} 1 & \varphi & 1/\varphi \\ -\varphi & 1/\varphi & 1 \\ 1/\varphi & -1 & \varphi \end{pmatrix}$$

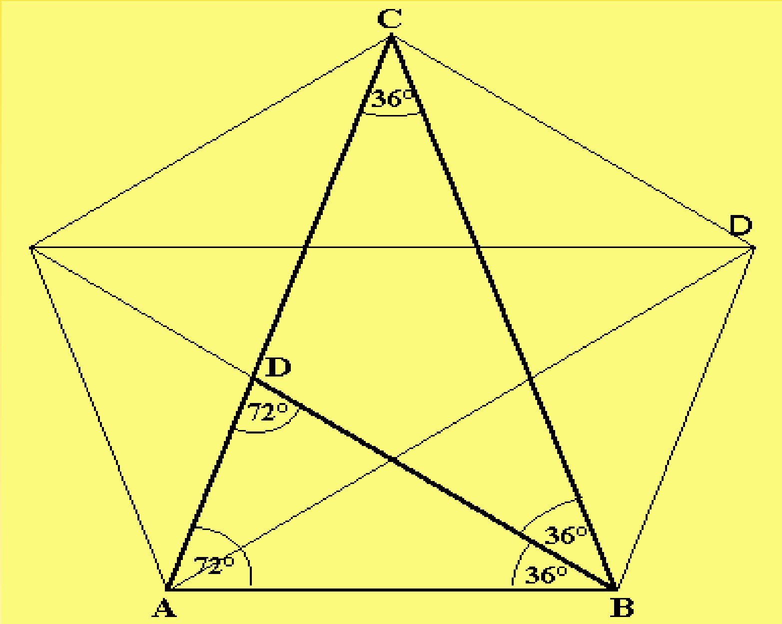
Everett, Stuart, PRD **79**, 085005 (2009)

Golden Ratio Prediction φ_2

$$\boxed{\cos\theta_{12} = \varphi/2} \quad \text{or: } \theta_{12} = \frac{\pi}{5}$$

$$\sin^2\theta_{12} = \sin^2\pi/5 = \tfrac{5-\sqrt{5}}{8} \simeq 0.345 \text{ (best-fit: 0.32)}$$

(W.R., PLB **671**, 267 (2009))

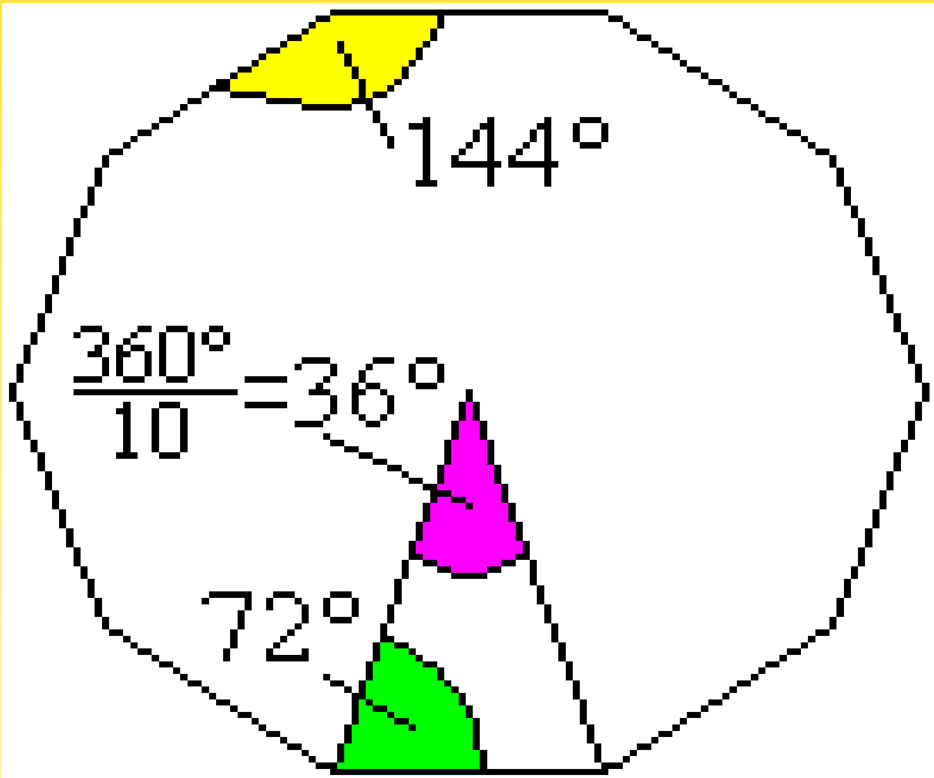


$$\overline{\mathrm{AD}} = \varphi\,\overline{\mathrm{AB}}$$

symmetry group of pentagon: D_5

Golden Ratio Prediction φ_2

symmetry group of decagon: D_{10}



Dihedral Groups

Blum, Hagedorn, Lindner, Hohenegger, PRD **77**, 076004 (2008):

D_n has several $\mathbf{2_j}$, generated by

$$A = \begin{pmatrix} e^{2\pi i \frac{j}{n}} & 0 \\ 0 & e^{-2\pi i \frac{j}{n}} \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and Z_2 is generated by

$$B\,A^k = \begin{pmatrix} 0 & e^{-2\pi i \frac{j}{n}\,k} \\ e^{2\pi i \frac{j}{n}\,k} & 0 \end{pmatrix}$$

Thus, break D_n such that m_ν invariant under $B\,A^{k_\nu}$ and m_ℓ under $B\,A^{k_\ell}$:

$$|U_{e1}|^2 = \left|\cos \pi \frac{j}{n} \, (k_\nu - k_\ell)\right|^2$$

Again, D_5 or D_{10} to obtain $\pi/5$

A Model based on D_{10}

Adulpravitchai, Blum, W.R., New J. Phys. **11**, 063026 (2009)

Field	$l_{1,2}$	l_3	$e^c_{1,2}$	e^c_3	$h_{u,d}$	σ^e	$\chi^e_{1,2}$	$\xi^e_{1,2}$	$\rho^e_{1,2}$	σ^ν	$\varphi^\nu_{1,2}$	$\chi^\nu_{1,2}$	$\xi^\nu_{1,2}$
D_{10}	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$
Z_5	ω	ω	ω^2	ω^2	1	ω^2	ω^2	ω^2	ω^2	ω^3	ω^3	ω^3	ω^3

A Model based on D_{10}

$$\begin{pmatrix} \langle \chi_1^e \rangle \\ \langle \chi_2^e \rangle \end{pmatrix} = v_e \begin{pmatrix} 1 \\ e^{\frac{2\pi i k}{5}} \end{pmatrix}, \begin{pmatrix} \langle \xi_1^e \rangle \\ \langle \xi_2^e \rangle \end{pmatrix} = w_e \begin{pmatrix} 1 \\ e^{\frac{3\pi i k}{5}} \end{pmatrix}, \begin{pmatrix} \langle \rho_1^e \rangle \\ \langle \rho_2^e \rangle \end{pmatrix} = z_e \begin{pmatrix} 1 \\ e^{\frac{4\pi i k}{5}} \end{pmatrix}$$

where k is an odd integer between 1 and 9, and

$$\begin{pmatrix} \langle \varphi_1^\nu \rangle \\ \langle \varphi_2^\nu \rangle \end{pmatrix} = v_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \chi_1^\nu \rangle \\ \langle \chi_2^\nu \rangle \end{pmatrix} = w_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \xi_1^\nu \rangle \\ \langle \xi_2^\nu \rangle \end{pmatrix} = z_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\langle \sigma^e \rangle = x_e, \langle \sigma^\nu \rangle = x_\nu$$

$$U_\ell = \mathrm{diag}(e^{-2i\Phi}, 1, e^{-i(\Phi+\delta)}) \begin{pmatrix} -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

where $\Phi = \frac{4\pi}{5}$

$$U_\nu = \begin{pmatrix} -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

- $\theta_{12} = \pi/5$
- vanishing U_{e3}
- in general non-maximal θ_{23}

VEV alignment

SUSY and "driving fields"

Field	ψ^{0e}	$\varphi^{0e}_{1,2}$	$\xi^{0e}_{1,2}$	$\psi^{0\nu}$	$\chi^{0\nu}_{1,2}$	$\xi^{0\nu}_{1,2}$
D_{10}	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$
Z_5	ω	ω	ω	ω^4	ω^4	ω^4

flavon superpotential $w_f = w_{f,e} + w_{f,\nu}$

flavor symmetry broken at high scale, thus minimize in supersymmetric limit determine supersymmetric minimum by setting F-terms of driving fields to zero:

$$
\begin{aligned}
\frac{\partial w_{f,e}}{\partial \psi^{0e}} &= a_e \left(\chi_1^e \, \xi_1^e + \chi_2^e \, \xi_2^e\right) = 0 \\
\frac{\partial w_{f,e}}{\partial \varphi_1^{0e}} &= b_e \, \chi_1^e \, \xi_2^e + c_e \, \xi_1^e \, \rho_2^e = 0 \\
\frac{\partial w_{f,e}}{\partial \varphi_2^{0e}} &= b_e \, \chi_2^e \, \xi_1^e + c_e \, \xi_2^e \, \rho_1^e = 0 \\
\frac{\partial w_{f,u}}{\partial \xi_1^{0e}} &= d_e \, \xi_2^e \sigma^e + f_e \, \xi_1^e \, \rho_1^e = 0 \\
\frac{\partial w_{f,u}}{\partial \xi_2^{0e}} &= d_e \, \xi_1^e \, \sigma^e + f_e \, \xi_2^e \, \rho_2^e = 0
\end{aligned}
$$

solved by vev configuration given above. . .

Alternatives to TBM

Bi-maximal

$$U_{\rm BM} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

S_4: Altarelli, Feruglio, Merlo, JHEP **0905**, 020 (2009) (needs large NLO corrections)

CKM(-like) charged lepton corrections may also resurrect it:

- $\text{QLC}_0 : \theta_{12} = \frac{\pi}{4} - \theta_C \Rightarrow \sin^2\theta_{12} \simeq 0.280$
- $\text{QLC}_1 : U = V^\dagger\, U_{\rm BM} \Rightarrow \sin^2\theta_{12} \simeq \frac{1}{2} - \lambda/\sqrt{2}\,\cos\phi \simeq 0.331\ldots0.670$
- $\text{QLC}_2 : U = U_{\rm BM}\, V^\dagger \Rightarrow \sin^2\theta_{12} \simeq \frac{1}{2} - \lambda\,\cos\phi' \simeq 0.276\ldots0.762$

"Quark-Lepton Complementarity"

Alternatives to TBM

Tri-maximal Mixing(s)

- TM$_2$ $(S_{3,4}, \Delta(27))$

$$\begin{pmatrix} |U_{e2}|^2 \\ |U_{\mu 2}|^2 \\ |U_{\tau 2}|^2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

(Lam; Grimus, Lavoura)

- TM$_1$, TM$_3$, TM^1, TM^2, TM^3, e.g.,

$$\text{TM}^1: \qquad \left(|U_{e1}|^2,\ |U_{e2}|^2,\ |U_{e3}|^2\right) = \left(\tfrac{2}{3},\ \tfrac{1}{3},\ 0\right)$$

$$\text{TM}_1: \qquad \begin{pmatrix} |U_{e1}|^2 \\ |U_{\mu 1}|^2 \\ |U_{\tau 1}|^2 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 1/6 \\ 1/6 \end{pmatrix}$$

(Lam; Albright, W.R.; Friedberg, Lee)

Alternatives to TBM

- tetra-maximal (Xing)

$$U = \mathrm{diag}(1,1,i)\,\tilde{R}_{23}(\pi/4;\pi/2)\,\tilde{R}_{13}(\pi/4;0)\,\tilde{R}_{12}(\pi/4;0)\,\tilde{R}_{13}(\pi/4;\pi)$$

- symmetric mixing $U = U^T$ (Joshipura, Smirnov; Hochmuth, W.R.)

$$|U_{e3}| = \frac{\sin\theta_{12}\,\sin\theta_{23}}{\sqrt{1-\sin^2\delta\,\cos^2\theta_{12}\,\cos^2\theta_{23}}+\cos\delta\,\cos\theta_{12}\,\cos\theta_{23}}$$

- hexagonal mixing (D_6)

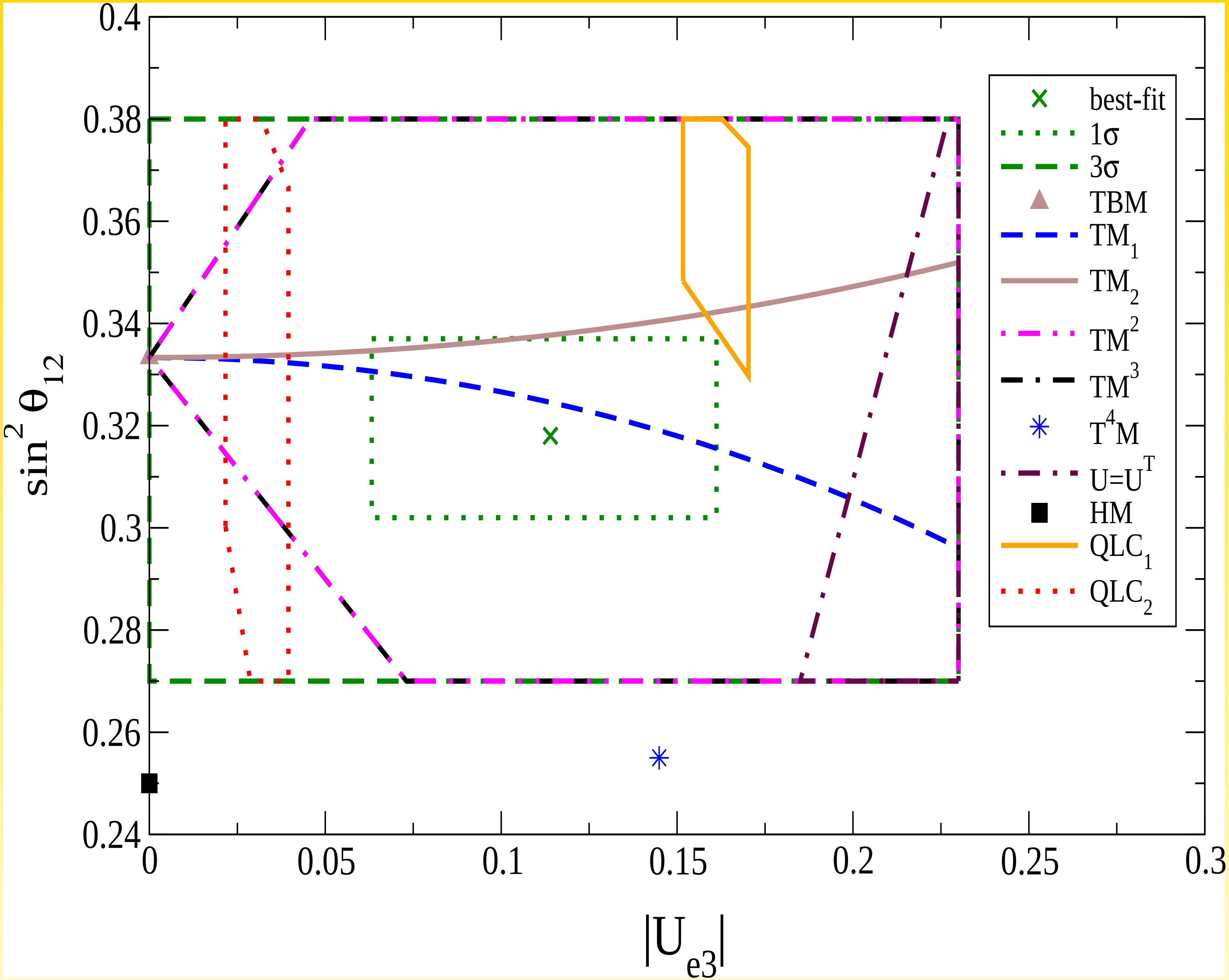
$$\theta_{12} = \pi/6 \Rightarrow \sin^2\theta_{12} = \frac{1}{4}$$

(Albright, Dueck, W.R.; Kim, Seo)

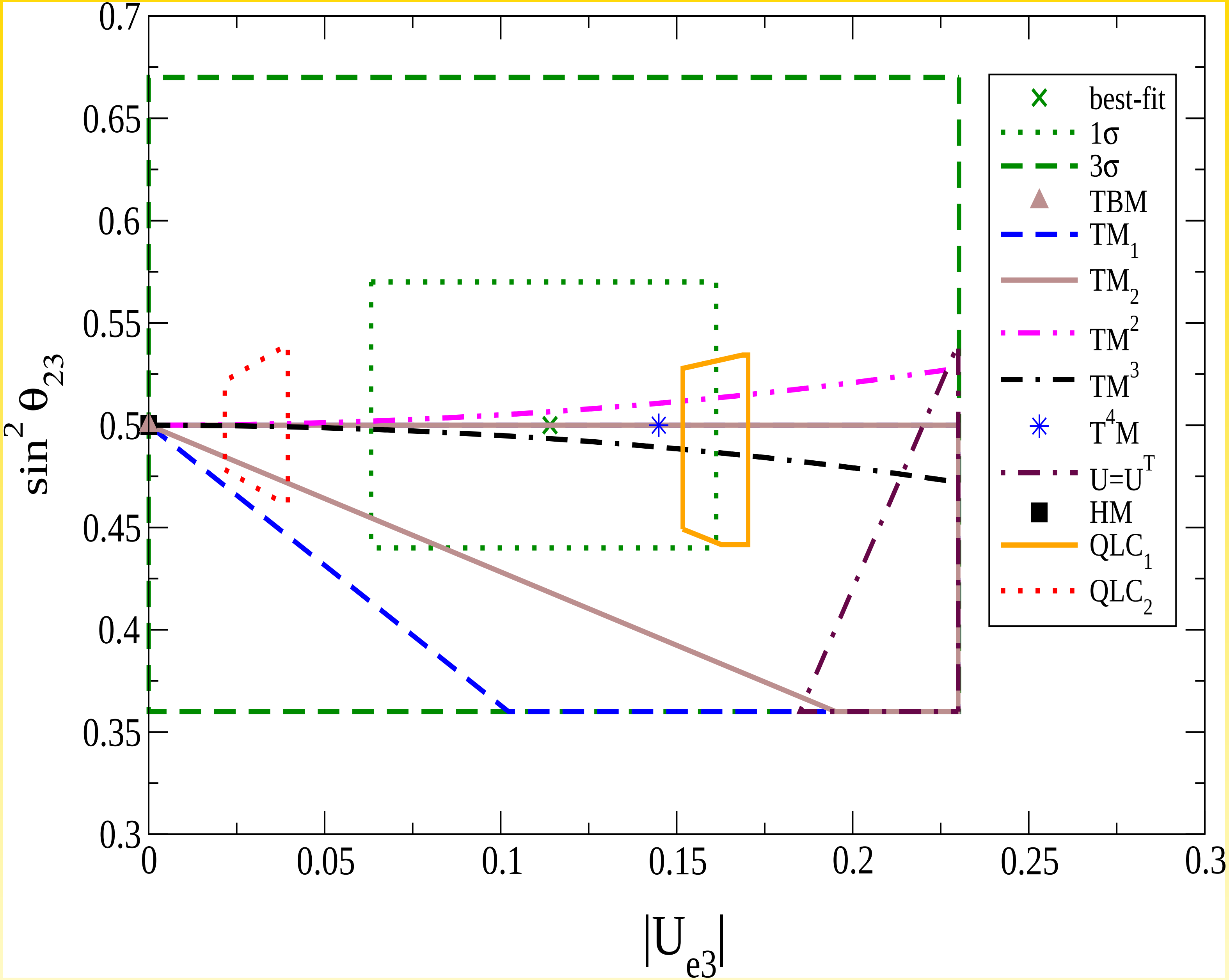
$\theta_{12} = \pi/6$, $\theta_C = \pi/12$: "dodecal" (D_{12} model)

Scenario	$\sin^2\theta_{12}$		$\sin^2\theta_{23}$		$\sin^2\theta_{13}$	
TBM	0.333		0.500		0.000	
$\mu - \tau$	—		0.500		0.000	
TM_1	0.296	0.333	**		—	
TM_2	0.333	0.352	**		—	
TM_3	—		0.500		0.000	
TM^1	0.333		—		0.000	
TM^2	**		0.500	0.528	—	
TM^3	**		0.472	0.500	—	
T^4M	0.255		0.500		0.021	
$U=U^T$	0.000	0.389	0.000	0.504	0.0343	0.053
BM	0.500		0.500		0.000	
HM	0.250		0.500		0.000	
φ_1	0.276		0.500		0.000	
φ_2	0.345		0.500		0.000	
QLC_0	0.280		0.459		—	
QLC_1	0.331	0.670	0.442	0.534	0.023	0.029
QLC_2	0.276	0.726	0.462	0.540	0.0005	0.0016

