Gromov-Witten Invariants

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A Gromov-Witten invariant of a projective manifold X is the virtual count of stable maps from a Riemann surface of a fixed genus g to X satisfying certain incidence constraints. Namely, we select several points on the Riemann surface, several cycles in X, and we require that the marked points are mapped to the cycles in such a way that the surface is tangent to the cycles with a certain degree of tangency. When we impose sufficiently many incidence constraints, the number of such maps becomes finite. Selecting different constraints we obtain an infinite sequence of numbers called the Gromov-Witten invariants of X. Probably the most mysterious conjecture about them is that they satisfy a recursion relation which can be formulated in terms of the representation theory of the Virasoro algebra. If in addition X has sufficiently many rational curves; then the recursion becomes stronger and one of the greatest achievements in that case is the discovery of new systems of differential equations called integrable hierarchies. One of the main challenges is to understand the significance of these integrable hierarchies in Mathematics. For example, there are some indications that such integrable hierarchies appear naturally in the settings of complex geometry as well as in the settings of representation theory of infinite dimensional Lie algebras.