Special Contribution

A Visit from the Mathemagician: John Horton Conway

Siobhan Roberts

On a recent visit to Sensoji Temple in Tokyo, I suspended all scientific rationality for a moment and pulled my fortune from a drawer. To paraphrase, it read: "You will make the acquaintance of an eminent individual and find your way in the world." Oddly enough, this nicely explained how I found myself in Japan in the first place. I was tailing John Horton Conway as I researched his biography, and Conway was the keynote speaker at the "Moonshiney" workshop held at IPMU this past May.

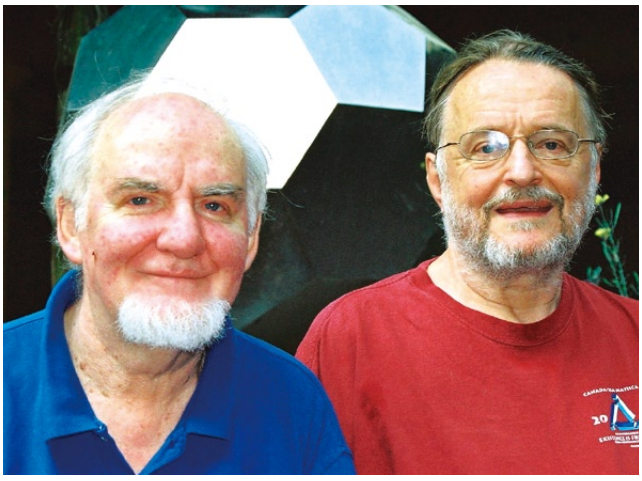
The John Von Neumann Professor of Mathematics at Princeton University, Conway is a breed of mathematician more aptly characterized as a "mathemagician." One of his sleight-of-hand tricks involves carefully balancing a penny on the tip of a coat hanger hook, then in one grand swoop spinning the entire contraption over his head, whirling it around like a helicopter rotor, and, eventually (and if the penny doesn't fall off, first), bringing it to a graceful swinging stop — a trick he performs at kids' math camps, which he attends religiously every summer.

Conway is the man responsible for so fabulously naming the Monster, and (together with Simon Norton) the Monstrous Moonshine conjecture — moonshine, due to the preposterous chance it could be true (as it was proven to be by Richard Borcherds). The Monster, to put it simply, is a rather impressive entity in group theory, the mathematical study of symmetry. Bernd Fischer and Bob Griess predicted its existence in 1973. Nearly a decade later Griess constructed the Monster (that is to say, confirmed it to exist), and Griess's construction was subsequently simplified by Conway.

"I have been fighting with the Monster for the last 25 years," said Conway during his lecture at the Moonshiney workshop. "Before I die, I really want to understand WHY the Monster exists. But I'm almost certain I won't."

"This is one of those things that perpetually intrigues me," he added later. "There are these abstract objects that are as real as trees or cats, but

John Conway (right) and John McKay (left) on the occasion of the Moonshiney Workshop at IPMU, the University of Tokyo. (Photo by Siobhan Roberts)



we can only access them by thinking about them. One feels the Monster can't exist without a very real reason. But I don't have any idea what that reason is."

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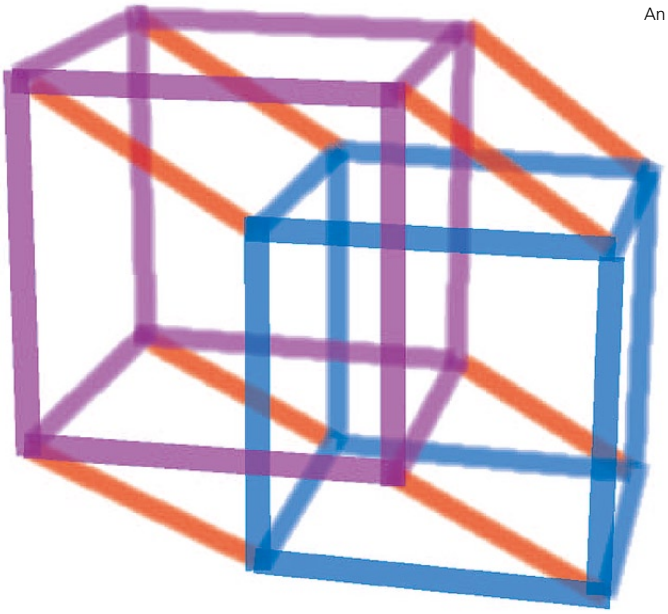
I first met Conway when writing *King of Infinite Space*, my biography of the classical geometer Donald Coxeter. Conway generously agreed to be my tutor, my translator, as it were, as I traveled through the foreign world of mathematics. I had taken all the math and sciences courses in high school, but thereafter my focus had been history, and then writing. My interest in math and science, however, endured — perhaps because it seems so preposterous, like moonshine itself, that the universal language of mathematics somehow expresses and describes the laws of the physical world. So on Coxeter's coattails I became a fulltime curious observer, and then, at one remove, a translator to the wider popular audience — attempting to reconcile and unite what the English physicist and novelist C.P. Snow once lamented were the "two cultures" of the sciences and humanities.

