FEATURE

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Research Area: Theoretical Physics

Putting the Universe on a Computer

1. Experimental Astronomy?

My physics colleagues get very excited when they perform an experiment that proves or falsifies some theory. Such a process is typical in the natural sciences such as chemistry, biology, and physics, where an experiment in the lab is an important step to developing a general theory. Unfortunately for astronomers, however, it is often (or almost always) impossible to conduct a laboratory experiment that can test their ideas, for example "What happens if the sun spins 100 times faster?" One can come up with many intriguing ideas like this, but astronomical objects are too large to handle and so we can't set up desk-top experiments. It is impossible to spin the sun around, and it is impossible to let two black holes collide. Appropriate materials to make the large-scale structure of the universe, we can't find even in a Tokyu-Hands store, which sells everything but the kitchen sink.

Instead, astronomers look up at the sky. Observations using multiple wave-bands, from radio to X-ray, enable us to look closely at planets, stars and galaxies. Thanks to the advancement of modern telescope instruments, we can see the detailed three-dimensional structure of a nebula, and can catch a glimpse of a young galaxy, for example.

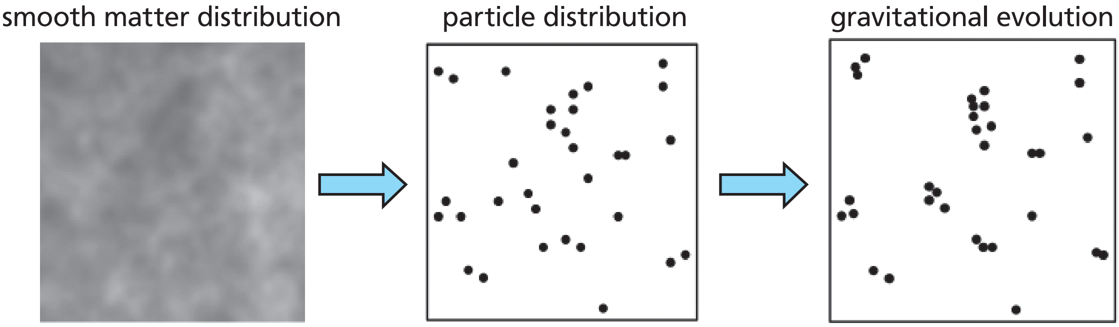
Yet, observations using a telescope do not always satisfy us. Sometimes we wish to see an object from different directions, and other times we wish to see the evolution of a galaxy over a billion years. Honestly, I often think "How nice if I could create a small galaxy in my office and look at it from any direction!"

Certainly, real experiments are impossible, but there's actually a nice way of performing a sort of experiment, on a computer.

2. Virtual Universe

Computer simulations have been established as a powerful method in science. Not only in physics and astronomy, we see computer simulations used in a variety of situations. Most familiar are perhaps weather forecasts and earthquake simulations. Fine movies generated from computer simulations are shown on the TV news, by which we appreciate that such forecasts are based on realistic large-scale computations.

Computer simulations play different roles, depending on the research area. In astronomy and cosmology, they are used to study the formation and evolution of astronomical objects and of the universe itself. Simulations are also used when



$$\frac{d^2 x_i}{dt^2} = \sum_{j \neq i} G m_j \frac{x_j - x_i}{|x_j - x_i|^3}$$

Figure 1. A schematic diagram of cosmic structure formation simulations. The initial density fluctuations (left panel) are represented by the distribution of mass elements (particles in the middle panel). The particles mutually interact via gravity, to form dense clumps (right panel).

planning a large observational program. For instance, the Kavli IPMU is leading a large sky survey using the Subaru Telescope. The so-called mock galaxy catalogs that closely resemble the real galaxy distribution are generated from structure formation simulations and are used extensively to explore a variety of scientific results to be obtained by the survey.

The reason I use computer simulations as my primary research tool is simple. In cosmology, the initial conditions are known observationally. This fact makes most of the cosmological simulations actually more than they could literally mean. It would probably be more appropriate to call them “numerical integrations” or simply “calculations.” The known initial conditions that are unambiguously described by mathematics and statistics, and the small number of physical processes involved, make the whole problem of cosmic structure formation a well-defined and actually tractable one.

One would guess that there are a number of physical processes involved in the formation of stars and galaxies. Even the basic elements such as general relativity and nuclear physics are difficult enough to implement in a computer program. Interestingly, however, the only relevant physics to the formation of large-scale structure of the universe is gravity. Newtonian gravity that we learn about in high school or in the first year of college suffices. For very large cosmic structures, gravity dominates over other forces known in nature, including familiar electromagnetism, weak interaction and strong interaction that are important at the microscopic level. To summarize, it is expected that cosmic evolution can be realized and followed on a computer, if a fast enough computer is used.