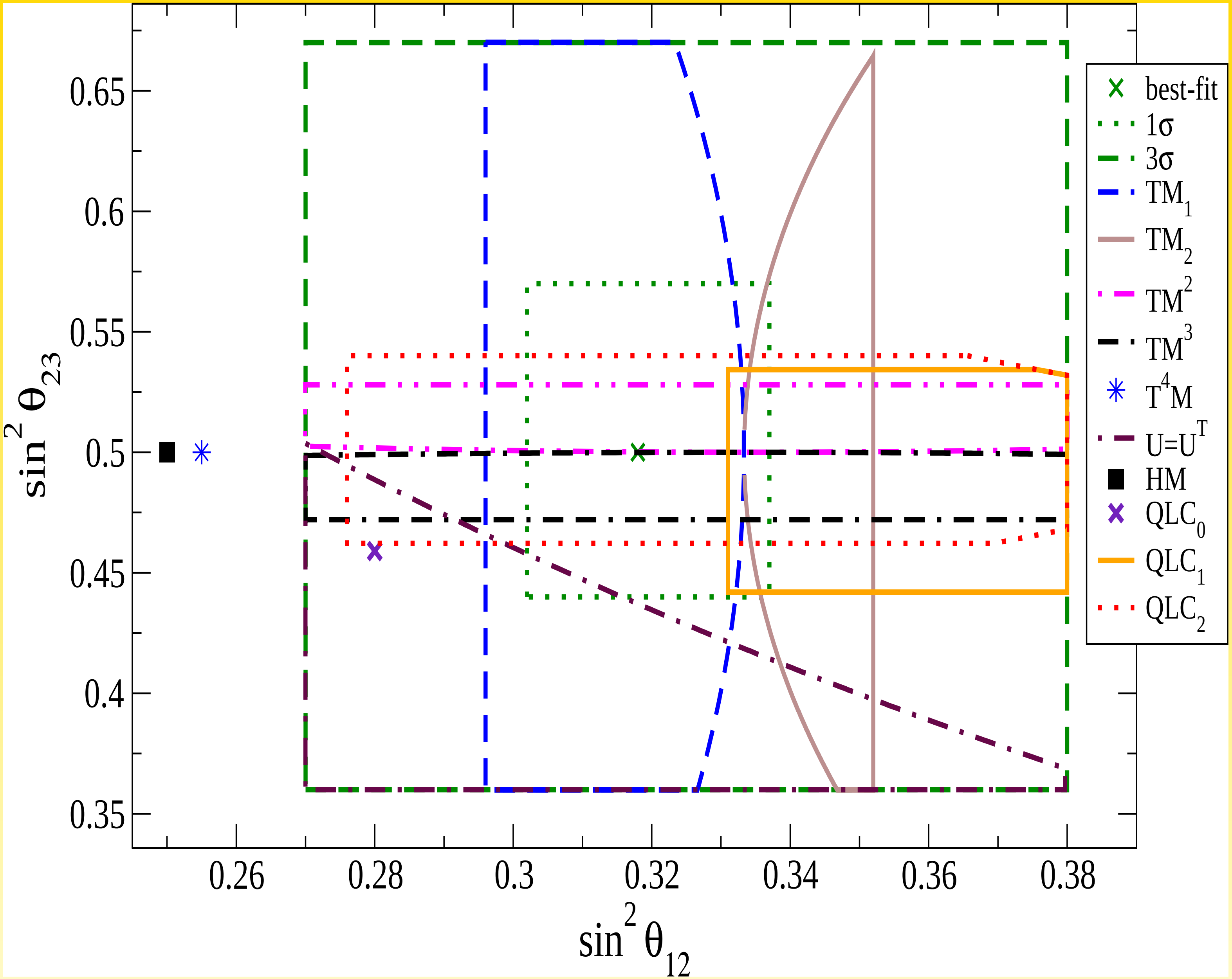
Albright, Dueck, W.R., 1004.2798



0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0
0.05
0.1
0.15
0.2
0.25
0.3
$\sin^2\theta_{12}$
$|U_{e3}|$
best-fit
1σ
3σ
TBM
TM_1
TM_2
TM^2
TM^3
T^4M
$U=U^T$
HM
QLC_1
QLC_2

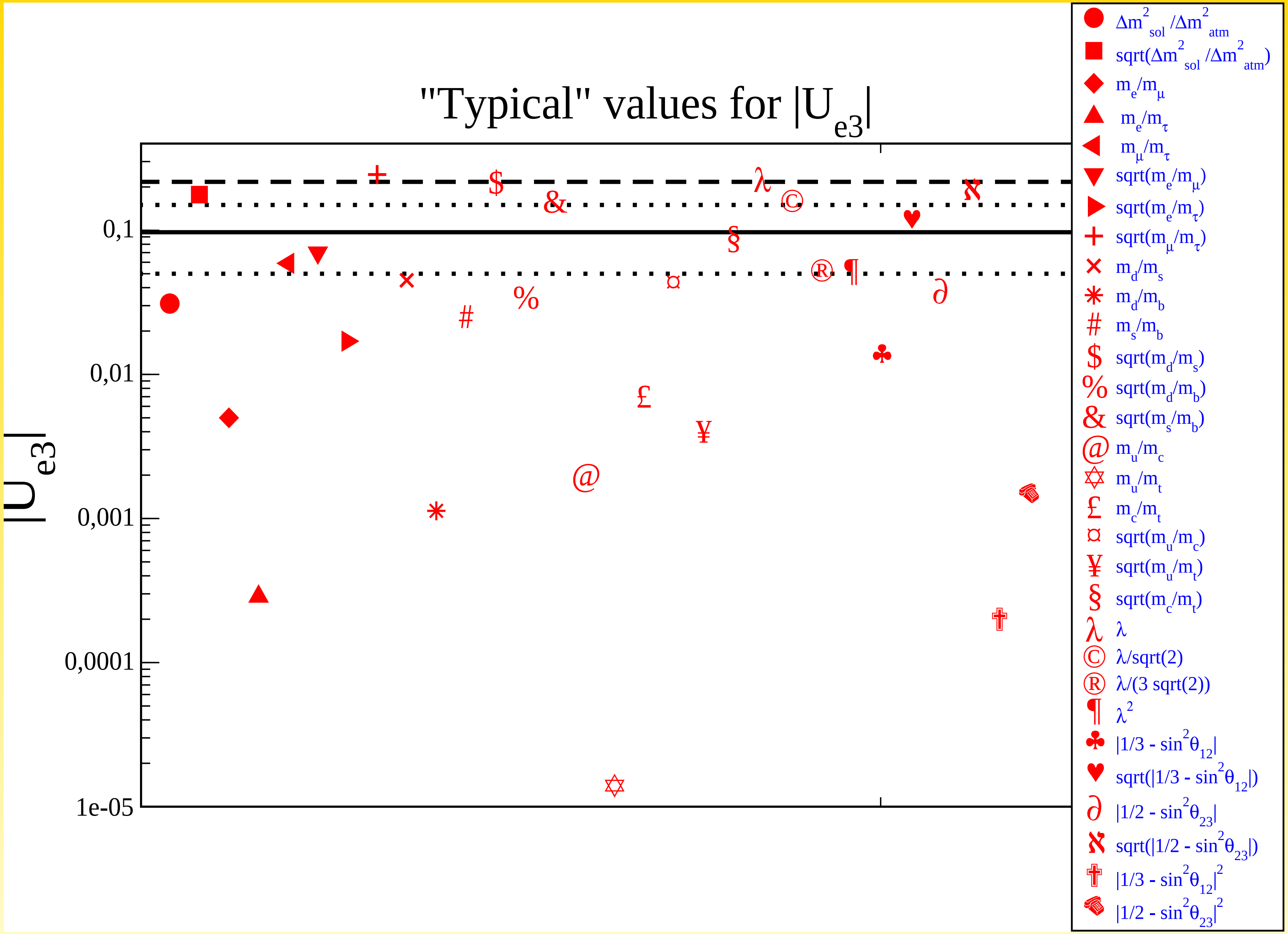


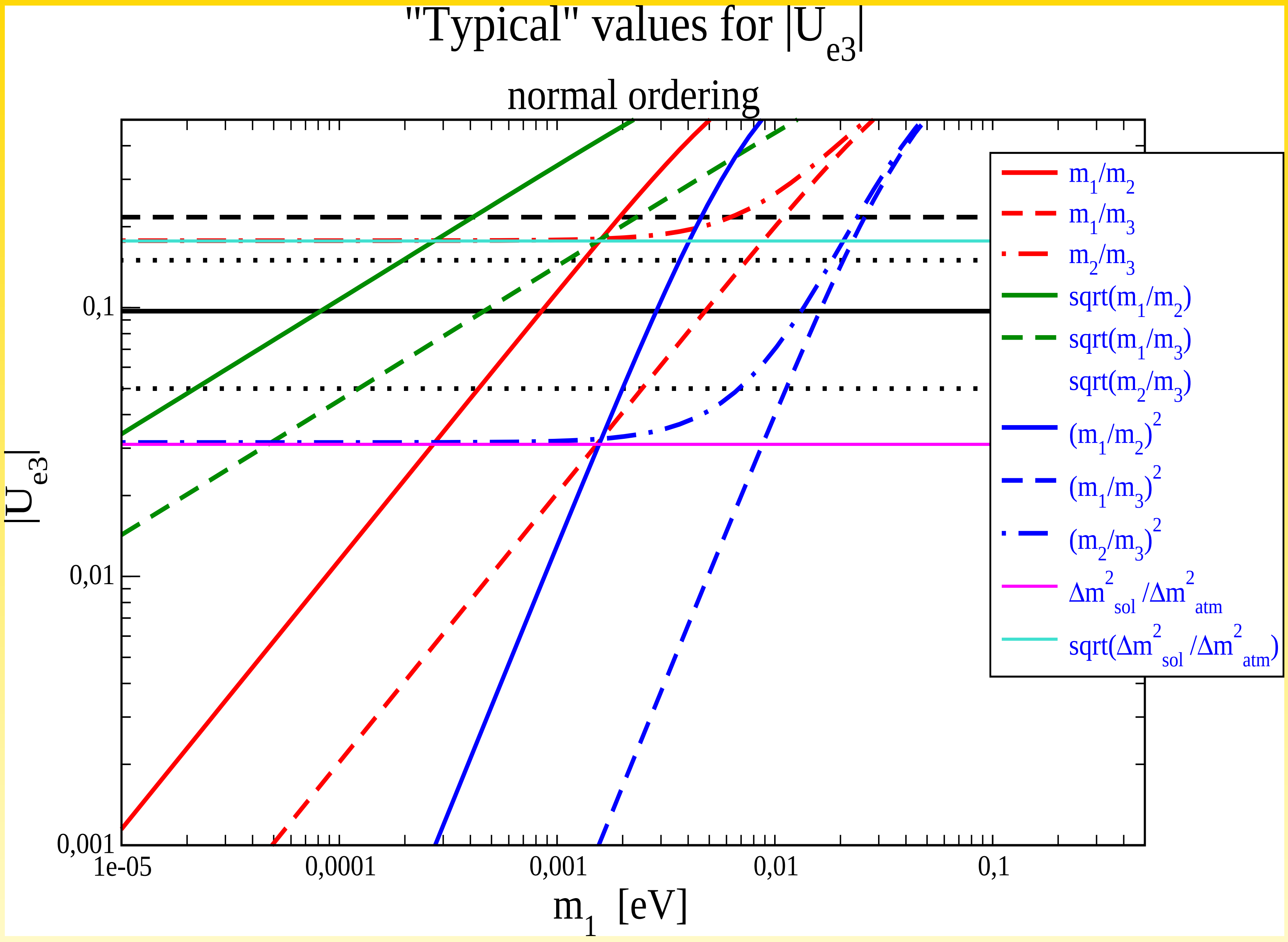
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0
0.05
0.1
0.15
0.2
0.25
0.3
sin² θ23
|Ue3|
best-fit
1σ
3σ
TBM
TM1
TM2
TM²
TM³
T⁴M
U=U^T
HM
QLC1
QLC2



0.65
0.6
0.55
0.5
0.45
0.4
0.35
$\sin^2\theta_{23}$
0.26
0.28
0.3
0.32
0.34
0.36
0.38
$\sin^2\theta_{12}$
best-fit
1σ
3σ
TM_1
TM_2
TM^2
TM^3
T^4M
$U=U^T$
HM
QLC_0
QLC_1
QLC_2

"Typical" values for $|U_{e3}|$

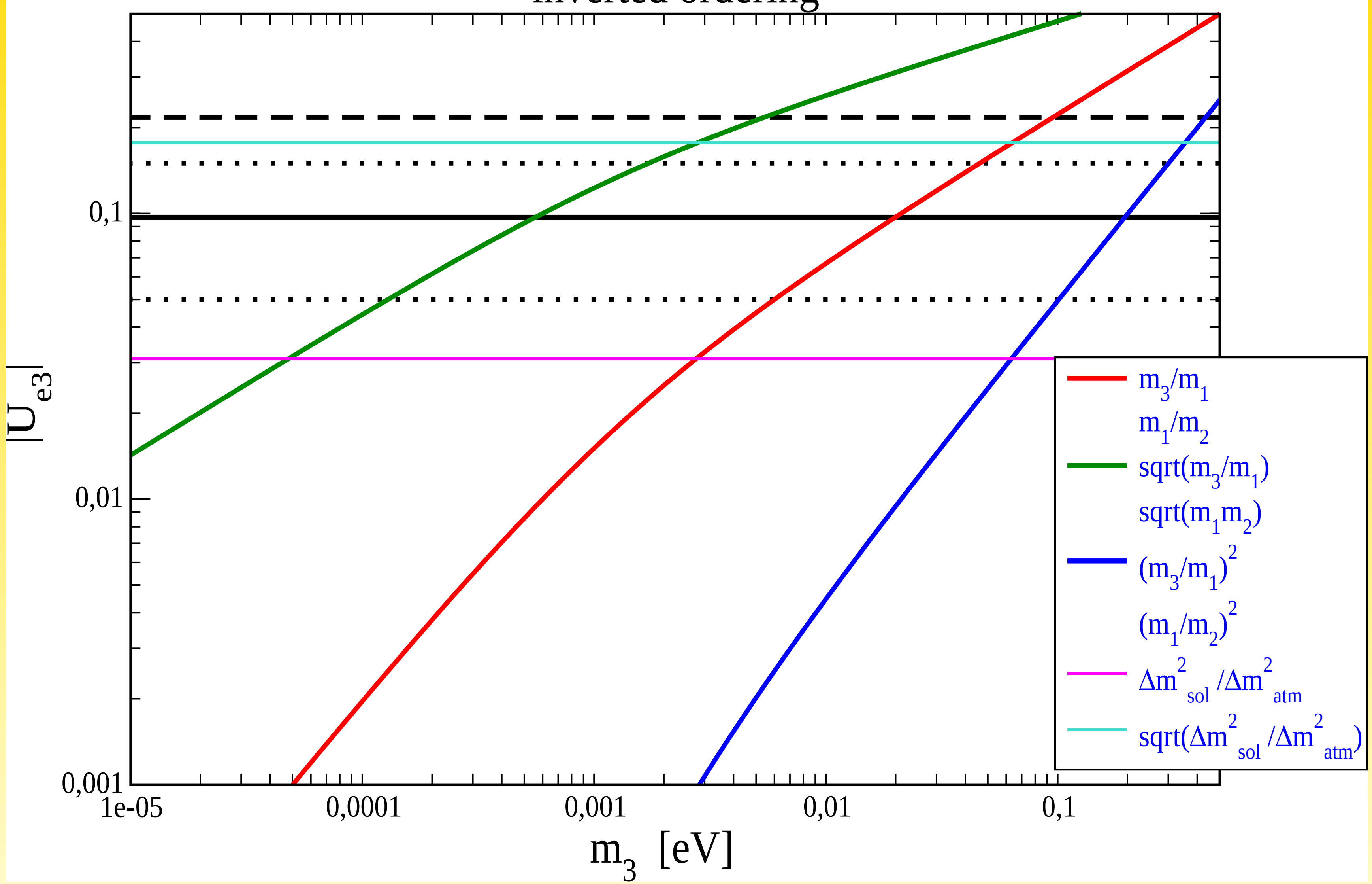


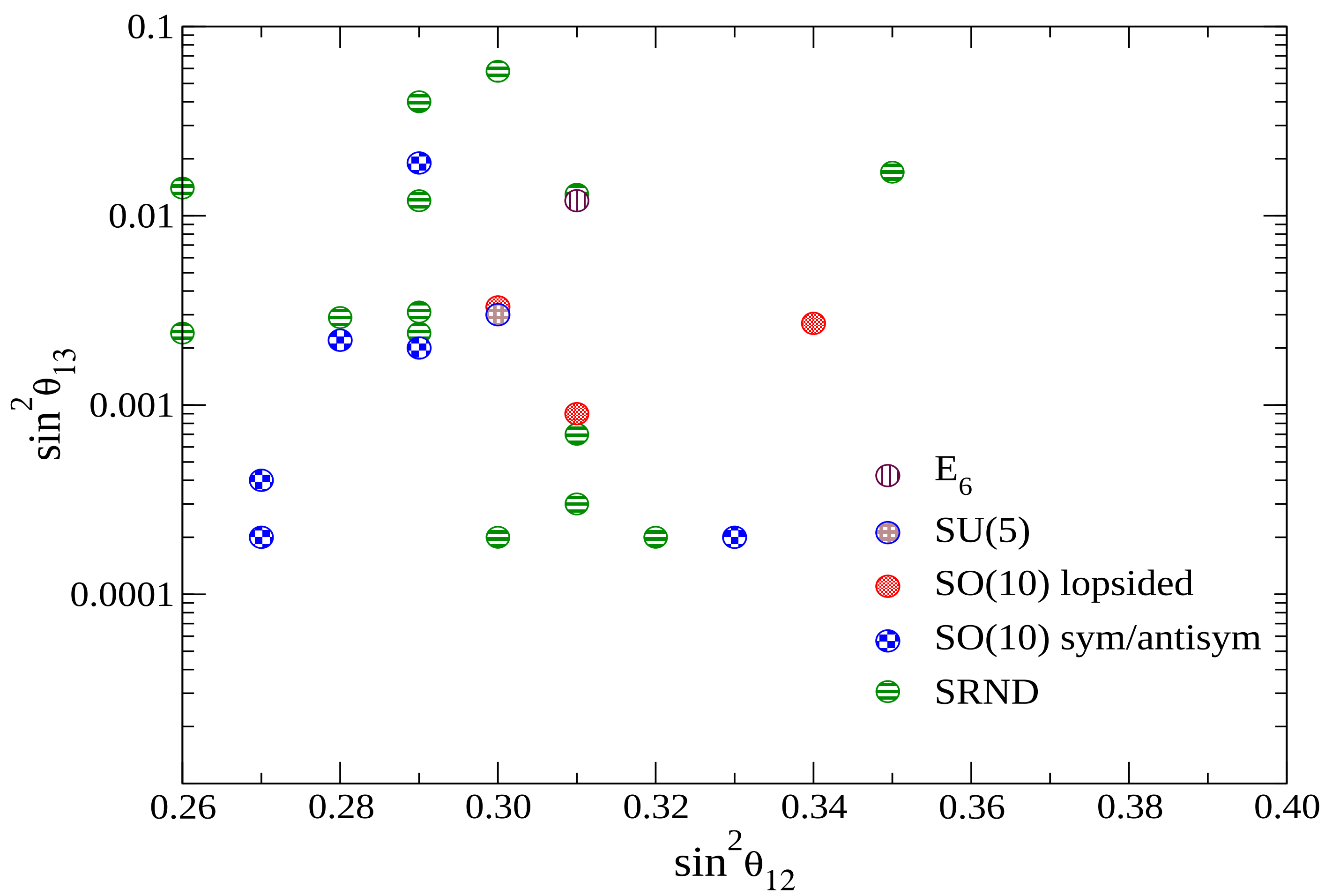


"Typical" values for |U_e3|
normal ordering
|U_e3|
m_1 [eV]
0,1
0,01
0,001
1e-05
0,0001
0,001
0,01
0,1
m_1/m_2
m_1/m_3
m_2/m_3
sqrt(m_1/m_2)
sqrt(m_1/m_3)
sqrt(m_2/m_3)
(m_1/m_2)^2
(m_1/m_3)^2
(m_2/m_3)^2
Δm^2_sol /Δm^2_atm
sqrt(Δm^2_sol /Δm^2_atm)