Confronted with this mathematical dilettante, Conway sat me down in front of a blackboard and provided personalized lessons on polytopes and hyperdimensions and how to draw a 4-dimensional cube. The latter triggered some rather surreal stories from his past.

Put the question of how to think in four dimensions to Conway, and he snaps back, "None of your business! That's personal!" But if one probes a little further, he happily recounts a nutty escapade while he was at Cambridge circa 1960, when he made an earnest attempt to think in four dimensions. He did not expect to see the fourth dimension, as if it were a physical reality. Time, of course is most often thought of as the fourth dimension (and time, the physicists are telling us lately, may not survive much longer). Higher dimensions, however, can measure any value or feature of existence. The fourth dimension could be temperature or wind direction,



An image of a 4-dimensional cube.

the fifth dimension could be the rate of interest on your credit card, and the sixth dimension could be your age, and on and on and on as you please. Each characteristic measured adds another "dimension" — the dimensions become coordinates, a navigational tool that quantifies our existence, our position in the world. Being a geometer, Conway naturally preferred contemplating a fourth dimension in terms of space.

In attempting to visualize a fourth coordinate or dimension in space, Conway built a device that allowed him to see with what he called "double parallax" — in addition to the displacement that occurs horizontally when you look at an object by closing one eye and then the other, he tried to train himself to see vertical parallax. If he could experience both horizontal and vertical parallax, he would have four coordinates for every point in space, and thus would be seeing four dimensions. In his attempt to do so, Conway donned a recycled motorcycle helmet, adapted with a flat visor and cheap, old war-surplus periscopes. The periscopes were bolted to the visor (not very well; they rattled when he walked) and extended from his right eye up to his forehead and his left eye down toward his chin. The only name Conway had for the helmet was "that damned contraption" because it was rather uncomfortable, his nose pressed up against the visor, as a child's to a toy shop window at Christmas.

Conway had a strong desire to see four dimensions, which he truly believed was possible (and still does). He regularly walked around wearing his helmet in the Fellows Garden of his college at Cambridge, and in a flash of daring (or stupidity) during one Saturday in the downtown streets busy with shoppers. "I suppose I had a limited amount of success in that quixotic quest," he told me. "I got to the point where I could see four dimensions, but there was no hope of going beyond, so what's the point?" Conway's discoveries since his helmet days are in dimensions much, much higher — the Conway group is in twenty-four dimensions, and the Monster group exists in 196, 883 dimensions.

Siobhan Roberts is a Toronto-based journalist and author. Her first book *King of Infinite Space: Donald Coxeter, The Man Who Saved Geometry*, will be published in Japan by Nikkei BP.

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Conway is perhaps most widely known as the inventor of the Game of Life. Though when he randomly flips through indexes of mathematics texts actually looking for his name, what he hopes to find is reference to his favourite piece of work to date: discovering surreal numbers.

Discoveries like these Conway says are "white hot." With the surreals, he walked around in a world all his own for weeks. He characterizes such a delectable discovery with what he has dubbed the "Hotspur property," in reference to a character in Shakespeare's *King Henry IV*. In act 3, Glendower says, "I can call spirits from the vasty deep..." To which Hotspur replies, "Why, so can I, or so can any man; But will they come when you do call for them?"

His one disappointment with the surreals is that they have yet found application. The late Princeton physicist Martin Kruskal (known for his discovery and theory of solitons, solitary waves that hold their shape as the travel at a constant speed), dedicated the latter part of his career to constructing the theory of surreals as applied to analysis, but he died in 2006 with the work incomplete.

I spent last year as a Director's Visitor at the Institute for Advanced Study in Princeton, and in speaking to a few Institute mathematicians they seem to have no doubt that someday Conway's surreals will find their application. That always seems to be the way it is with mathematics: what is beautiful becomes useful, eventually. The Monster holds promise to inform string theory, the so-called Theory of Everything. It may take a year or a century, or two, as well as a few passionate scientists working in ideal think tank conditions, like those at IPMU.