3. From Big Bang Ripples to Large-Scale Structure

Cosmological simulations have a long history. They were pioneered nearly 40 years ago, and hence have a history as long as my own! In the early 1970’s, researchers used just a few hundred particles

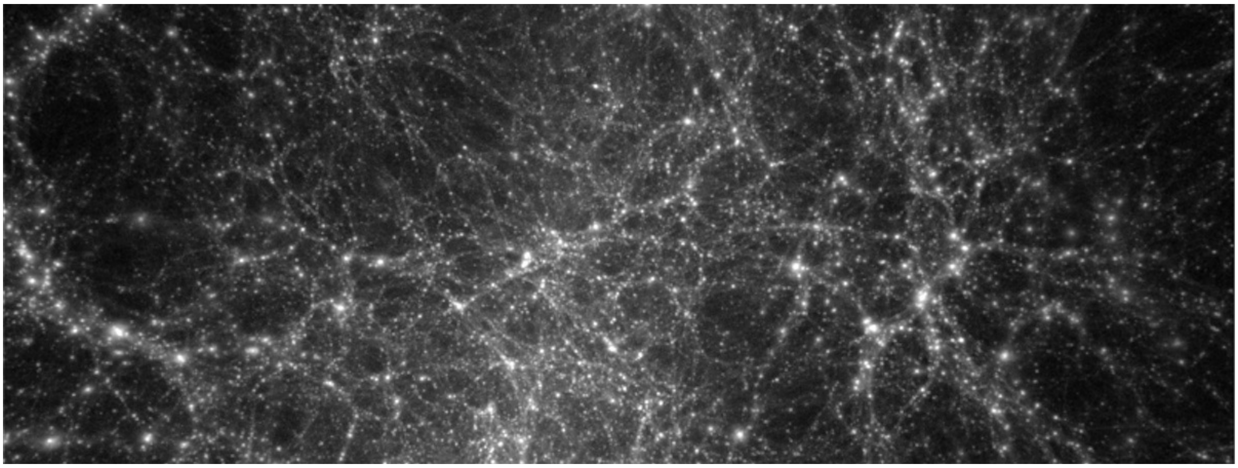


Figure 2. The distribution of dark matter in a modern cosmological simulation. Shown is a projected 100 x 300 million light-years area. The bright parts have high densities of dark matter.

("galaxies") to simulate the formation of galaxy clusters. It is worth mentioning that two Japanese physicists, Taro Kihara and Kazunori Miyoshi, made an important early contribution. The solid-state physics professor Kihara, then at the University of Tokyo, proposed using the two-point correlation function to describe the distribution of galaxies. The correlation function has been a primary element in statistics in cosmology since then. The most recent data from SDSS3 survey that the Kavli IPMU is participating in enabled precise measurement of the geometric structure of our universe by means of the two-point correlation function.

Simulations with a few hundred bodies (particles) were used in the early 1970's. Rapid advancement in computer technology as well as sophisticated calculation algorithms made it possible to run simulations with larger and larger numbers of particles. Currently, the largest simulations employ nearly a trillion particles, achieving a billion times increase in 40 years (Figure 2).

The progress of such cosmological simulations is shown in Figure 3. The simulation size has been increasing exponentially. Projecting the increase over the next 60 years, I would expect that simulations with 10^{23} particles will be performed in the year 2073, when I celebrate my 100th birthday! The number, close to the Avogadro number that is important in basic thermodynamics, also corresponds roughly to the number of stars in our observable universe. So, in about 60 years, one can represent all the stars in the universe in a computer simulation. This is something I would call "The Whole Universe Simulation."

4. New Approach

While the idea of putting the whole universe on a computer is very exciting, I don't think it will actually be realized, for several reasons. When one is able to follow the dynamics of the Avogadro number of particles, there should be a better way of describing the system in a macroscopic manner or in terms of a few gross quantities, just as thermodynamics

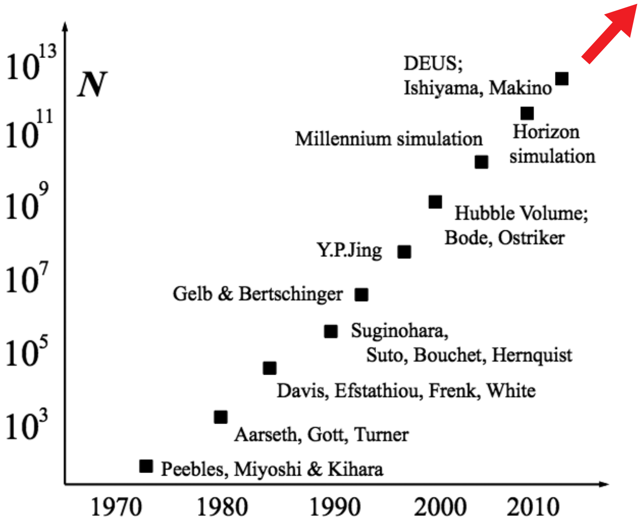


Figure 3. The evolution of cosmological simulations. We plot the number of particles used in the simulations against publication year. The names of the researchers or of the simulation projects are indicated. This author (NY) contributed to the two projects in the early 2000's.

provides a handful of quantities such as temperature and entropy that are useful and indeed quite enough to describe the state of a gas. Gravitational dynamics is peculiar and so such usual approaches do not seem to work, but still there is a better way of describing the behavior of a many-particle system.