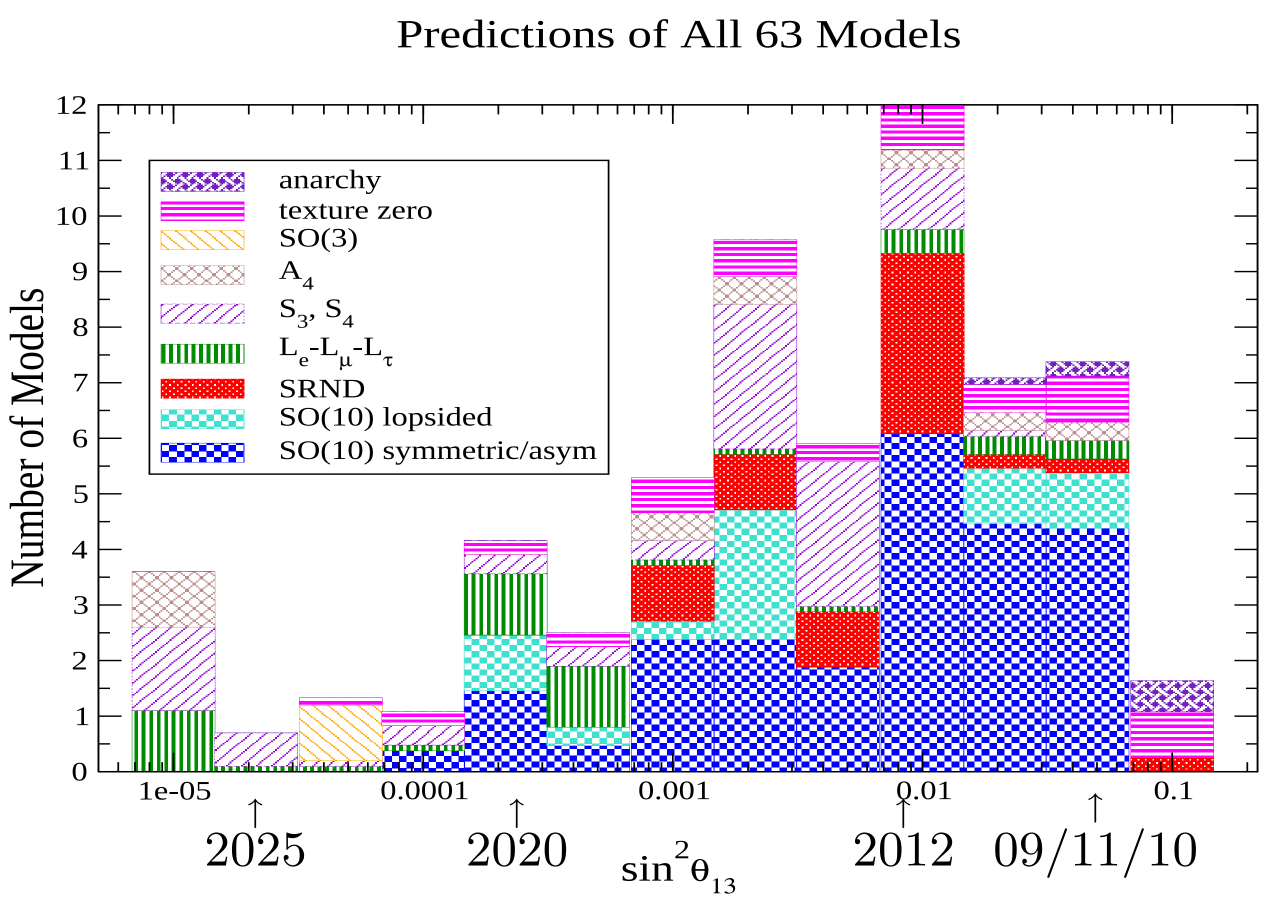
"Typical" values for $|U_{e3}|$

inverted ordering





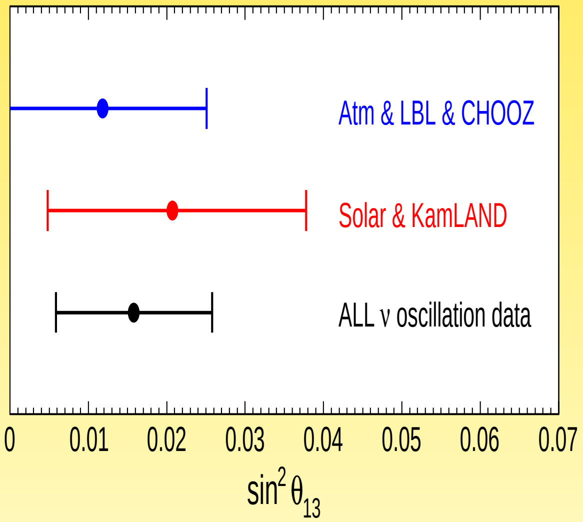
Albright, Chen

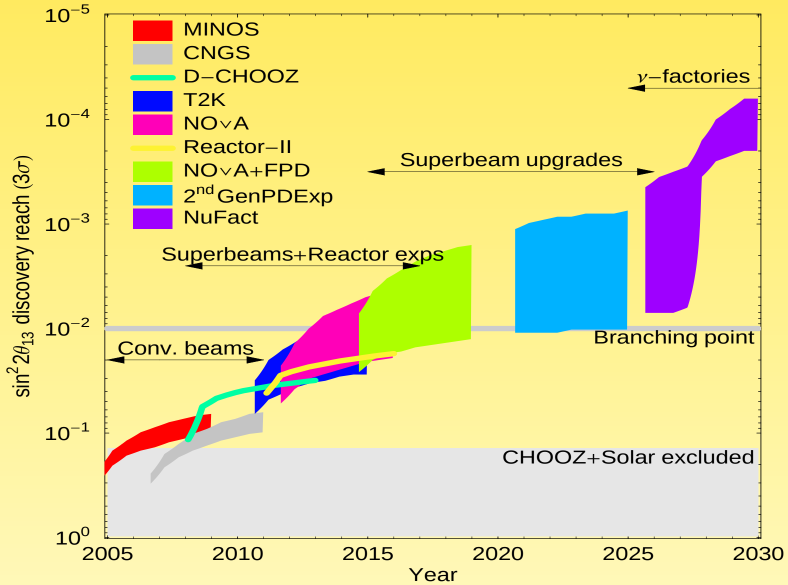


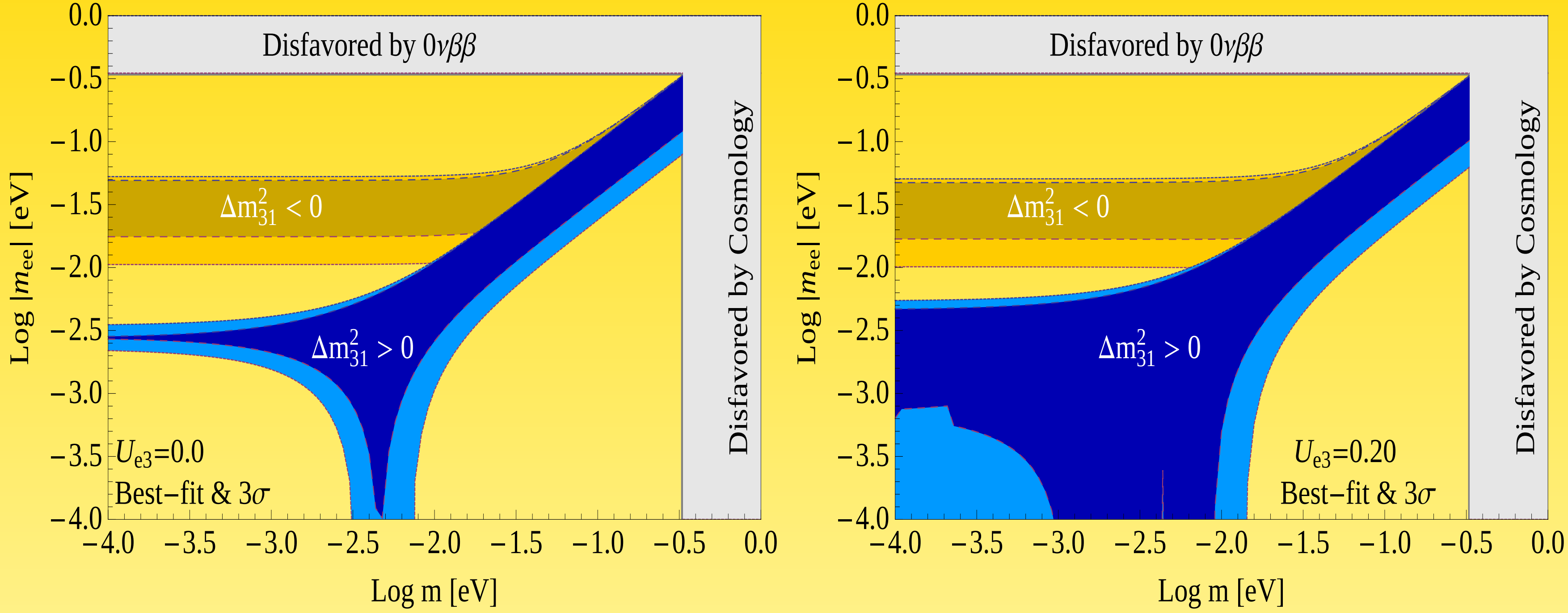
Albright, Chen, PRD **74**, 113006 (2006)

The value $|U_{e3}| \simeq 0.1$ seems to be interesting:

- little bit smaller than current limit
- "Bari hint"
- two numbers a, b of order one: $a/b \gtrsim 0.1$
- experimentally probed very soon







importance of U_{e3} in neutrino-less double beta decay

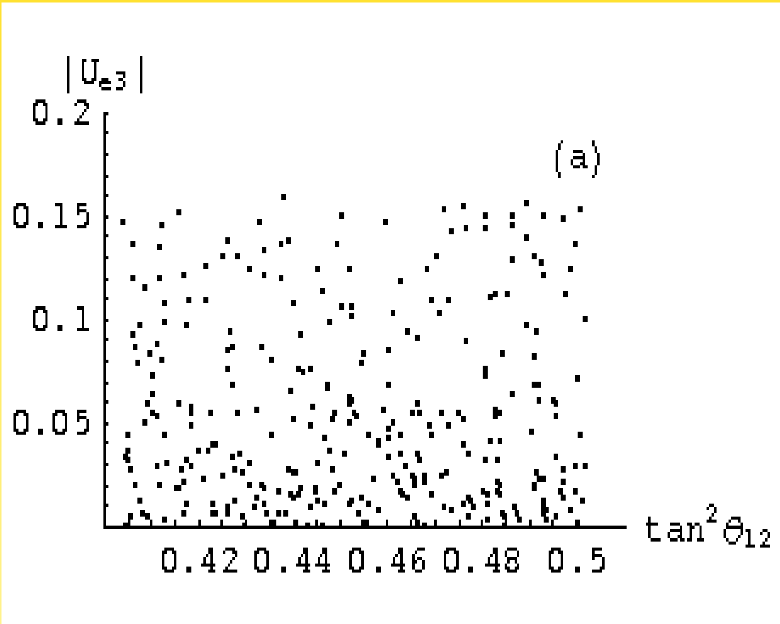
How to perturb a mixing scenario/model

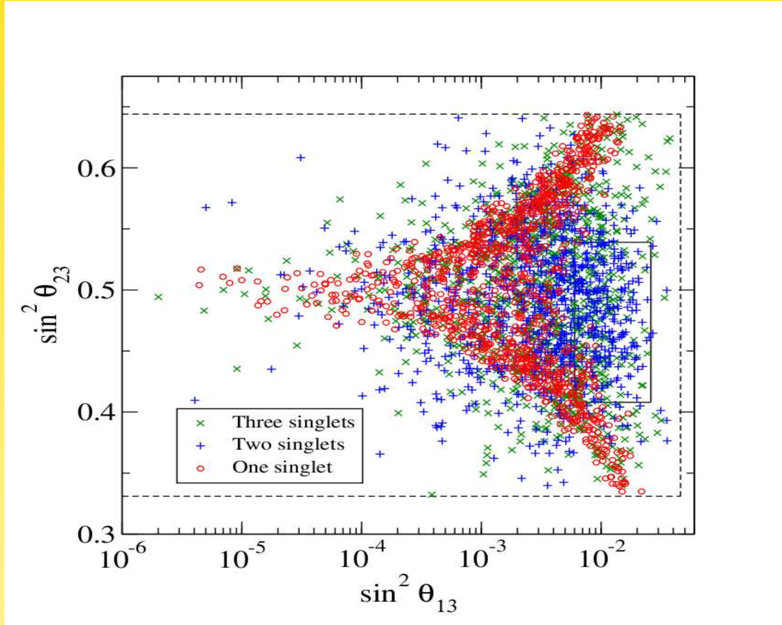
- VEV misalignment, NLO terms
- explicit naive breaking
- renormalization
- charged leptons

VEV misalignment, NLO terms

- **"naive misalignment"**:

  if $\langle \text{flavon} \rangle = (1, 1, 1)^T$, perturb it to $\langle \text{flavon} \rangle = (1, 1 + \epsilon_1, 1 + \epsilon_2)^T$





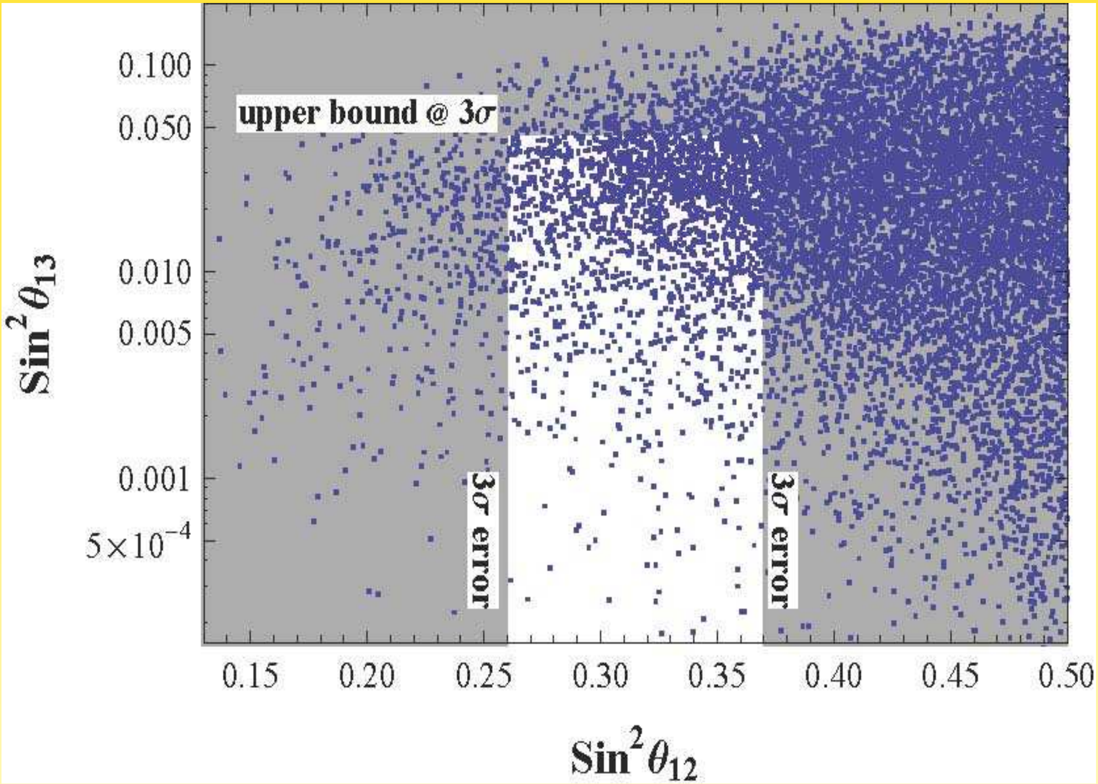
Honda, Tanimoto

Barry, W.R.

- typically of the same order for θ_{23} and $|U_{e3}|$
- of order $\langle \text{flavon} \rangle / \Lambda$ or $\langle \text{flavon} \rangle / M_R$, typically $\mathcal{O}(0.1)$ or $\mathcal{O}(\lambda_C)$ or $\mathcal{O}(0.01)$
- solar neutrino mixing angle receives larger corrections

VEV misalignment, NLO terms

- NLO terms, VEV misalignment due to terms allowed by the symmetry
  $\Rightarrow$ model-dependent!
  - Altarelli, Feruglio, Merlo, JHEP 0905:



$\delta \sin^2\theta_{12} \simeq \delta|U_{e3}| = \mathcal{O}(\lambda)$ and $\delta \sin^2\theta_{23} = \mathcal{O}(\lambda^2)$

- Altarelli, Feruglio, Hagedorn, JHEP 0803:
  corrections $\mathcal{O}(\lambda^2)$ to all mixing angles
- Lin, NPB 824:
  $\delta|U_{e3}| = \mathcal{O}(\lambda)$ and $\delta\sin^2\theta_{12} \simeq \delta\sin^2\theta_{23} = \mathcal{O}(\lambda^2)$
- Hagedorn, Ziegler, 1007.1888:
  $\delta|U_{e3}|^2 = \mathcal{O}(\lambda^2)$ and $\delta\sin^2\theta_{12} = \mathcal{O}(\lambda)$
- Ishimori *et al.*, 1004.5004:
  $\delta|U_{e3}|^2 = \mathcal{O}(\epsilon^2)$ and $\delta\sin^2\theta_{12} = \mathcal{O}(\epsilon)$ and $\delta\sin^2\theta_{23} = \mathcal{O}(\epsilon^2)$
- etc.:
  etc.

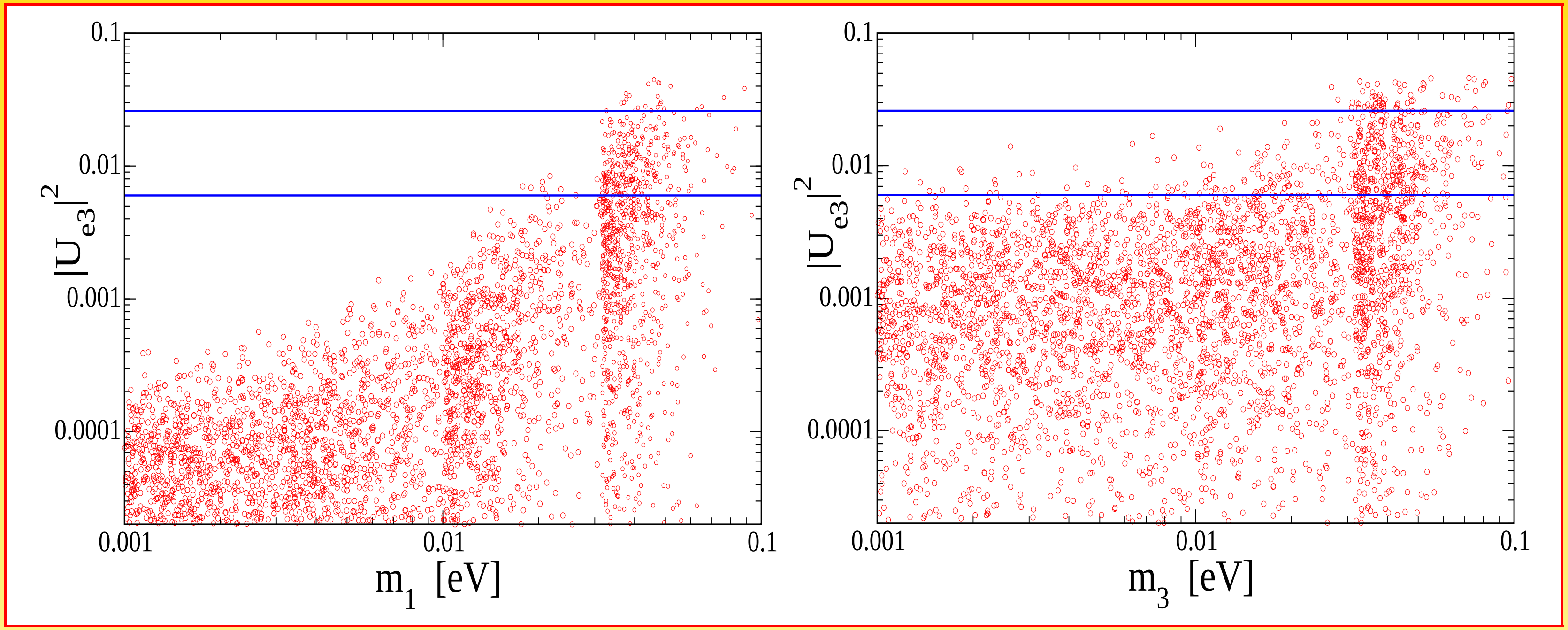
How to perturb a mixing scenario/model

- "explicit" (naive) breaking

$$m_\nu = \begin{pmatrix} A\,(1+\epsilon_1) & B\,(1+\epsilon_2) & B\,(1+\epsilon_3) \\ \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_4) & \frac{1}{2}(A+B-D)\,(1+\epsilon_5) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_6) \end{pmatrix}$$

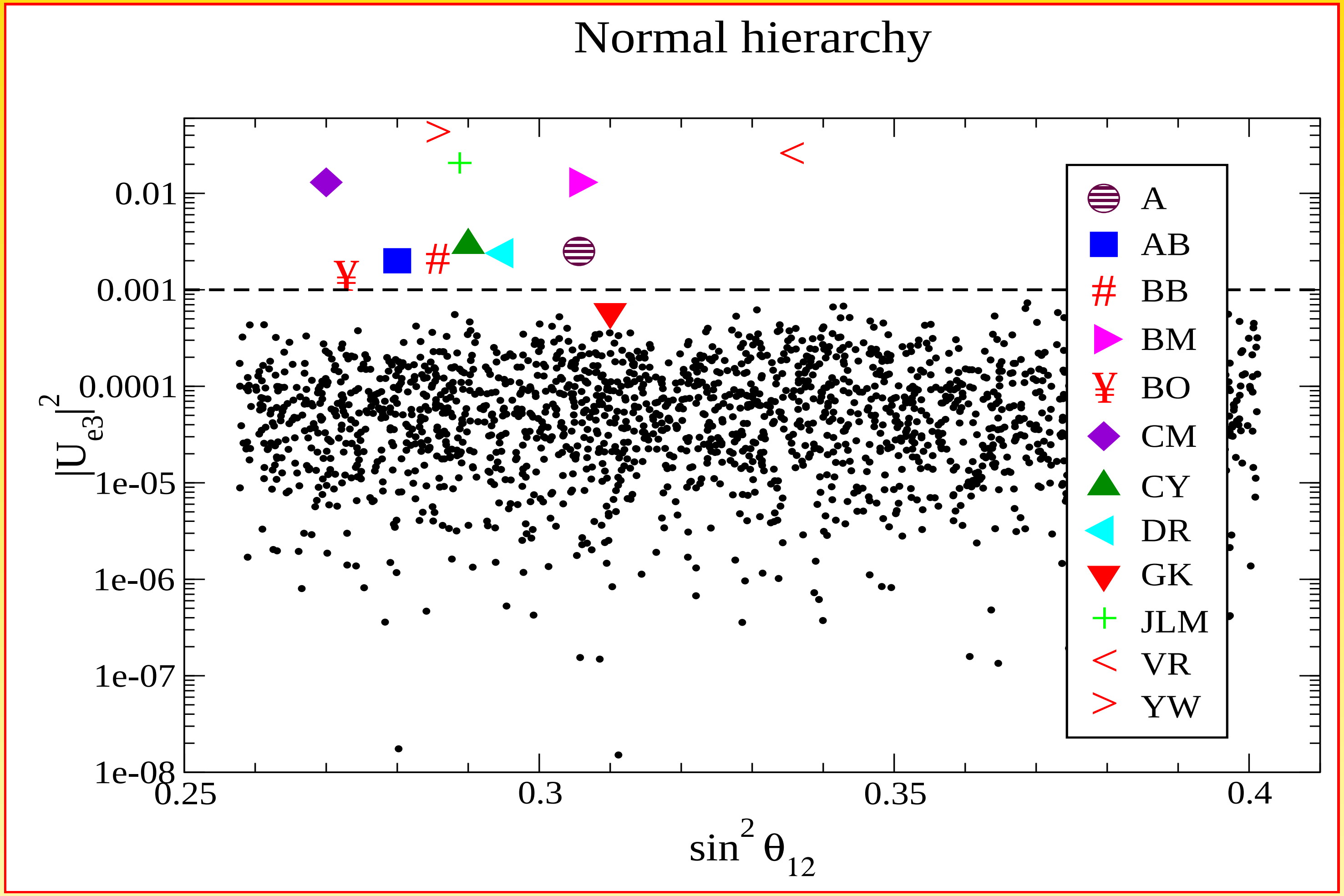
small complex parameters $\epsilon_i = |\epsilon_i|\, e^{i\phi_1}$ with $|\epsilon_i| \leq 0.2$

Albright, W.R., Phys. Lett. B **665**, 378 (2008)



$|U_{e3}|^2 \simeq 0.01$ requires

- $m_1 \gtrsim 0.02$ eV in normal ordering ($\propto \epsilon^2 \, (m_1^2 + \Delta m_\odot^2)/\Delta m_\mathrm{A}^2$)
- nothing in inverted ordering ($\propto \epsilon^2$)



$\sin^2 \theta_{12}$ very unstable $\leftrightarrow$ connected to two very close eigenvalues

How to perturb a mixing scenario/model

- Radiative corrections

$$\theta_{ij} \simeq \theta_{ij}^{\rm TBM} + k_{ij}\,\epsilon_{\rm RG}$$

$$k_{12} = \frac{\sqrt{2}}{3}\,\frac{\left|m_1 + m_2\,e^{i\alpha_2}\right|^2}{\Delta m_\odot^2}$$

$$k_{23} = -\left(\frac{2}{3}\,\frac{\left|m_2 + m_3\,e^{i(\alpha_3-\alpha_2)}\right|^2}{m_3^2 - m_2^2} + \frac{1}{3}\,\frac{\left|m_1 + m_3\,e^{i\alpha_3}\right|^2}{m_3^2 - m_1^2}\right)$$

$$k_{13} = -\frac{\sqrt{2}}{3}\left(\frac{\left|m_2 + m_3\,e^{i(\alpha_3-\alpha_2)}\right|^2}{m_3^2 - m_2^2} - \frac{\left|m_1 + m_3\,e^{i\alpha_3}\right|^2}{m_3^2 - m_1^2}\right)$$

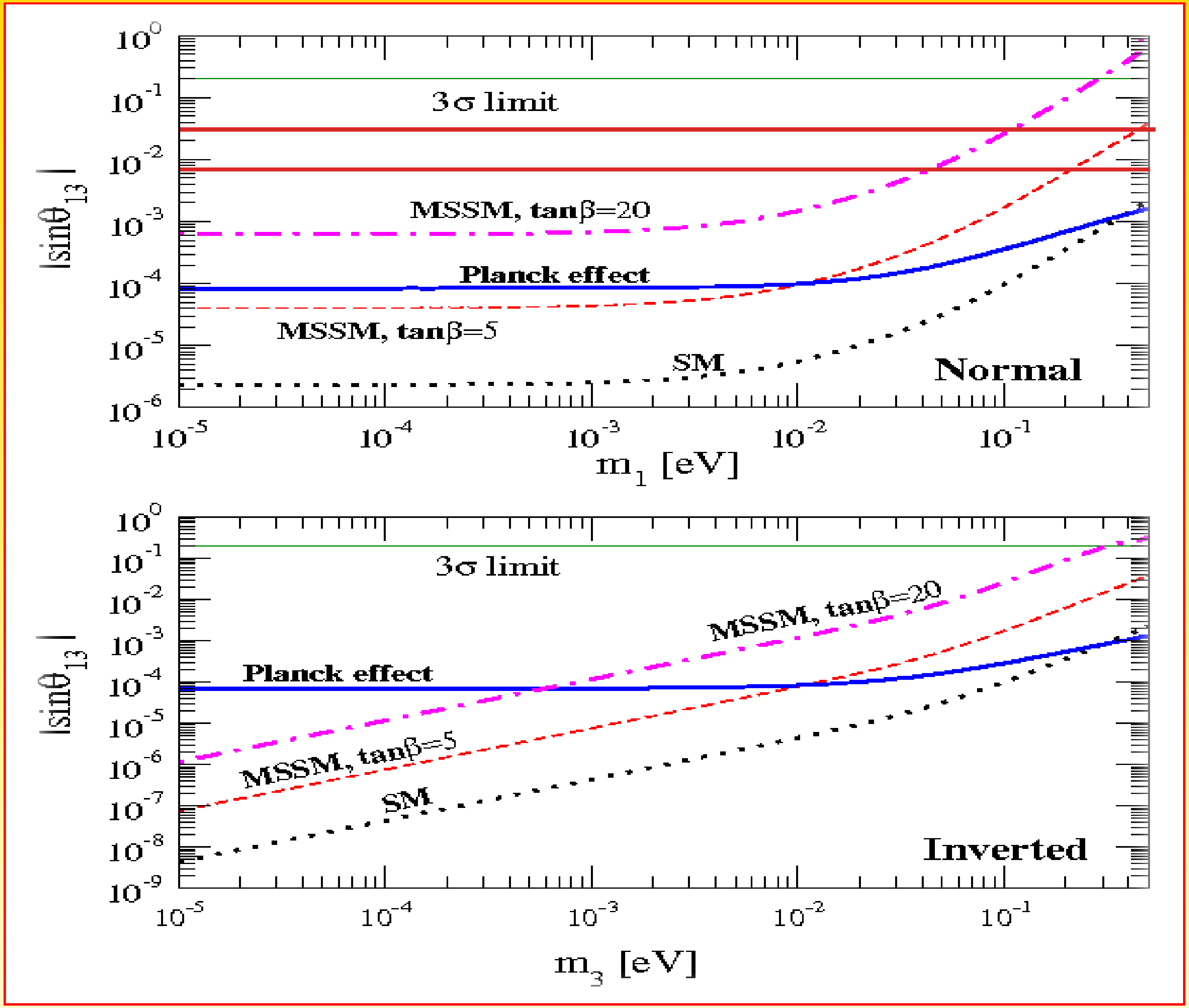
$$\epsilon_{\rm RG} = c\,\frac{m_\tau^2}{16\pi^2\,v^2}\ln\frac{M_X}{m_Z}$$ and $c = -3/2$ or $1 + \tan^2\beta$

Sign of RG correction (all best-fits have $\sin^2\theta_{12} \leq \frac{1}{3}\ldots$):

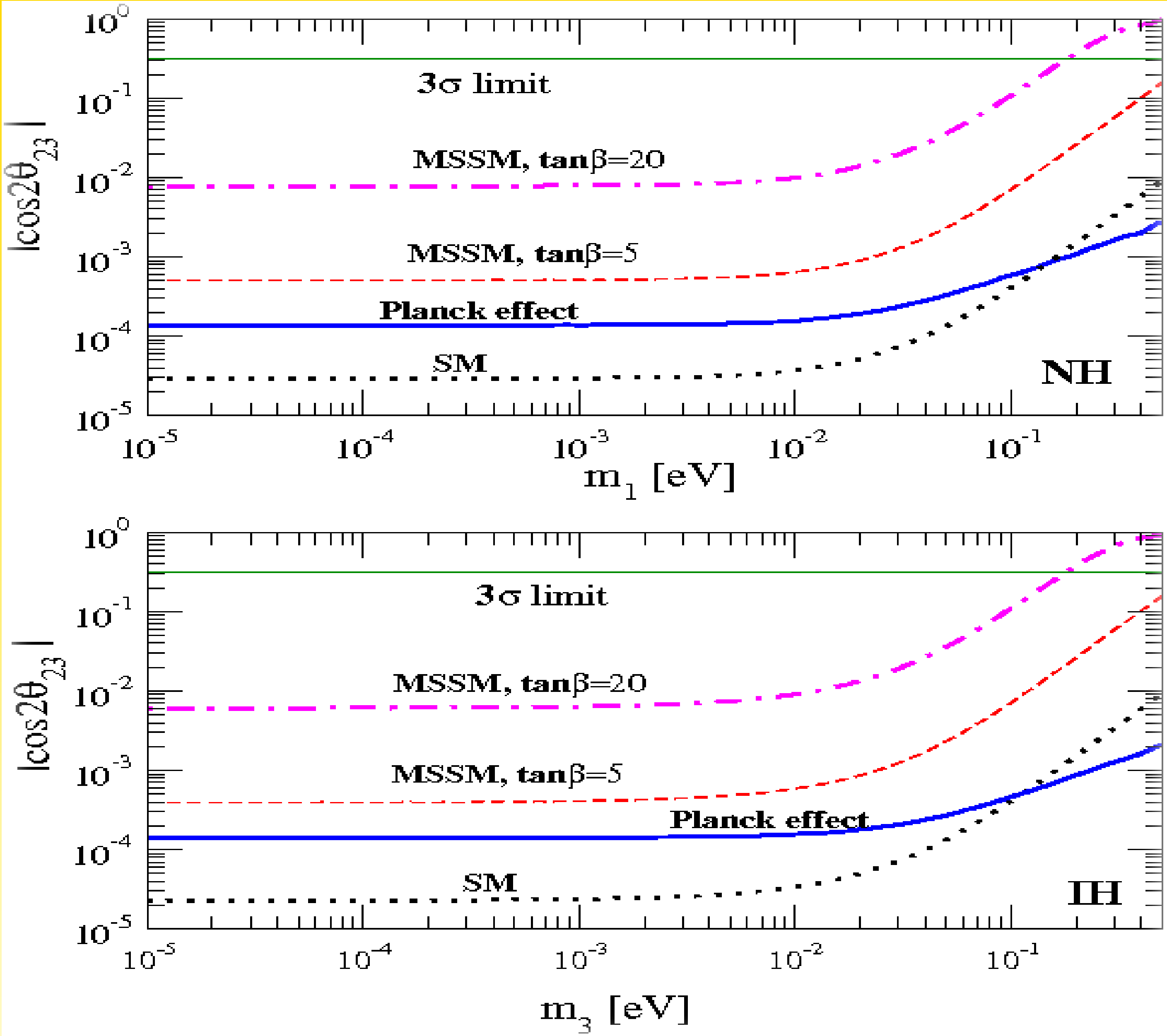
Model	mass ordering	θ_{12}	θ_{23}
SM	$\Delta m^2_{31} > 0$	$\searrow$	$\searrow$
	$\Delta m^2_{31} < 0$	$\searrow$	$\nearrow$
MSSM	$\Delta m^2_{31} > 0$	$\nearrow$	$\nearrow$
	$\Delta m^2_{31} < 0$	$\nearrow$	$\searrow$

Size of RG correction (phases ignored. . .):

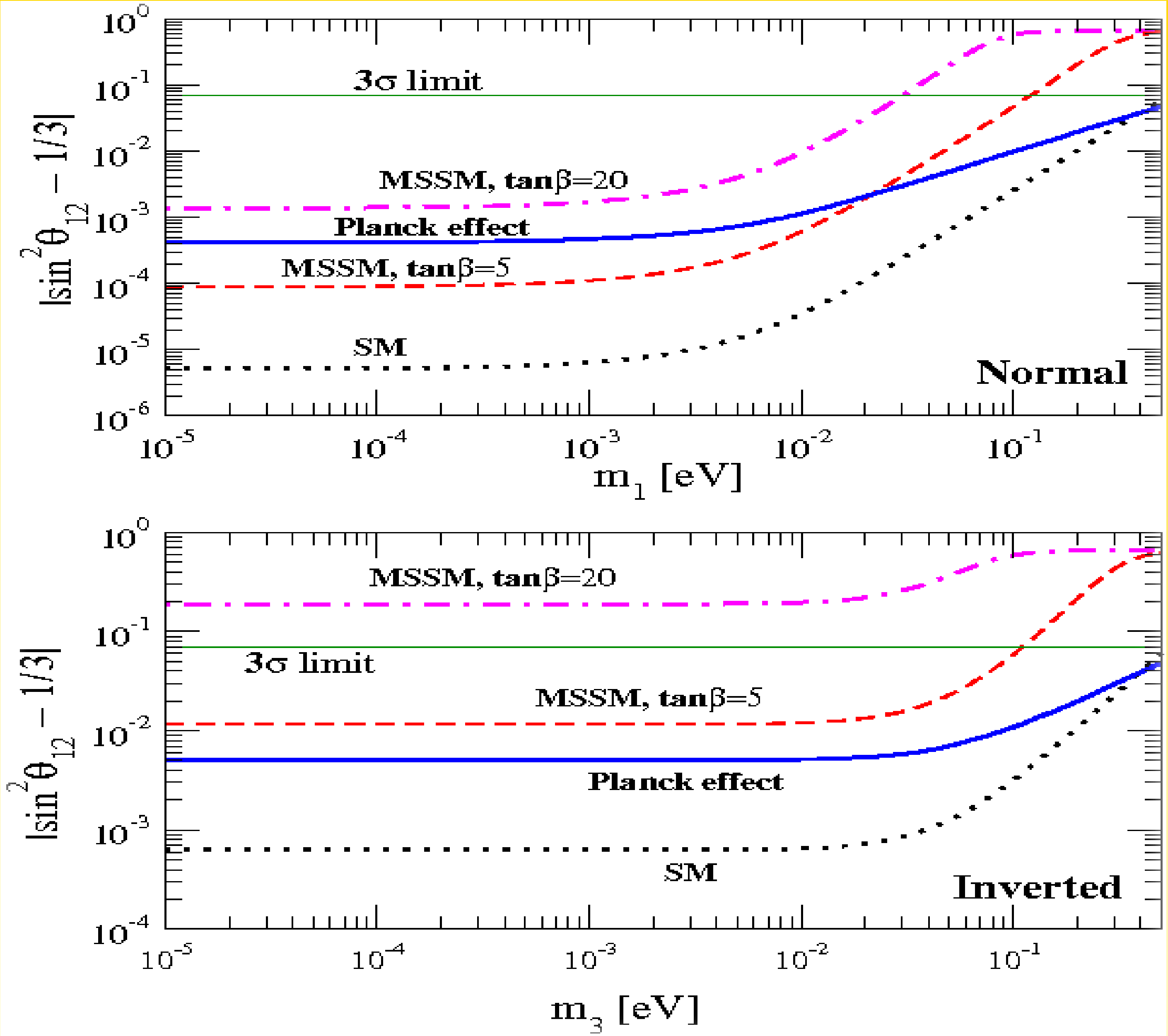
Angle	NH	IH	QD
$\Delta\theta_{12}$	1	$\Delta m^2_{\rm A}/\Delta m^2_\odot$	$m_0^2/\Delta m^2_\odot$
$\Delta\theta_{13}$	1	1	$m_0^2/\Delta m^2_{\rm A}$
$\Delta\theta_{23}$	1	1	$m_0^2/\Delta m^2_{\rm A}$



3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Normal
$|\sin\theta_{13}|$
m_1 [eV]
3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Inverted
$|\sin\theta_{13}|$
m_3 [eV]



3σ limit
MSSM, tanβ=20
MSSM, tanβ=5
Planck effect
SM
NH
|cos2θ23|
m1 [eV]
3σ limit
MSSM, tanβ=20
MSSM, tanβ=5
Planck effect
SM
IH
|cos2θ23|
m3 [eV]



3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Normal
$|\sin^2\theta_{12} - 1/3|$
m_1 [eV]
MSSM, tanβ=20
3σ limit
MSSM, tanβ=5
Planck effect
SM
Inverted
$|\sin^2\theta_{12} - 1/3|$
m_3 [eV]

Large $|U_{e3}|$ and RG

aim: get $|U_{e3}| = 0.1$ from TBM

- constraint: keep $\sin^2\theta_{12}$ close to TBM value
- what is $\sin^2\theta_{23}$?

`Goswami, Petcov, Ray, W.R., PRD` **80** `(2009) 053013`

Effect on θ_{12}

$$k_{12} = \frac{\sqrt{2}}{3} \frac{\left|m_1 + m_2\, e^{i\alpha_2}\right|^2}{\Delta m^2_\odot} \propto \begin{cases} 1 & \text{NH} \\ \frac{\Delta m^2_{\rm A}}{\Delta m^2_\odot}\,(1 + e^{i\alpha_2}) & \text{IH} \\ \frac{m_0^2}{\Delta m^2_\odot}\,(1 + e^{i\alpha_2}) & \text{QD} \end{cases}$$

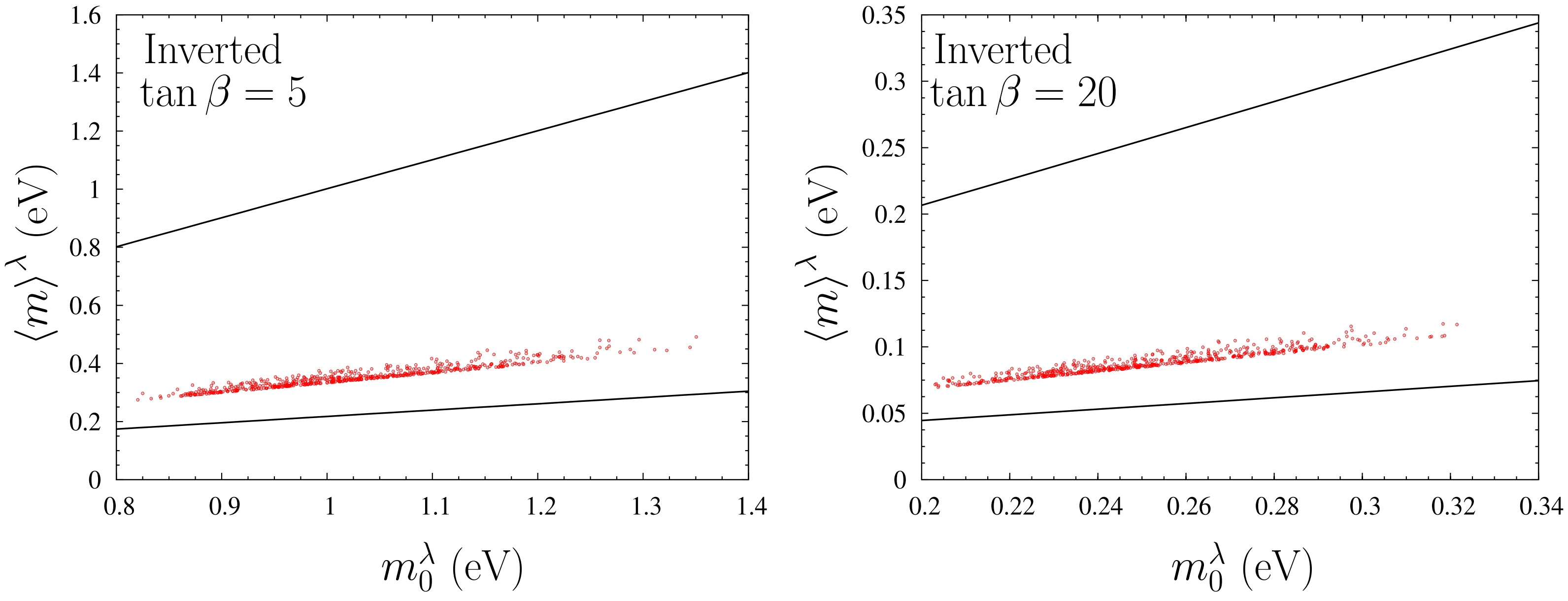
$\Rightarrow$ strong effect for IH and QD

$\Rightarrow$ suppress with $\alpha_2 = \pi$

$$|m_{ee}| \simeq m_0\, \sqrt{1 - \sin^2 2\theta_{12}\, \sin^2 \alpha_2/2} \stackrel{\alpha_2=\pi}{\longrightarrow} m_0\, \cos 2\theta_{12}$$

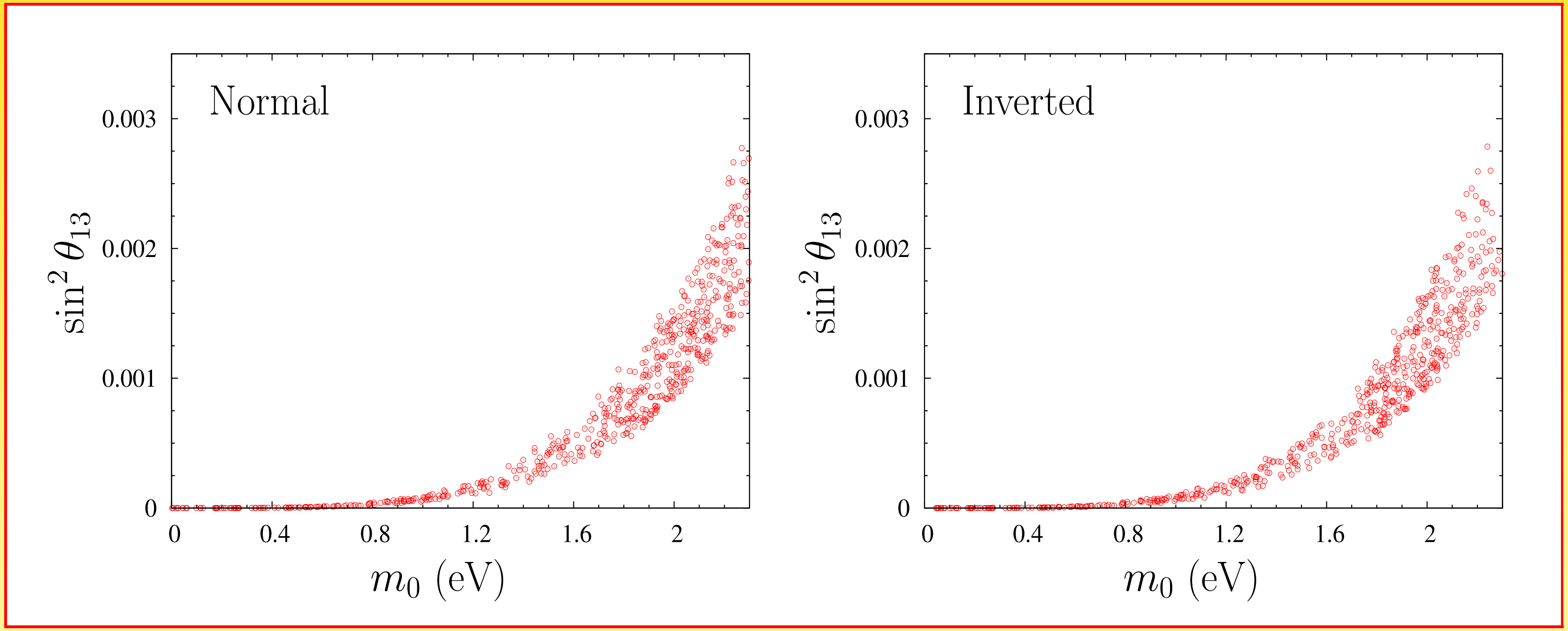
large cancellations in $0\nu\beta\beta$!