The basic equations that govern the dynamics of matter (mostly dark matter) in the universe is given by

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$\nabla^2 \phi = 4\pi G \rho = 4\pi G \int f d^3 \vec{v}$$

Here, the so-called velocity distribution function $f = f(x, y, z, u, v, w)$ describes the number (fraction) of particles that are in an infinitesimal volume at a position (x, y, z) and have velocity (u, v, w). Integrating f over the velocity space gives the particle number density ρ from which the gravitational field ϕ is obtained via the second equation, which is called the Poisson equation.

The time evolution of the velocity distribution function f on a six-dimensional space is completely determined by the coupled equations. A dream of computer physicists like me is to perform such a multi-dimensional integration efficiently on a massively parallel computer. Only last year (2013), our research group made a first important step toward realizing that goal. We were allowed to use about ten percent of the full capability of a supercomputer at Tsukuba University and so we performed the integration of the velocity distribution function f of dark matter particles on 64 to the power 6th grids. The results are shown in Figure 4. The spatial resolution is not impressive; in fact it is actually quite poor when compared with modern N-body particle simulations as shown in Figure 2. However, the velocity distribution is well described by a smooth function as seen in the middle panel of Figure 4. For reference, the corresponding result from a million particle simulation is shown in the right panel. Despite the impressive structure in the configuration space (x, y, z), the overall resolution

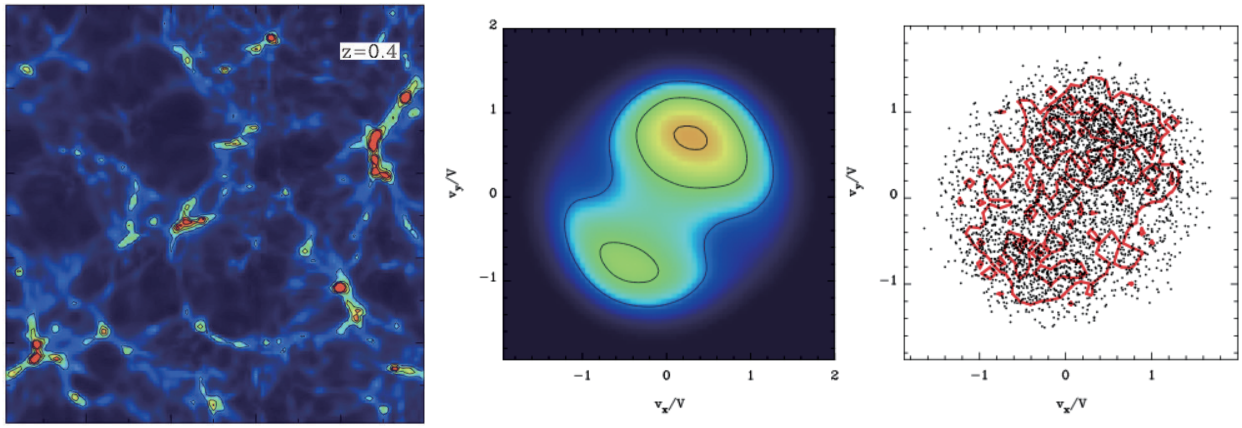


Figure 4. Large-scale structure formed in a Boltzmann simulation. Although the spatial resolution in the left panel is not as impressive as in the modern particle simulation (Figure 2), the velocity distribution function is accurately described by a smooth function (middle panel). The corresponding plot from a million particle simulation (right) shows no clear structure. The red lines show the smoothed distribution of the particles.

in the velocity space is poor, even with so many particles employed.

5. Future Prospects

The rapid progress in high performance computing makes it impossible for me to even imagine what one will be able to do in 30-50 years. My guess is that eventually cosmological simulations will become technically very simple. Computer power will be used to do integrations of the kind I explained above, or to do something similarly simple in concept. In fact, sophisticated algorithms are developed in order to overcome the shortcomings of current computers. When computers that are fast enough become available sometime in the near future, simple methods will be the best choice. Simulations will then be pure calculations, rather than sophisticated modelling with approximations. In this article, I have described an exceptionally clear problem of gravitational dynamics in cosmology. There are many other interesting phenomena in the universe that involve more physics such as gas dynamics, chemistry, radiation transfer, and nuclear reactions. Only a small portion of such things have been explored so far by means of computational physics. It will be very exciting to put all these physical processes into a computer program and run it on a big machine. However, a crucial role of such grand-challenge simulations is, from my viewpoint, to identify some missing element in our understanding of how things work in the universe, rather than to reproduce what is already understood. I believe that there is also a vast discovery space where we can explore with telescopes *and* computer simulations. Surely I will be much more excited if I find a new telescope observation showing a structure of the universe that looks completely different from my computer simulations. I will then appreciate the breadth and the depth of the physics of the universe all over again.