Renormalization and $|U_{e3}| \simeq 0.1$



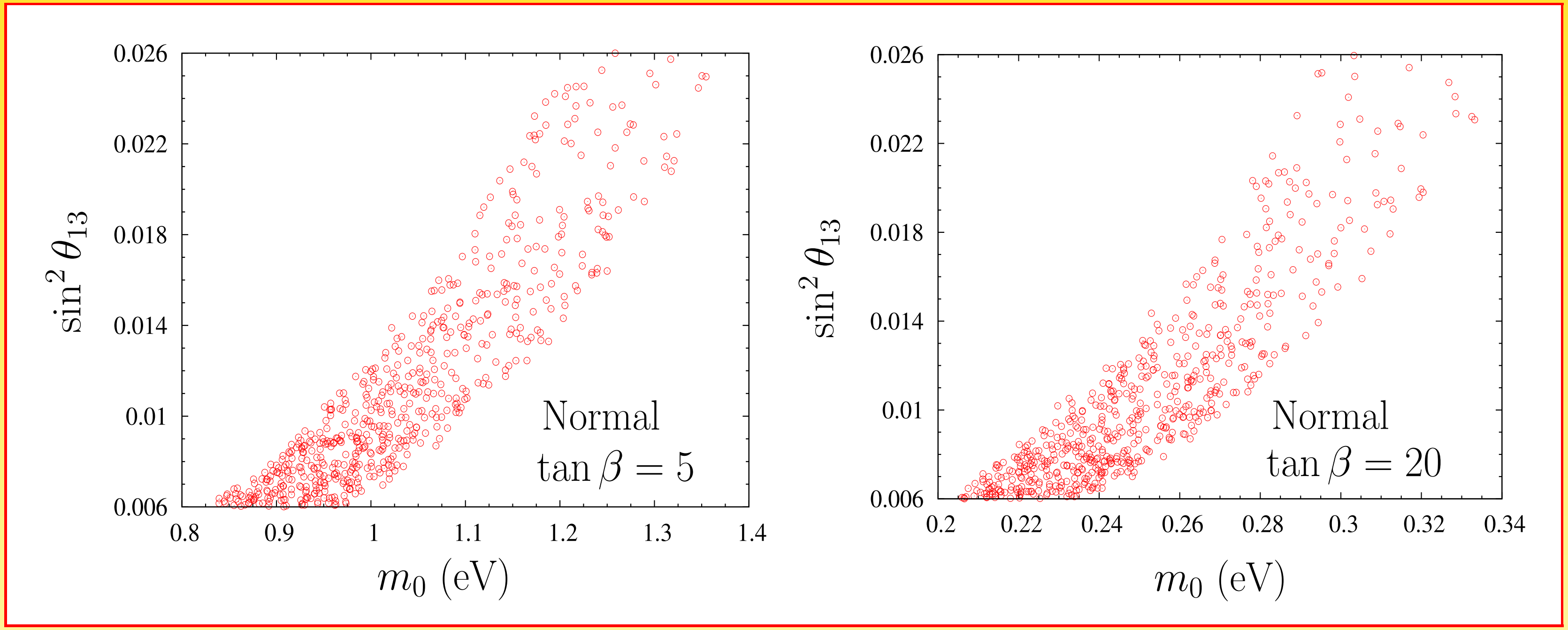
- $|m_{ee}| \simeq c_{13}^2\, m_0\, |c_{12}^2 + s_{12}^2\, e^{i\alpha_2}|$
- $\tan\beta = 5$: $|m_{ee}|$ takes values between 0.26 and 0.50 eV; general upper and lower limits: 0.2 eV and 1.4 eV
- $\tan\beta = 20$: $|m_{ee}|$ takes values between 0.07 and 0.11 eV; general upper and lower limits: 0.05 eV and 0.34 eV

Renormalization and $|U_{e3}| \simeq 0.1$



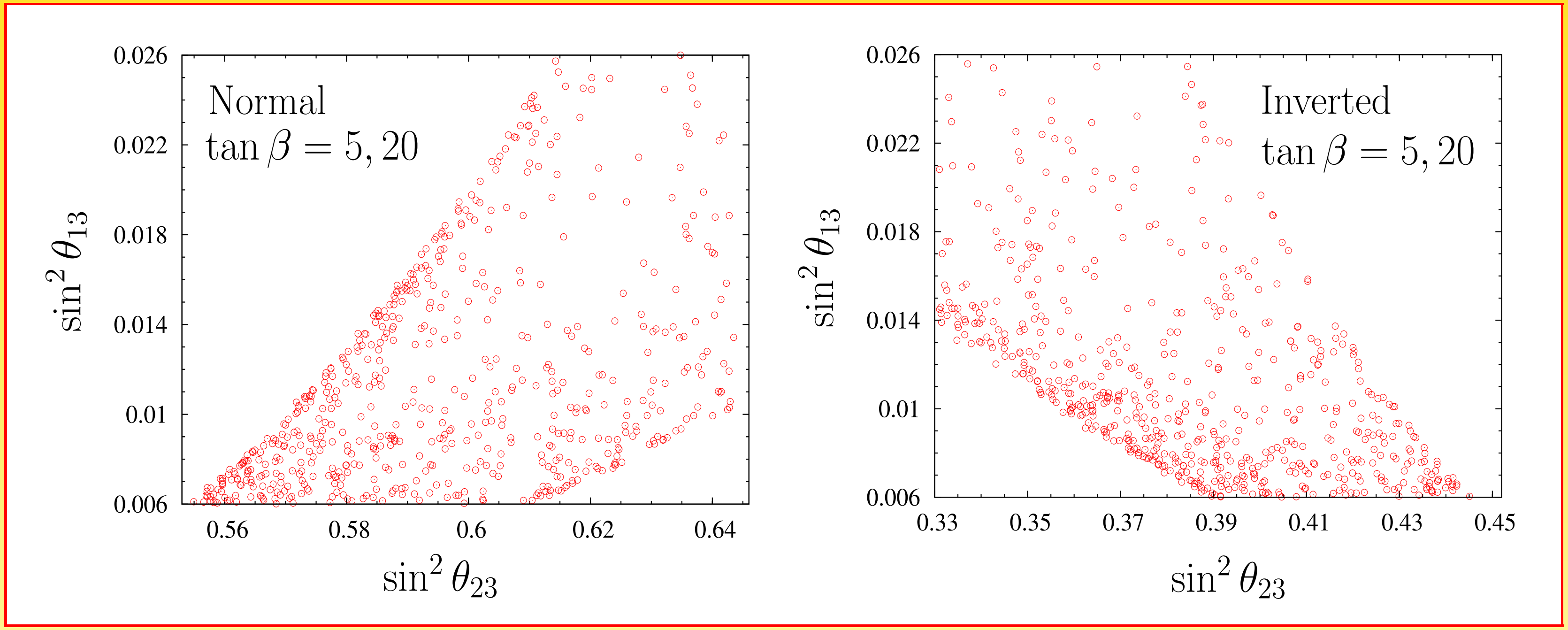
- SM: doesn't work

Renormalization and $|U_{e3}| \simeq 0.1$



- MSSM: $4 \lesssim (m_0/\mathrm{eV}) \tan\beta \lesssim 7$

Renormalization and $|U_{e3}| \simeq 0.1$



- $|\theta_{23} - \pi/4| = \mathcal{O}(|U_{e3}|)$
- can NOT be maximal

Lower Limits on θ_{13}?

- Planck-scale?

$$\mathcal{L} = \frac{1}{M_{\text{Pl}}}(L\,\Phi)^2 \Rightarrow |U_{e3}| \lesssim \frac{v^2}{M_{\text{Pl}}}\frac{1}{\sqrt{\Delta m^2_{\text{A}}}} \simeq 5\cdot 10^{-5}$$

but can be zero

- RG for $m_1 = 0$:

$$|U_{e3}| \gtrsim \frac{y_\tau^2}{32\,\pi^2}\frac{\sin 2\theta_{12}\,\sin 2\theta_{23}\,\sqrt{\Delta m^2_\odot}}{\sqrt{\Delta m^2_\odot + \sqrt{\Delta m^2_{\text{A}}}}}\ln\frac{\Lambda}{\lambda} \simeq \begin{cases} 1.1\cdot 10^{-6} & \text{SM} \\ 1.6\cdot 10^{-6}\,(1+\tan^2\beta) & \text{MSSM} \end{cases}$$

increases for non-zero neutrino masses with a factor

$$\frac{8\,m_0^2}{\left(\sqrt{\Delta m^2_\odot} + \sqrt{\Delta m^2_{\text{A}}}\right)\sqrt{\Delta m^2_\odot}} \simeq 1600\left(\frac{m_0}{0.3\,\text{eV}}\right)^2$$

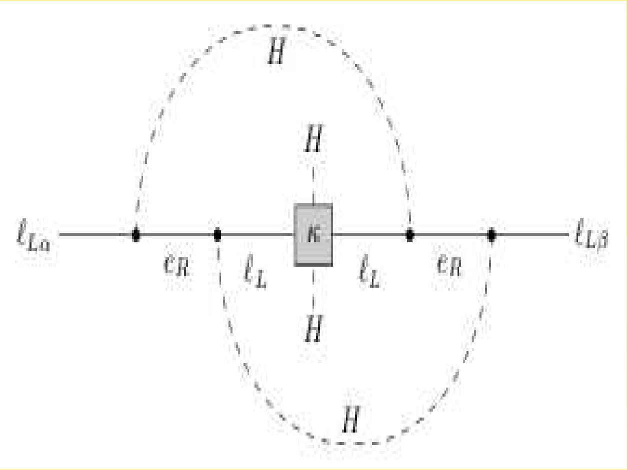
A case without RG?

$$\dot{\theta}_{13} \propto \dot{m}_3 \propto m_3$$

$\Rightarrow$ inverted hierarchy with $m_3 = \theta_{13} = 0$ receives no corrections!

$\Rightarrow$ need 2-loop RGEs, to obtain **most minimal value of** $|U_{e3}|$

$$\dot{m}_\nu = \frac{2}{(16\pi^2)^2} Y^T m_\nu Y \text{ with } Y = \text{diag}(y_e^2, y_\mu^2, y_\tau^2)$$

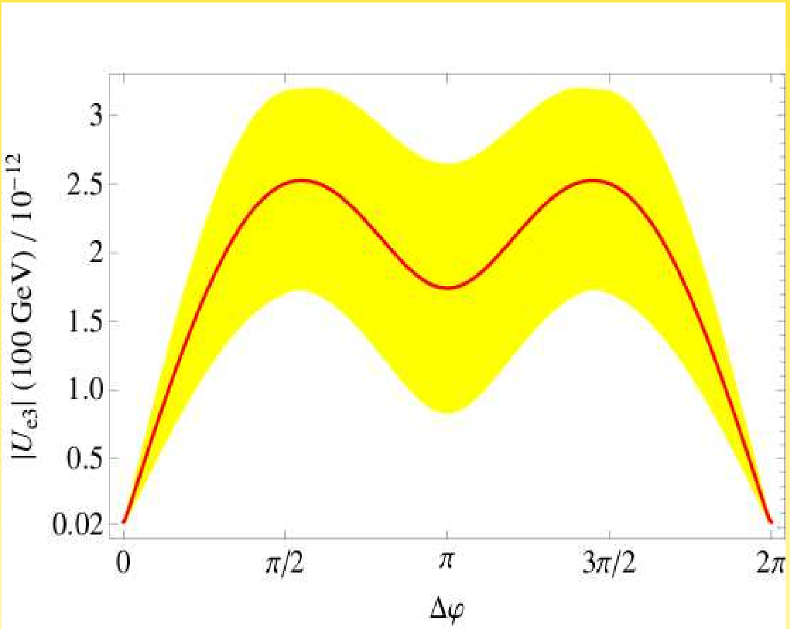


(Davidson, Isidori, Strumia, PLB **646** (2007) 100 have used it to get smallest neutrino mass $\sim 10^{-13}$ eV)

Most minimal value of $|U_{e3}|$

$$|\dot{U}_{e3}| \simeq \frac{y_\tau^4}{2\,(16\pi^2)^2}\,\frac{s_{23}^2}{m_1\,m_2}\,\sin 2\theta_{12}\,\sin 2\theta_{23}\big|m_1 - m_2\,e^{i\Delta\varphi}\big|\,\big|m_1\,s_{12}^2 + m_2\,c_{12}^2\,e^{i\Delta\varphi}\big|$$

lies between 10^{-12} and 10^{-14}



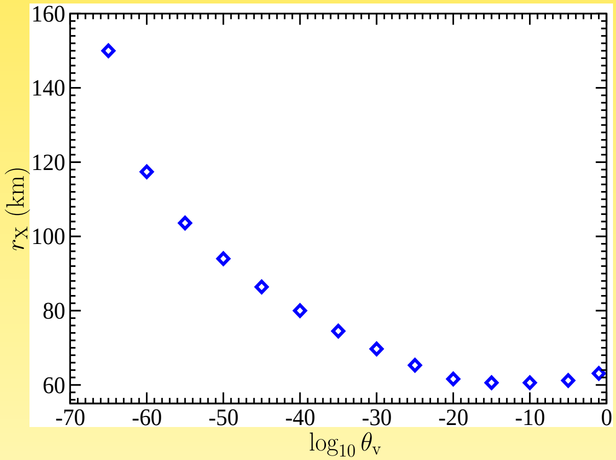
Ray, W.R., Schmidt, 1010.1206

Most minimal value of $|U_{e3}|$

- MSSM: from SUSY breaking with sleptons in the loop, or additive terms $\tilde{L}H_u\tilde{L}H_u$

$$|U_{e3}|^{\rm MSSM} \sim |U_{e3}|^{\rm SM}\,(1+\tan^2\beta)^2\,/\,\ln\frac{\Lambda}{\lambda}$$

- Supernova physics (collective effects)



Fuller *et al.*, PRL **99**, 241802 (2007)

How to perturb a mixing scenario/model

- Charged lepton corrections $U = U_\ell^\dagger \, U_{\rm (T)BM}$

$$U_\ell \simeq \left(\begin{array}{ccc} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right) \text{ gives}$$

$$\boxed{\sin^2 \theta_{12} \simeq \frac{1}{3} - \frac{2}{3} \, \lambda \cos \phi} \text{ or } \boxed{\sin^2 \theta_{12} \simeq \frac{1}{2} - \frac{1}{\sqrt{2}} \, \lambda \cos \phi}$$

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}} \, \lambda \quad , \quad J_{\rm CP} = \frac{\lambda}{6} \sin \phi$$

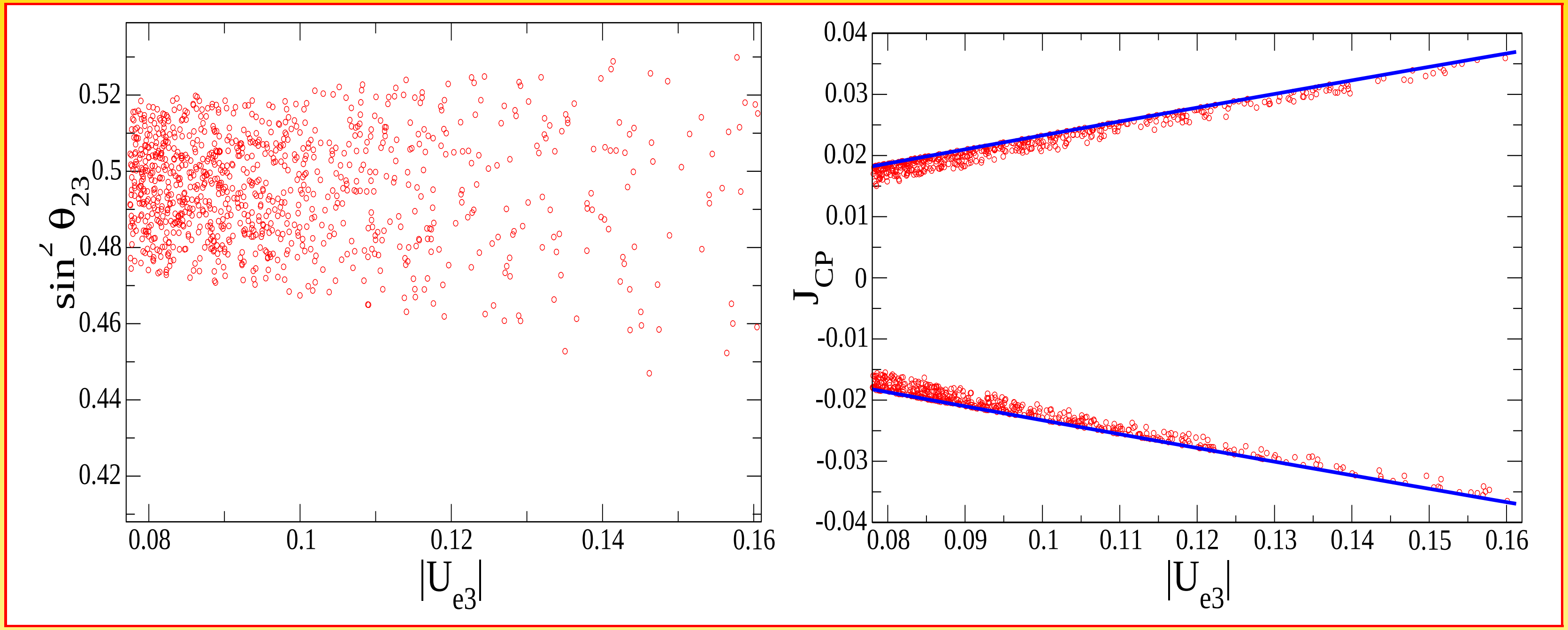
$$\sin^2 \theta_{23} \simeq \frac{1}{2} - \mathcal{O}(\lambda^2)$$

Direct correlation between U_{e3}, $\sin^2 \theta_{12}$ and CP violation!

For BM: small CP violation

For TBM: large CP violation

Charged Lepton Corrections and $|U_{e3}| \simeq 0.1$



- CP violation maximal!
- $|\theta_{23} - \pi/4| = \mathcal{O}(|U_{e3}|^2)$
- θ_{23} can be maximal

Goswami, Petcov, Ray, W.R., PRD **80**, 053013 (2009)

	charged leptons	renormalization (MSSM)	explicit breaking
$\sin^2\theta_{23}$	$0.44 - 0.53$	$0.55 - 0.64 \quad (\Delta m_A^2 > 0)$ $0.33 - 0.45 \quad (\Delta m_A^2 < 0)$	—
$\lvert U_{e3}\rvert$	$\simeq \frac{\lambda}{\sqrt{2}}$	$\propto \frac{m_0^2}{\Delta m_A^2}\,(1+\tan^2\beta)$	$\propto \epsilon \quad$ (IH) $\propto \epsilon\, m_1/\sqrt{\Delta m_A^2} \quad$ (PD/QD)
mass	—	QD: $m_0 \tan\beta \simeq (4-7)$ eV	IH, PD, QD
$\lvert m_{ee}\rvert$	—	$m_0\, c_{13}^2\, \cos 2\theta_{12}$	$m_0\, c_{13}^2\, \cos 2\theta_{12} \quad$ (QD) $\sqrt{\Delta m_A^2}\, c_{13}^2\, \cos 2\theta_{12} \quad$ (IH)
CP	oscillations: almost maximal CP violation	$\alpha_2 \simeq \pi$	large $\lvert U_{e3}\rvert$ requires suppressed $\lvert m_{ee}\rvert$ only when initially $\alpha_2 \simeq \pi$

$$0.077 \leq |U_{e3}| \leq 0.161$$

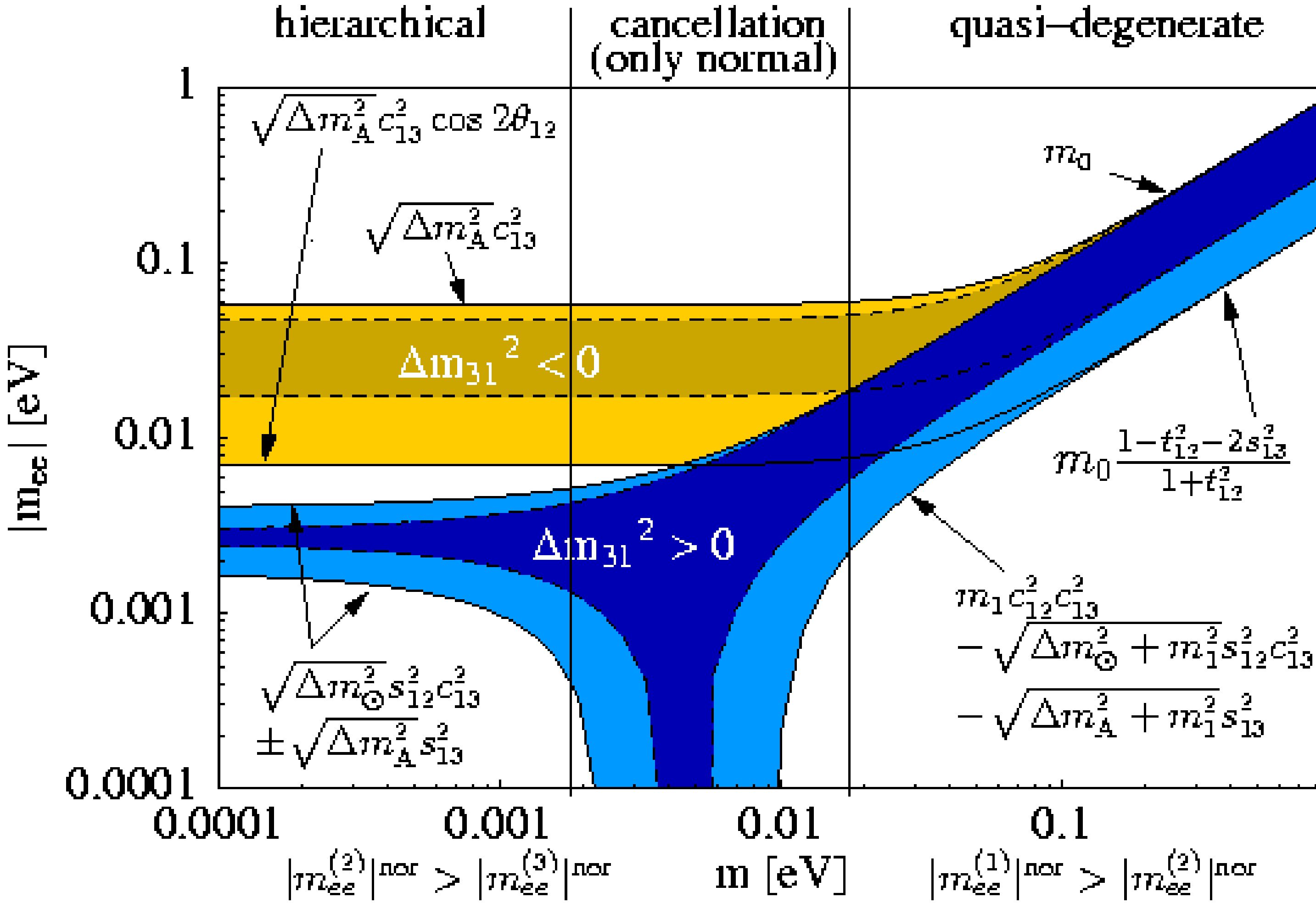
Summary so far

- Corrections to U_{e3} and θ_{23} similar
- do we need precision experiments for θ_{12}?
  - could distinguish very different approaches

$$\sin^2\theta_{12} = \frac{1}{2} - \lambda/\sqrt{2} \simeq 0.339 \quad \text{vs.} \quad \sin^2\theta_{12} = \frac{1}{3}$$

  - BUT: Corrections to θ_{12} tend to be largest...

where else is θ_{12} important?


hierarchical
cancellation (only normal)
quasi–degenerate
$\sqrt{\Delta m_A^2}\, c_{13}^2 \cos 2\theta_{12}$
$\sqrt{\Delta m_A^2}\, c_{13}^2$
m_0
$\Delta m_{31}{}^2 < 0$
$\Delta m_{31}{}^2 > 0$
$m_0 \frac{1 - t_{12}^2 - 2 s_{13}^2}{1 + t_{12}^2}$
$m_1 c_{12}^2 c_{13}^2 - \sqrt{\Delta m_\odot^2 + m_1^2 s_{12}^2 c_{13}^2} - \sqrt{\Delta m_A^2 + m_1^2 s_{13}^2}$
$\sqrt{\Delta m_\odot^2}\, s_{12}^2 c_{13}^2 \pm \sqrt{\Delta m_A^2}\, s_{13}^2$
$|m_{ee}|$ [eV]
1
0.1
0.01
0.001
0.0001
0.0001
0.001
0.01
0.1
1
$|m_{ee}^{(2)}|^{nor} > |m_{ee}^{(3)}|^{nor}$
m [eV]
$|m_{ee}^{(1)}|^{nor} > |m_{ee}^{(2)}|^{nor}$

Testing Inverted Ordering

Nature gives us a scale:

$$|m_{ee}|_{\rm min}^{\rm IH} = (1 - |U_{e3}|^2)\,\sqrt{|\Delta m_{\rm A}^2|}\,(1 - 2\sin^2\theta_{12}) = \begin{cases} (0.015 \ldots 0.020)\ {\rm eV} & 1\sigma \\ (0.010 \ldots 0.024)\ {\rm eV} & 3\sigma \end{cases}$$

Desiderata:

- small $|U_{e3}|$
- large $|\Delta m_{\rm A}^2|$
- small $\sin^2\theta_{12}$

Recall: a limit $|m_{ee}|_{\rm lim}$ scales with $\left(\frac{\Delta E\, B}{M\, t}\right)^{\frac{1}{4}}$

Talk by Kishimoto

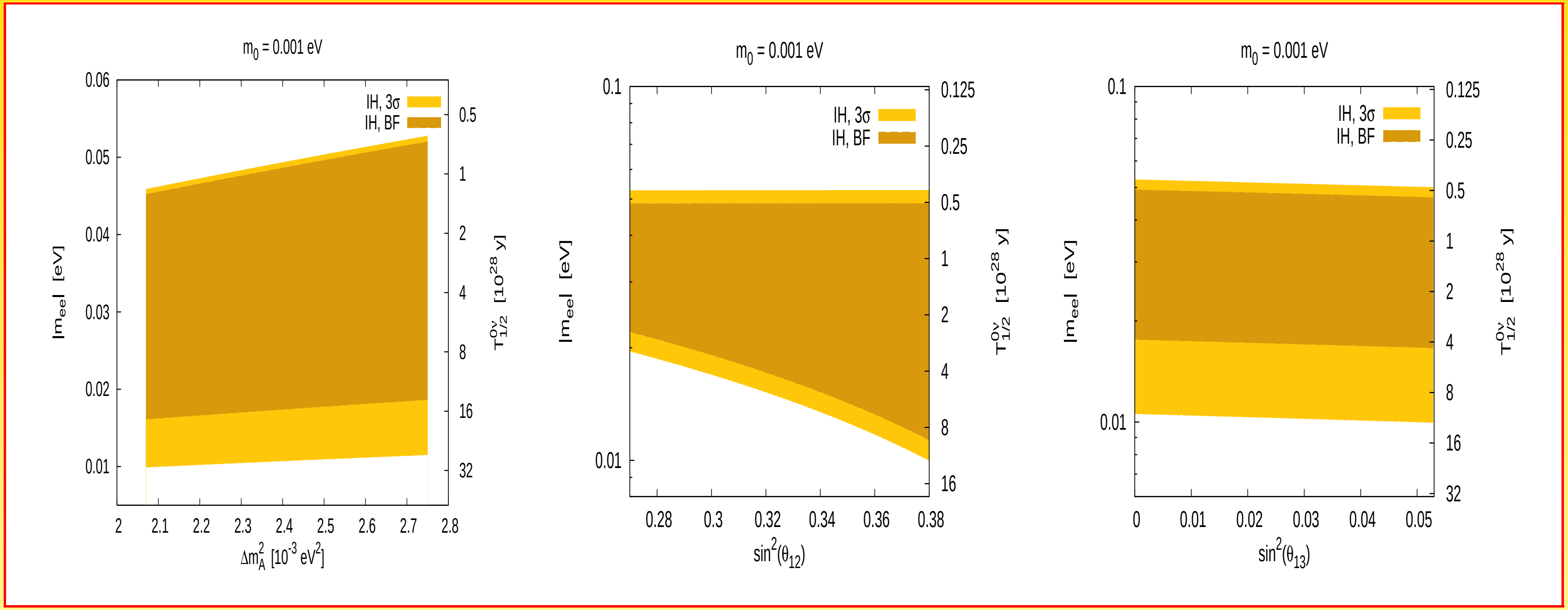
Testing Inverted Ordering

Nature gives us another scale:

$$|m_{ee}|^{\rm IH}_{\rm max} = \left(1 - |U_{e3}|^2\right) \sqrt{|\Delta m^2_{\rm A}|} = \left\{ \begin{array}{cc} (0.047 \ldots 0.050)~{\rm eV} & 1\sigma \\ (0.043 \ldots 0.052)~{\rm eV} & 3\sigma \end{array} \right.$$

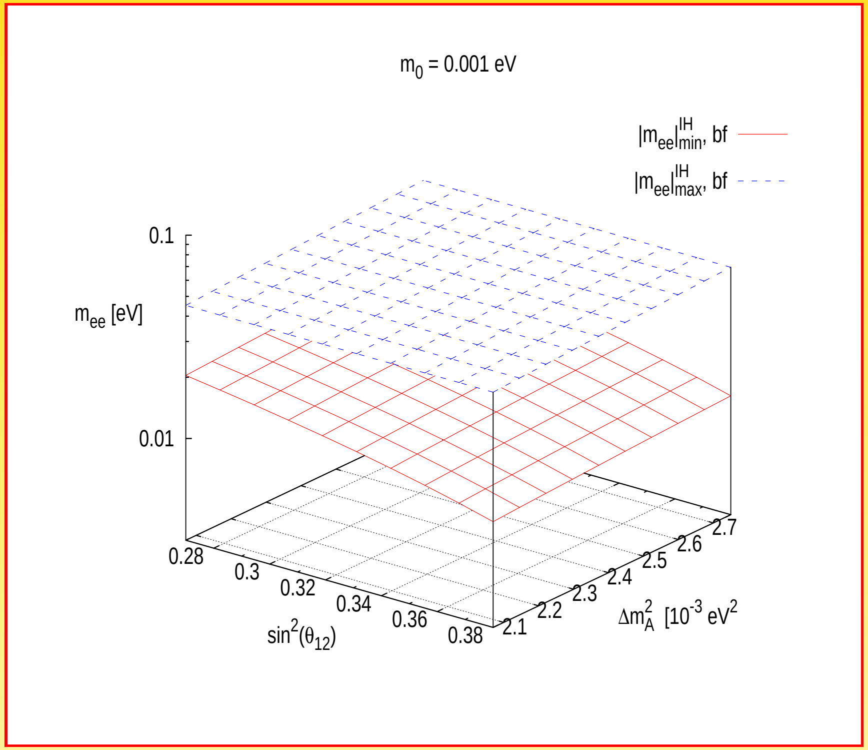
Desiderata:

- small $|U_{e3}|$
- large $|\Delta m^2_{\rm A}|$



$|m_{ee}|$ vs. $\Delta m^2_{\rm A}$ $|m_{ee}|$ vs. $\sin^2\theta_{12}$ $|m_{ee}|$ vs. $|U_{e3}|^2$

$\sin^2\theta_{12}$ gives strongest dependence!



$|m_{ee}|$ vs. $\sin^2\theta_{12}$ and $|U_{e3}|^2$

Non-Standard Neutrino Physics

- sterile neutrinos
  - LSND/MiniBooNE, return of the 3+1 scenarios
  - cosmology: $N_{\text{eff}} \simeq 4$
  - phenomenology: mass-related observables
  - *expected from theory?*
- Non-Standard Interactions
  - effective approach: $\mathcal{L} = \epsilon_{\alpha\beta}\, G_F\, (\overline{\nu}_\alpha \gamma_\mu \nu_\beta)\, (\overline{f} \gamma^\mu f)$
  - expectation $\epsilon_{\alpha\beta} = \mathcal{O}\left(\frac{m_W}{m_{\rm NP}}\right)^2 \times \text{flavor}_{\alpha\beta}$
  - gauge invariance reduces neutrino limits...

Unitarity violation $U\,U^\dagger \neq 1$, or $N = (1+\eta)U_0$

- sub-percent bounds already there (Antusch *et al.*)
- expected from type I and III seesaw, but tiny unless mild hierarchy in M_ν and/or extended see-saws, e.g. inverse see-saw (Mohapatra, Valle)

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & m_{RS}^T \\ 0 & m_{RS} & M_S \end{pmatrix} \quad M_S \ll m_D \ll m_{RS}$$

$$m_\nu \simeq \left(\frac{m_D}{10^2\,\mathrm{GeV}}\right)^2 \left(\frac{\mathrm{TeV}}{m_{RS}}\right)^2 \left(\frac{M_S}{0.1\,\mathrm{keV}}\right)\mathrm{eV}$$

$$\eta \simeq -\tfrac{1}{2} m_D^\dagger\,(m_{RS}^*)^{-1}\,(m_{RS}^T)^{-1}\,m_D$$

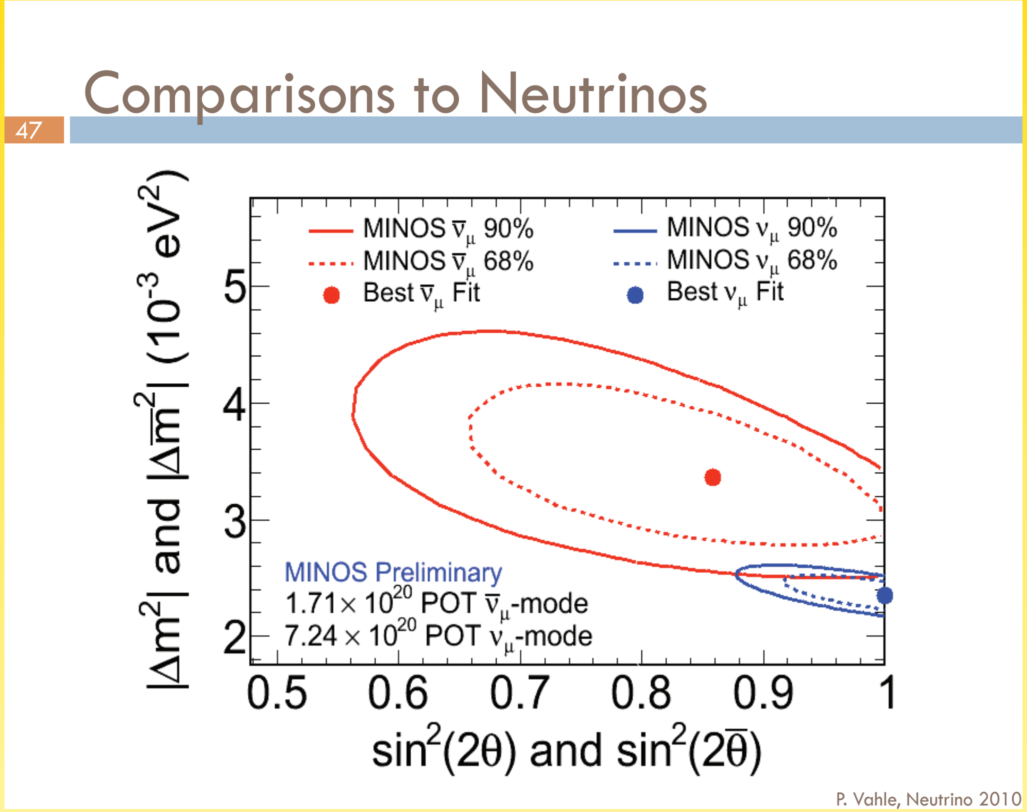
equivalent: NLO term $m_\nu^1 = \mathcal{O}(m_D^4\, m_{RS}^{-4}\, M_S)$ at percent level

- "natural" also in E_6 models with $\lesssim$TeV "exotic" new leptons (Stech)

Non-Standard Neutrino Physics

- new $U(1)$ generating interactions/potential for neutrinos (“leptonic forces”, Z', etc.)
  - no real expectation on scale
  - neutrinos can give best limits on such forces (better than equivalence principle)
- exotic exotics
  - Lorentz invariance violation
  - CPT violation
    - for mass scale m expect it to be order $m/M_{\rm Pl}$
    - see-saw: $M_R/M_{\rm Pl} \Rightarrow$ permille ?
  - Fermi-Dirac statistics violation
  - ...

Example MINOS anomaly



$$\Delta m^2 = \left(2.35^{+0.11}_{-0.08}\right) \times 10^{-3}\,\mathrm{eV}^2\,, \quad \sin^2 2\theta > 0.91$$

$$\overline{\Delta m^2} = \left(3.36^{+0.45}_{-0.40}\right) \times 10^{-3}\,\mathrm{eV}^2\,, \quad \sin^2 2\overline{\theta} = 0.86 \pm 0.11$$

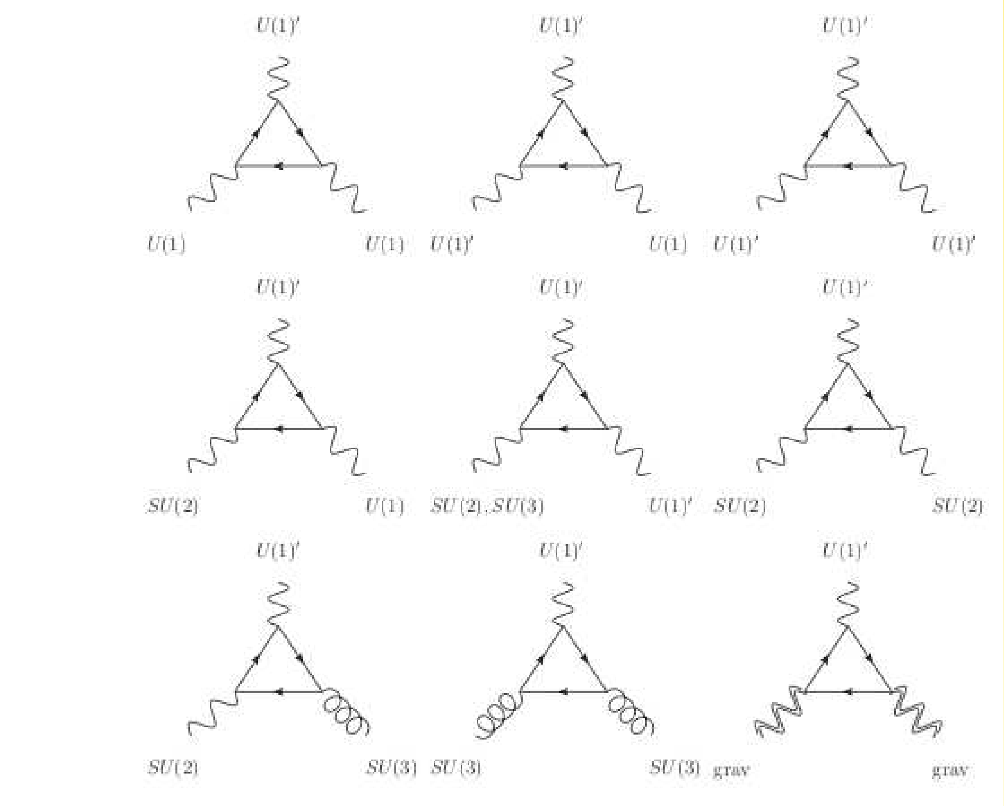
Is this...

- ...Non-Standard Interaction (Mann, Cherdack, Musial, Kafka, 1006.5720; Kopp, Machado, Parke, 1009.0014)?
- ...sterile neutrino (plus gauged Z' from $U(1)$ according to $B - L$) (Engelhardt, Nelson, Walsh, 1002.4452)?
- ...gauged ultra-light Z' from $U(1)$ according to $L_\mu - L_\tau$ (Heeck, W.R., 1007.2655)?
- ...CPT violation? (Barenboim, Lykken, 0908.2993; Choudhury, Datta, Kundu, 1007.2923)?
- ...nothing and will go away (common sense)?

Gauged $L_\alpha - L_\beta$

$L_e - L_\mu$ or $L_e - L_\tau$ or $L_\mu - L_\tau$ can be gauged **without anomaly** in SM

(Foot, 1991)



- SM: $\sum Y_i = 0$ in each family
- extra $U(1)$: anomalies cancel among different (lepton) generations
- example $L_e - L_\mu$: ν_e, L_e have $Q = 1$, ν_μ, L_μ have $Q = -1$
- there is an extra Z' which couples to ν_e, L_e and ν_μ, L_μ with coupling g'
- no expectation for mass scale. . .

- if Z' from $L_e - L_\alpha$ is ultra-light: particles in Sun (or Earth) create potential for terrestrial neutrinos (Joshipura, Mohanty, PLB **584**, 103 (2004))

$$V = \frac{g'^2}{4\pi} \frac{N_e}{R} \equiv \alpha \frac{N_e}{R}$$

Scale: $m'_Z \le 1/\mathrm{A.U.} \simeq 10^{-18}$ eV...

V must be added to Hamiltonian:

$$\Rightarrow \mathcal{H}_{e\mu} = \frac{\Delta m^2}{4\,E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix}$$

$\leftrightarrow$ looks like NSI, but **does not depend on matter density!**

$\leftrightarrow$ also works for vacuum oscillations!

- V changes sign for anti-neutrinos!

$$\Rightarrow P(\nu_\alpha \to \nu_\alpha) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) \text{ without CPT violation}$$

$$V = \alpha \frac{N_e}{R_{\rm A.U.}} = \alpha \frac{6 \times 10^{56}}{8 \times 10^{17}\,{\rm eV}^{-1}} \simeq 8 \times 10^{38}\, \alpha\, {\rm eV}$$

- atmospheric neutrinos

$$\frac{\Delta m^2_{\rm A}}{4E} \simeq 6 \times 10^{-13} \left(\frac{\rm GeV}{E}\right) {\rm eV}$$

Limits $\alpha_{e\mu} \leq 5.5 \times 10^{-52}$ and $\alpha_{e\tau} \leq 6.4 \times 10^{-52}$

(Joshipura, Mohanty, PLB **584**, 103 (2004))

- solar neutrinos

$$\frac{\Delta m^2_{\odot}}{4E} \simeq 2 \times 10^{-11} \left(\frac{\rm MeV}{E}\right) {\rm eV}$$

Limits $(\theta_{13} = 0)$ $\alpha_{e\mu} \leq 3.4 \times 10^{-53}$ and $\alpha_{e\tau} \leq 2.5 \times 10^{-53}$

(Bandyopadhyay, Dighe, Joshipura, PRD **75**, 093005 (2007))

stronger than limits from equivalence principle!

Gauged $L_\mu - L_\tau$

never considered for oscillation physics, but very interesting because

$$L_e - L_\tau : m_\nu = \begin{pmatrix} 0 & 0 & a \\ \cdot & b & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \text{ not successful}$$

but $L_\mu - L_\tau$ has zeroth order mass matrix

$$L_\mu - L_\tau : m_\nu = \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$$

masses $a, \pm b$ and $U_{e3} = 0$, $\theta_{23} = \pi/4$

automatically μ–τ symmetric!

flavor $\leftrightarrow$ gauge

Gauged $L_\mu - L_\tau$

$$\mathcal{L} = -\frac{1}{4} Z'_{\mu\nu}\, Z'^{\mu\nu} + \frac{1}{2} M'^2_Z\, Z'_\mu\, Z'^\mu - g'\, j'^\mu\, Z'_\mu - \frac{\sin\chi}{2}\, Z'^{\mu\nu}\, B_{\mu\nu} + \delta M^2\, Z'_\mu\, Z^\mu$$

with new current

$$j'^\mu = \bar{\mu}\,\gamma^\mu\,\mu + \bar{\nu}_\mu\,\gamma^\mu\, P_L\,\nu_\mu - \bar{\tau}\,\gamma^\mu\,\tau - \bar{\nu}_\tau\,\gamma^\mu\, P_L\,\nu_\tau$$

Diagonalizing kinetic and mass terms gives

$$\mathcal{L}_A = -e\,(j_{\mathsf{EM}})_\mu\, A^\mu$$

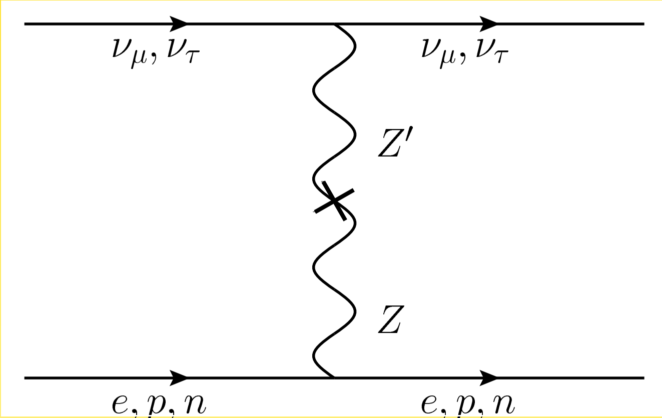
$$\mathcal{L}_{Z_1} = -\left(\frac{e}{s_W\, c_W}\left((j_3)_\mu - s_W^2\,(j_{\mathsf{EM}})_\mu\right) + g'\,\xi\,(j')_\mu\right) Z_1^\mu$$

$$\mathcal{L}_{Z_2} = -\left(g'\,(j')_\mu - \frac{e}{s_W\, c_W}\,(\xi - s_W\,\chi)\left((j_3)_\mu - s_W^2\,(j_{\mathsf{EM}})_\mu\right) - e\, c_W\,\chi\,(j_{\mathsf{EM}})_\mu\right) Z_2^\mu$$

$\Rightarrow$ Z-Z' mixing

Potential through Z-Z' mixing (Heeck, W.R., 1007.2655):

$$V = g'\,(\xi - s_W\,\chi)\,\frac{e}{4\,s_W\,c_W}\,\frac{N_n}{4\pi R_{A.U.}} \equiv \alpha\,\frac{e}{4\,s_W\,c_W}\,\frac{N_n}{4\pi R_{A.U.}}$$



With $\eta = 2\,E\,V/\Delta m^2$:

$$\sin^2 2\theta_V = \frac{\sin^2 2\theta}{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\Delta m_V^2 = \Delta m^2 \sqrt{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

Recall: V changes sign for anti-neutrinos!!

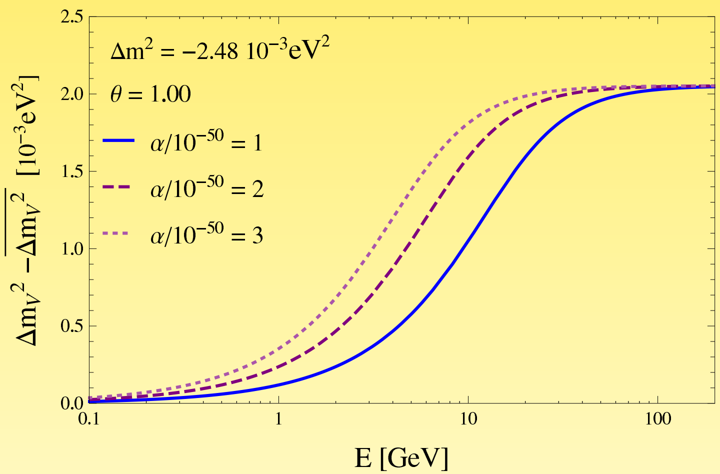
$$\sin^2 2\theta_V = \frac{\sin^2 2\theta}{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\Delta m_V^2 = \Delta m^2 \sqrt{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\Delta m_V^2 - \overline{\Delta m_V^2} = \Delta m^2 \sqrt{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2} - \Delta m^2 \sqrt{1 + 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\simeq -4\,\eta\,\Delta m^2\,\cos 2\theta$$

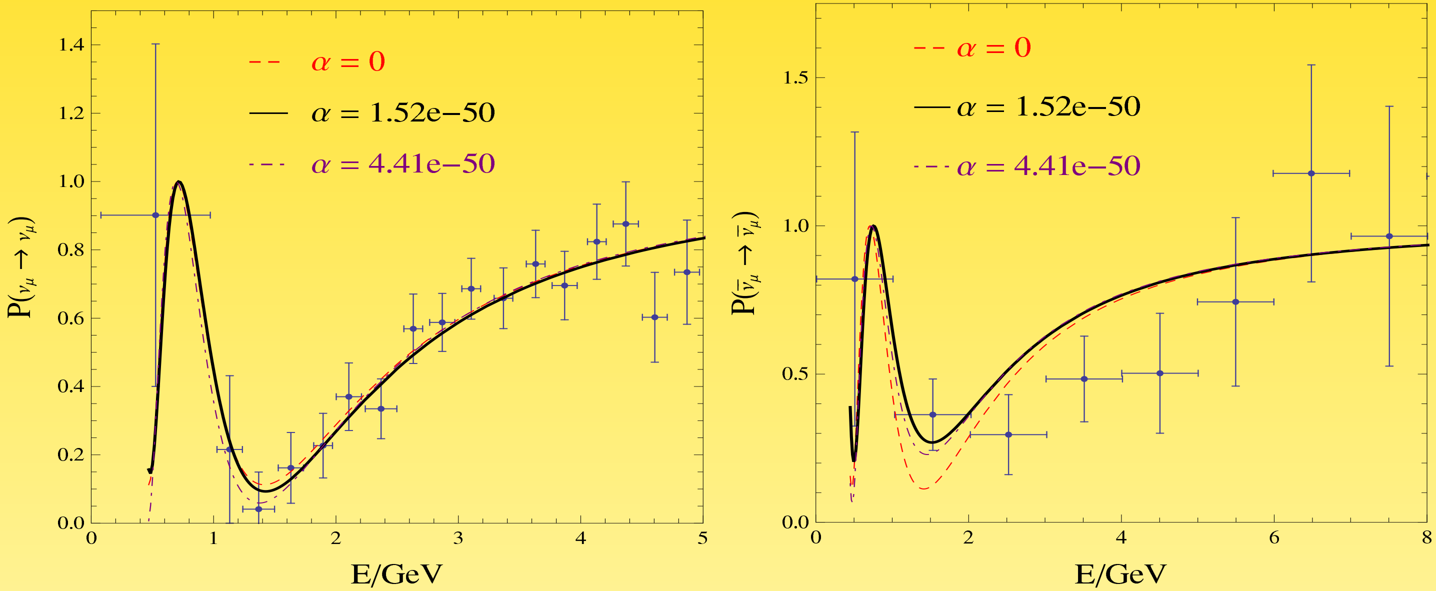
$\Rightarrow$ works only with non-maximal θ



Gauged $L_\mu - L_\tau$

$$\Delta m_V^2 - \overline{\Delta m_V^2} \simeq -4\,\eta\,\Delta m^2\,\cos 2\theta$$

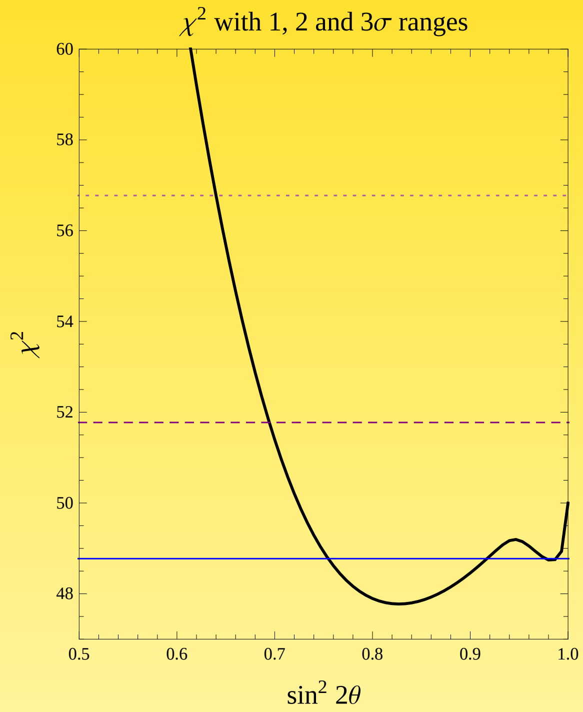
$\Rightarrow$ goes only with non-maximal θ

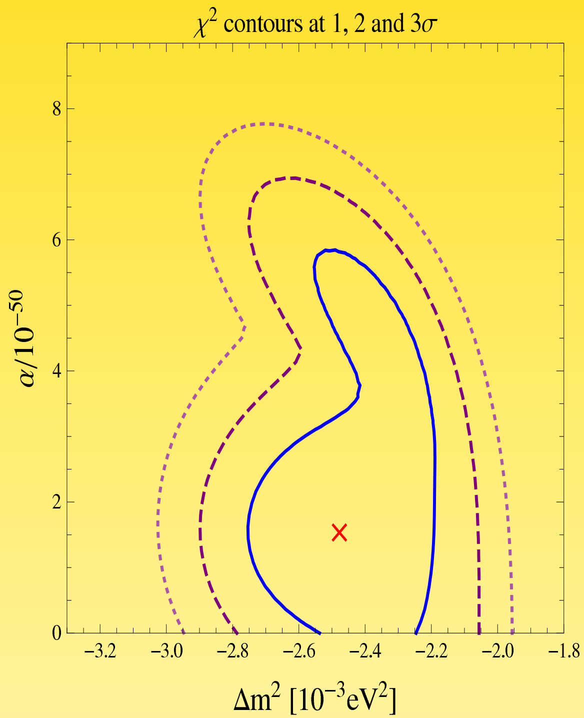


$$\sin^2 2\theta = 0.83 \pm 0.08\,,\quad \Delta m^2 = (-2.48 \pm 0.19) \times 10^{-3}\,\mathrm{eV}^2\,,\quad \alpha = \left(1.52^{+1.17}_{-1.14}\right) \times 10^{-50}$$

with $\chi^2_{\rm min}/N_{\rm dof} = 47.77/50$, (without α: $\chi^2_{\rm min}/N_{\rm dof} = 49.43/51$)

Heeck, W.R., 1007.2655


χ^2 with 1, 2 and 3σ ranges
χ^2
60
58
56
54
52
50
48
0.5
0.6
0.7
0.8
0.9
1.0
$\sin^2 2\theta$


χ^2 contours at 1, 2 and 3σ
$\alpha/10^{-50}$
8
6
4
2
0
−3.2
−3.0
−2.8
−2.6
−2.4
−2.2
−2.0
−1.8
Δm^2 [10^{-3}eV^2]

- looks in $\mathcal{H}$ like NSI, hence apply NSI limits

$$\alpha = 10^{-50} \Rightarrow |\epsilon_{\mu\mu}^{\oplus}| \simeq 0.25$$

current limit

$$|\epsilon_{\mu\mu}^{\oplus}| \lesssim 0.068 \Rightarrow \alpha \simeq 10^{-51}$$

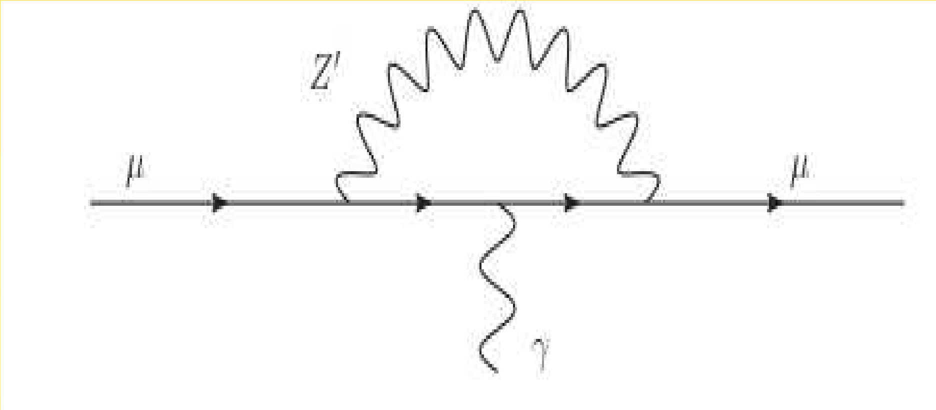
- 3-flavor effects...?

GLoBES

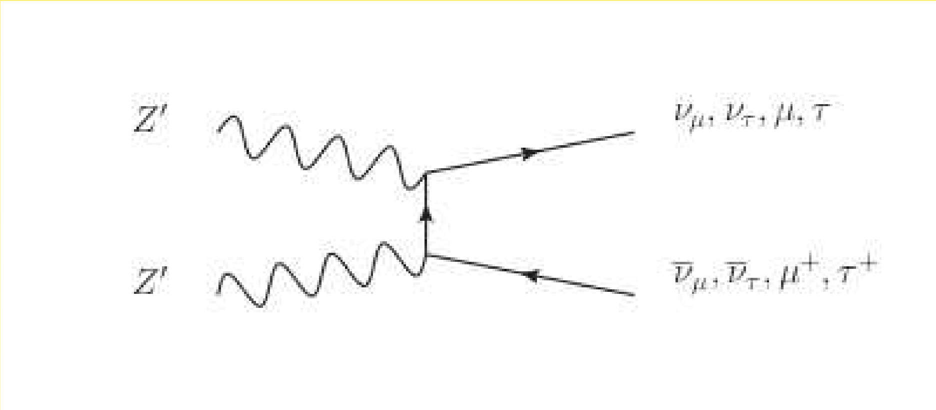
Experiment	Sensitivity to $\alpha/10^{-50}$ at 99.73% CL
T2K (ν-run)	11.8
T2K	4.3
T2HK	1.7
SPL	7.5
NOνA	1.9
Combined Superbeams	1.4
Nufact	0.53

Other aspects/limits of $L_\mu - L_\tau$

- $\Delta a_\mu = {g'}^2/(8\pi^2)$



- BBN: $\Gamma(Z'\,Z' \to \nu_{\mu,\tau}\,\nu_{\mu,\tau}) \propto g'^2\,T \Rightarrow g' \lesssim 10^{-5}$



- other EW precision: there are only $\sim 10^8$ $Z\ldots$

Other aspects/limits of $L_\mu - L_\tau$

- coupling of Z' with electromagnetic current gives modified charge

$$\frac{Q(\mu^+)}{Q(e^+)} \simeq 1 + \frac{g'}{e}\left((\xi - s_W\,\chi)(\frac{1}{4} - s_W^2)/(s_W\,c_W) + c_W\,\chi\right)$$

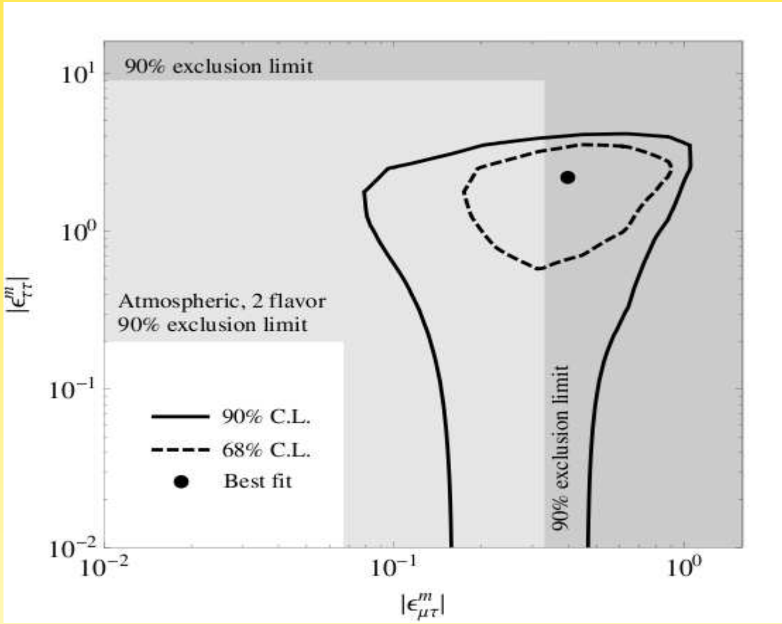
measured to be 1 ± 10^{-9}

Non-Standard Interactions

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}}\,\epsilon^f_{\alpha\beta}\,(\overline{\nu}_\alpha\gamma_\mu\nu_\beta)\,(\overline{f}\gamma^\mu f)$$

and $\epsilon_{\alpha\beta} \longrightarrow \epsilon^*_{\alpha\beta}$ for anti-neutrinos

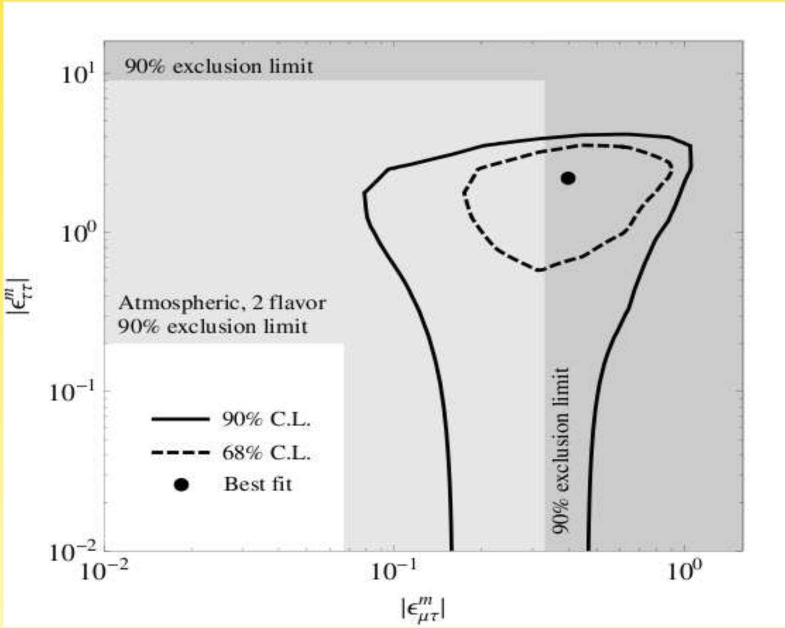
$$\mathcal{H} = \frac{1}{2E}\left[U\begin{pmatrix}0 & 0\\ 0 & \Delta m^2_{32}\end{pmatrix}U^\dagger + A\begin{pmatrix}\epsilon^\oplus_{\mu\mu} & \epsilon^\oplus_{\mu\tau}\\ \epsilon^{\oplus *}_{\mu\tau} & \epsilon^\oplus_{\tau\tau}\end{pmatrix}\right]$$



Kopp, Machado, Parke, 1009.0014 (only $\epsilon^\oplus_{\mu\tau}$: Mann *et al.*, 1006.5720)

NSIs

$$\mathcal{H} = \frac{1}{2E}\left[U\begin{pmatrix}0 & 0\\ 0 & \Delta m^2_{32}\end{pmatrix}U^\dagger + A\begin{pmatrix}\epsilon^{\oplus}_{\mu\mu} & \epsilon^{\oplus}_{\mu\tau}\\ \epsilon^{\oplus *}_{\mu\tau} & \epsilon^{\oplus}_{\tau\tau}\end{pmatrix}\right]$$

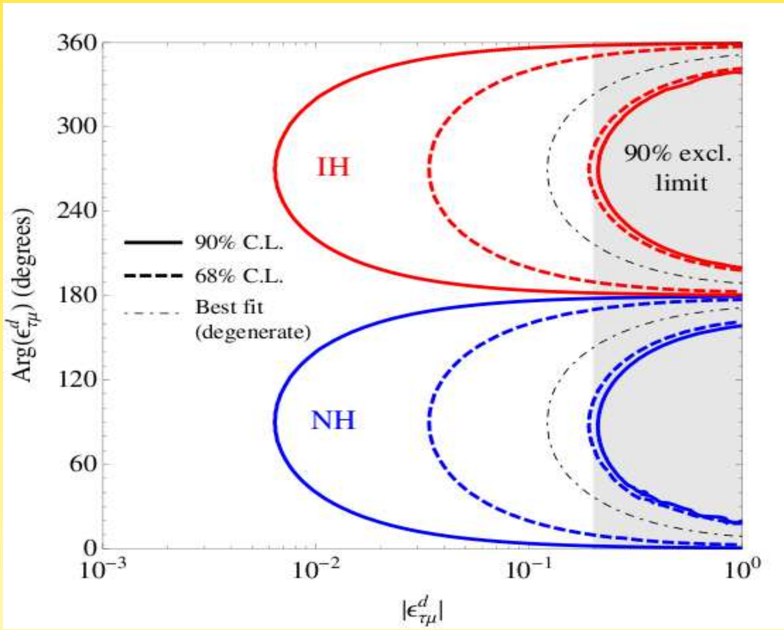


Charged Current NSIs

$$\mathcal{L}_{\rm NSI} \supset -2\sqrt{2}\, G_F\, \epsilon^d_{\tau\mu}\, V_{ud}\, [\bar{u}\gamma^\mu d]\, [\bar{\mu}\gamma_\mu P_L \nu_\tau]$$

leads to interference of

$\nu_\mu \rightsquigarrow \nu_\tau + N \to X + \mu$ and $\nu_\mu + N \to X + \mu$



Kopp, Machado, Parke, 1009.0014

Gauge Invariance strikes back!

$$\mathcal{L}_{\rm NSI} \supset -2\sqrt{2}\, G_F\, \epsilon^d_{\tau\mu}\, V_{ud}\, [\bar{u}\gamma^\mu d]\, [\bar{\mu}\gamma_\mu P_L \nu_\tau]$$

gives 1-loop diagram for $\tau \to \mu\,\pi^0$: $|\epsilon^d_{\tau\mu}| \leq 0.2$

BUT: gauge invariant term

$$\mathcal{L}_{\rm NSI} \supset -2\sqrt{2}\, G_F\, \epsilon^d_{\tau\mu}\, V_{ud}\, [\bar{U}\gamma^\mu U]\, [\bar{L}_\mu\gamma_\mu L_\tau]$$

gives tree-level diagram for $\tau \to \mu\,\pi^0$: $|\epsilon^d_{\tau\mu}| \leq 10^{-4}$

Gavela, Talk@NOW2010

$\Leftrightarrow$ **this argument does not apply to gauged $U(1)$!**

Mass Observables

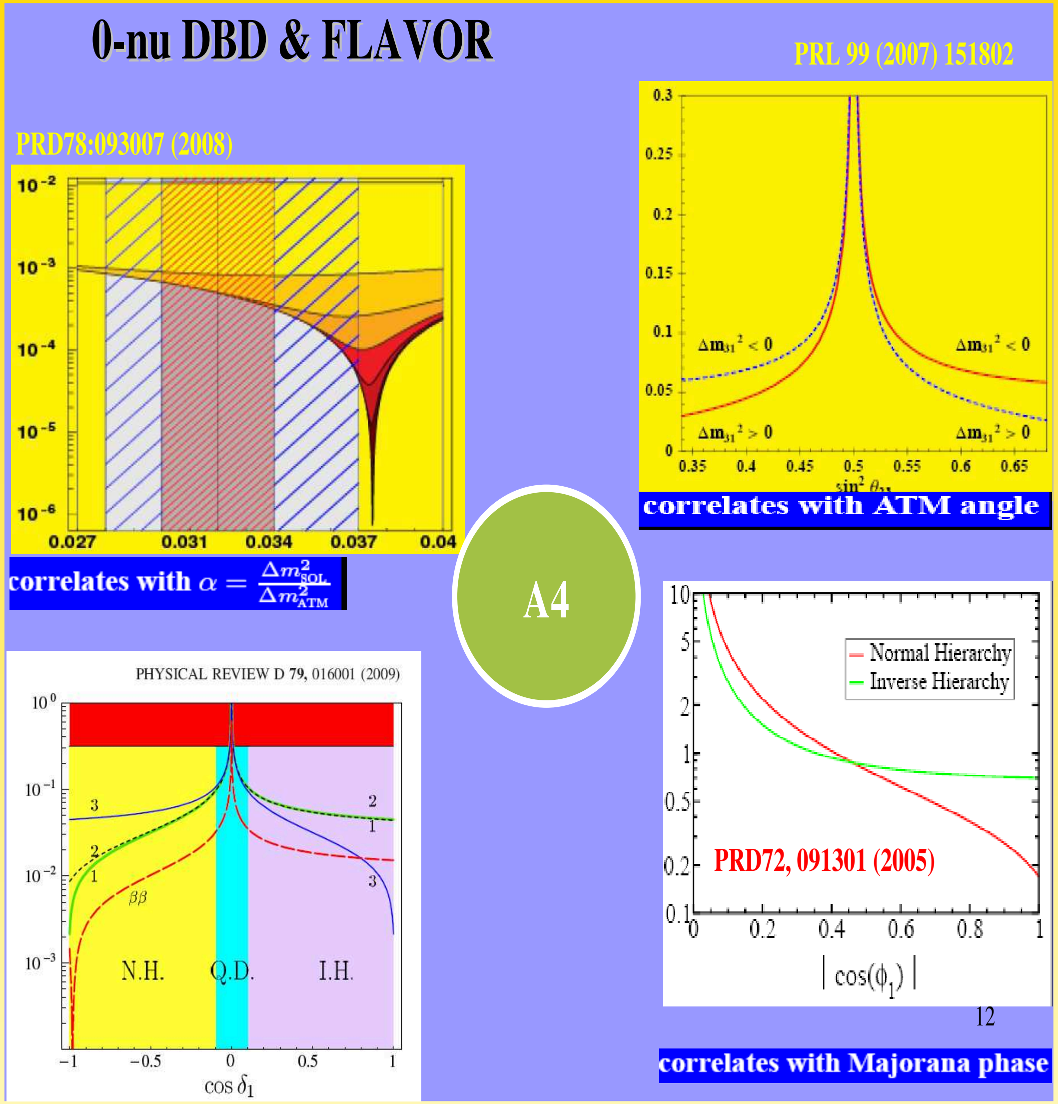
Example: 58 models based on A_4 leading to tri-bimaximal mixing:

Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	-	[1–11] [12]#
A2				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[13, 14]
A3				$\underline{1}, \underline{3}$	[15]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[4, 16–21]# [22, 23]* [24–35]
B2				$\underline{1}, \underline{3}$	[36]#
C1	$\underline{3}$	$\underline{3}$	-	-	[2]
C2				$\underline{1}$	[37, 38] [39]#
C3				$\underline{1}, \underline{3}$	[40]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[41]
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	-	[42, 43]* [44, 45]
D2				$\underline{1}$	[46] [47]*
D3				$\underline{1}'$	[48]*
D4				$\underline{1}', \underline{3}$	[49]*
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[50, 51]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[52]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[53]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[54]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[55]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[56, 57]
K	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	$\underline{1}$	[58]*
L	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}$	-	[59]*

Barry, W.R., PRD **81**, 093002 (2010)

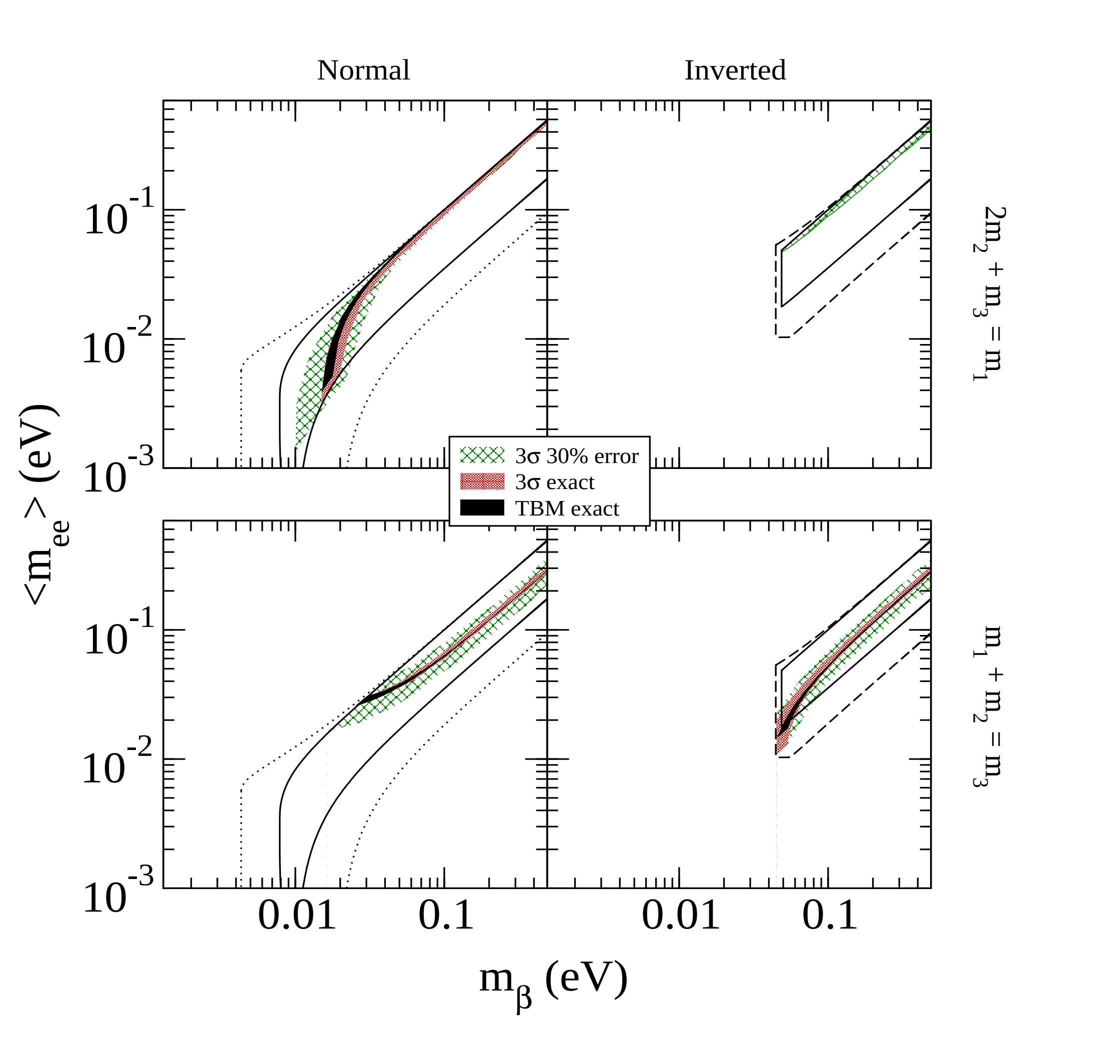
How to distinguish?

- LFV
- low scale scalars: Higgs, LFV
- compatible with GUTs?
- leptogenesis possible?
- neutrino mass sum-rules!


0-nu DBD & FLAVOR
PRL 99 (2007) 151802
PRD78:093007 (2008)
correlates with $\alpha = \frac{\Delta m^2_{SOL}}{\Delta m^2_{ATM}}$
correlates with ATM angle
A4
PHYSICAL REVIEW D 79, 016001 (2009)
N.H.
Q.D.
I.H.
cos δ_1
Normal Hierarchy
Inverse Hierarchy
PRD72, 091301 (2005)
$|\cos(\phi_1)|$
correlates with Majorana phase

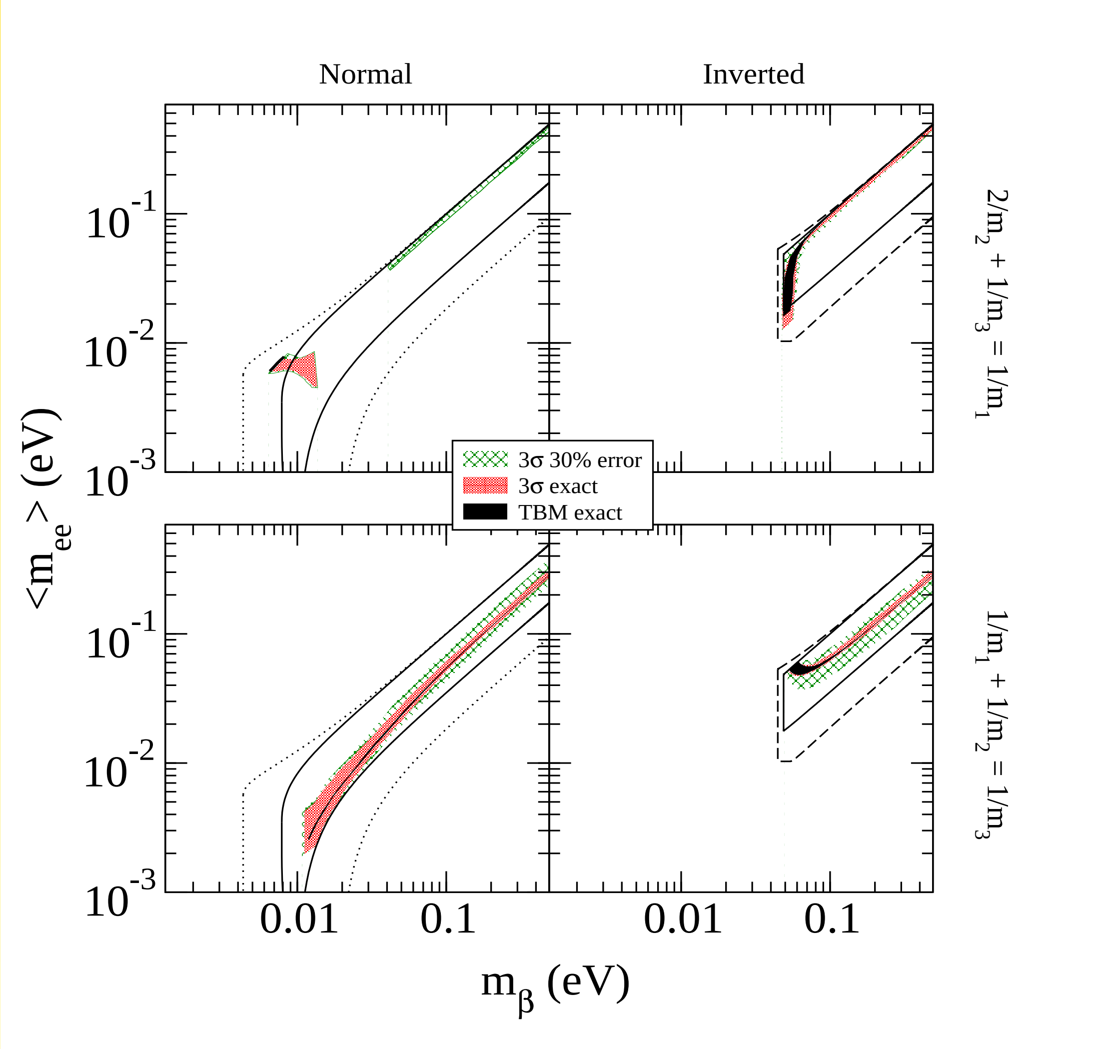
Valle

Sum-rules in Models and $0\nu\beta\beta$



Barry, W.R., NPB **842**, 33 (2011)

Sum-rules in Models and $0\nu\beta\beta$



Barry, W.R., NPB **842**, 33 (2011)

Mass Observables and New Physics

LSND/MiniBooNE/cosmology are compatible with a sterile neutrino having

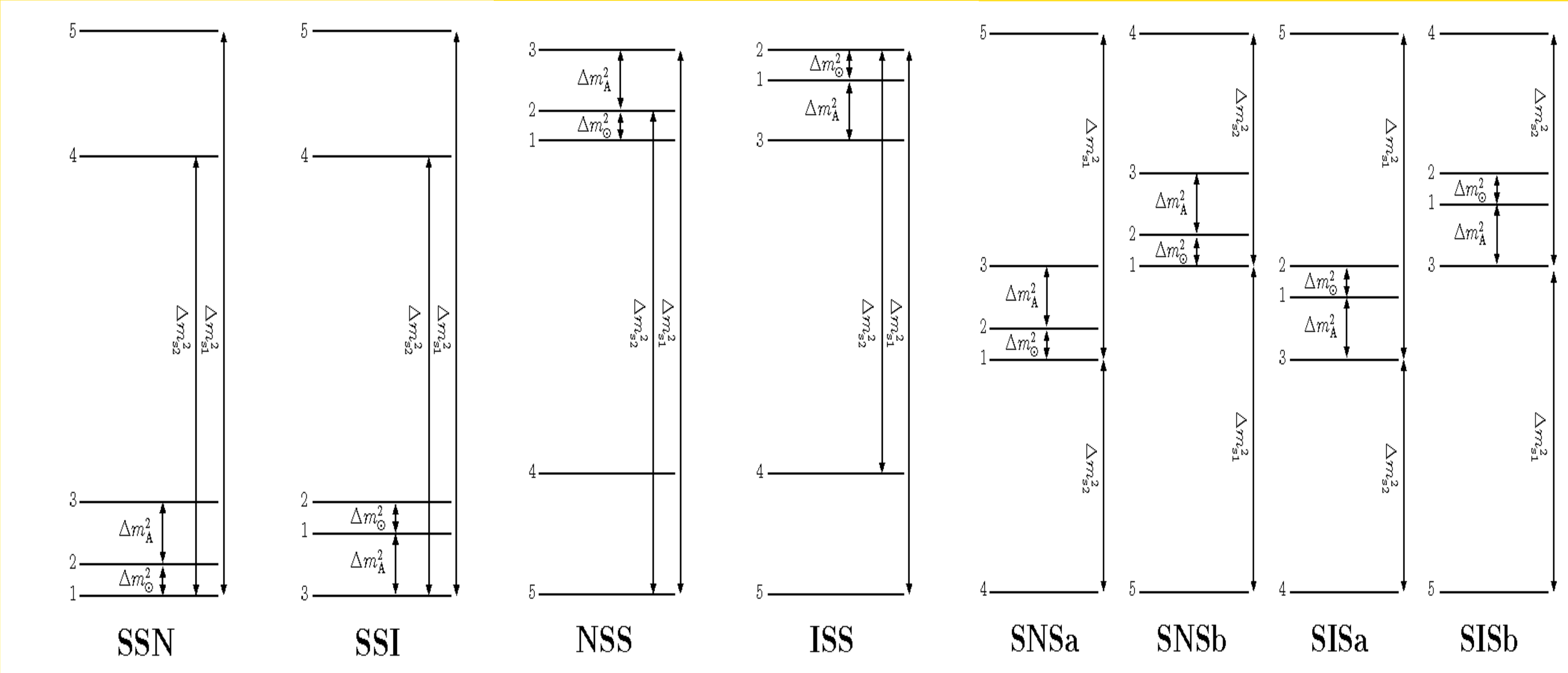
$\sim$ eV mass and $\sim$ 0.1 mixing

$$|m_{ee}|^{\rm st} \simeq (0.1)^2 \times 1 \sim 0.01 \text{ eV}$$

$$m_\beta^{\rm st} \simeq \sqrt{(0.1)^2 \times 1^2} \sim 0.1 \text{ eV}$$

is of order of inverted hierarchy contribution

Suppose there are 2 sterile neutrinos



8 Orderings → 4 phenomenologies

4 cases to the right usually NOT considered in fits. . .

scheme	KATRIN	$0\nu\beta\beta$	feature
SSN	maybe	maybe	NH plus ν_{s_1}, ν_{s_2}
SSI	maybe	maybe	IH plus ν_{s_1}, ν_{s_2}
NSS, ISS, SNSb, SISb	yes	yes	QD with $\sqrt{\Delta m^2_{\mathrm{s1}}}$
SNSa, SISa	yes	yes	QD with $\sqrt{\Delta m^2_{\mathrm{s2}}}$

Goswami, W.R., JHEP **0710**, 073 (2007)

Summary

- Corrections/Predictions for θ_{13} and $\theta_{23} - \pi/4$ are typically similar
- to test them on a level of 0.1 is a good idea...
- once θ_{13} will be determined, deviation from maximal θ_{23} will become crucial
- interesting theoretical speculations on θ_{12}, but receives typically large corrections (of phenomenological interest for $0\nu\beta\beta$)
- Non-standard neutrino physics: various speculations and hints, with different theoretical expectation/motivation
  - not really comparable to SUSY in quark flavor physics
  - but: neutrino mass generation different: keep on looking!

Go seisho arigato gozaimashita