IPMU Interview
with Maxim Kontsevich

Interviewer: Kyoji Saito

Received the special education in mathematics for gifted students in Russia

Saito: I really enjoyed your seminar today. In spite of its title, "Wall-Crossing", your talk covered many recent developments in mathematics in interaction with physics and, in particular, with string theory. It was really exciting and I enjoyed it very much. But let's come back to that later, and start with a more general story. Let's begin with your general background. You were born during the Soviet era. Later, during your studies, the system changed and now you are working in Western countries.

Kontsevich: Actually, the system changed after my studies.

Saito: Could you briefly describe your education? How did you become a scientist and mathematician?

Maxim Kontsevich is permanent professor at the Institut des Hautes Études Scientifiques (IHÉS) in France. He was awarded the Fields Medal, the world's highest honor in mathematics, in 1998. He was also awarded the 2008 Crafoord Prize in mathematics by the Royal Swedish Academy of Sciences, jointly with Edward Witten, for their important contributions to mathematics, inspired by modern theoretical physics.

Kontsevich: There was a system of special mathematics schools in Russia, created as early as the 1960s by Kolmogorov. The system was open to students with mathematical talent for their last three years of school, from the ages of 14 to 16. There were three or four schools in Moscow and some in Leningrad. My older brother also studied in one of the schools. It was he who gave me my interest in mathematics. My parents are not mathematicians. My mother was a rocket engineer and my father was a specialist in Korean language and history.

Saito: An academic family.

Kontsevich: It's academic, but first-generation academic. My grandfather on my father's side was from a peasant family, but he became a self educated engineer. He was a kind of inventor.

Saito: In the Soviet era?

Kontsevich: In the Soviet era, yes. My grandparents on my mother's side were accountants, but not academic. So my parents were first-generation... Actually, only my father, because my mother was not academic; she was in engineering. I also got some good results quite early, when I was only ten years old. I was a participant in some Olympiads and impressed my teachers, so I skipped one year in school. Then, also at the age of 14 or 15, I attended one of the special schools in Moscow. I participated in the Mathematical Olympiad and studied at Moscow University.

I have to say that this part of my life in Russia was quite happy. I was under the protection of some of my older colleagues. After university I worked for the Institute for Problems of Information Transmission for five years in a very good laboratory whose theme is the mathematics of information theory, coding theory, dynamics of large systems, etc. In 1988, at the beginning of Perestroika, I went to France for one month. I was 24. I went at that time as a specialist in statistical physics. So I went essentially to Marseilles, but also to the IHÉS in Paris. Then in 1990 I was invited for three months to the Max Planck Institute. This was kind of an interesting development. At the end of my stay, there was a traditional meeting of mathematicians in Europe called "Mathematische Arbeitstagung." Michael Atiyah gave the first talk and explained Witten's conjecture about matrix integrals and intersection theory. I somehow got an idea how to approach it. The next day I explained the idea to Atiyah. People got so excited, and they invited me to the Max Planck Institute.

Then in the beginning of 1991 I came to Max Planck. Actually I got my Ph.D. in Germany. For some reason I didn't get it in Moscow.

Saito: So the change in the system didn't have much of an influence on your development?

Kontsevich: No, I left kind of early, before that.

Omnivorous mathematician

Saito: Of course you had

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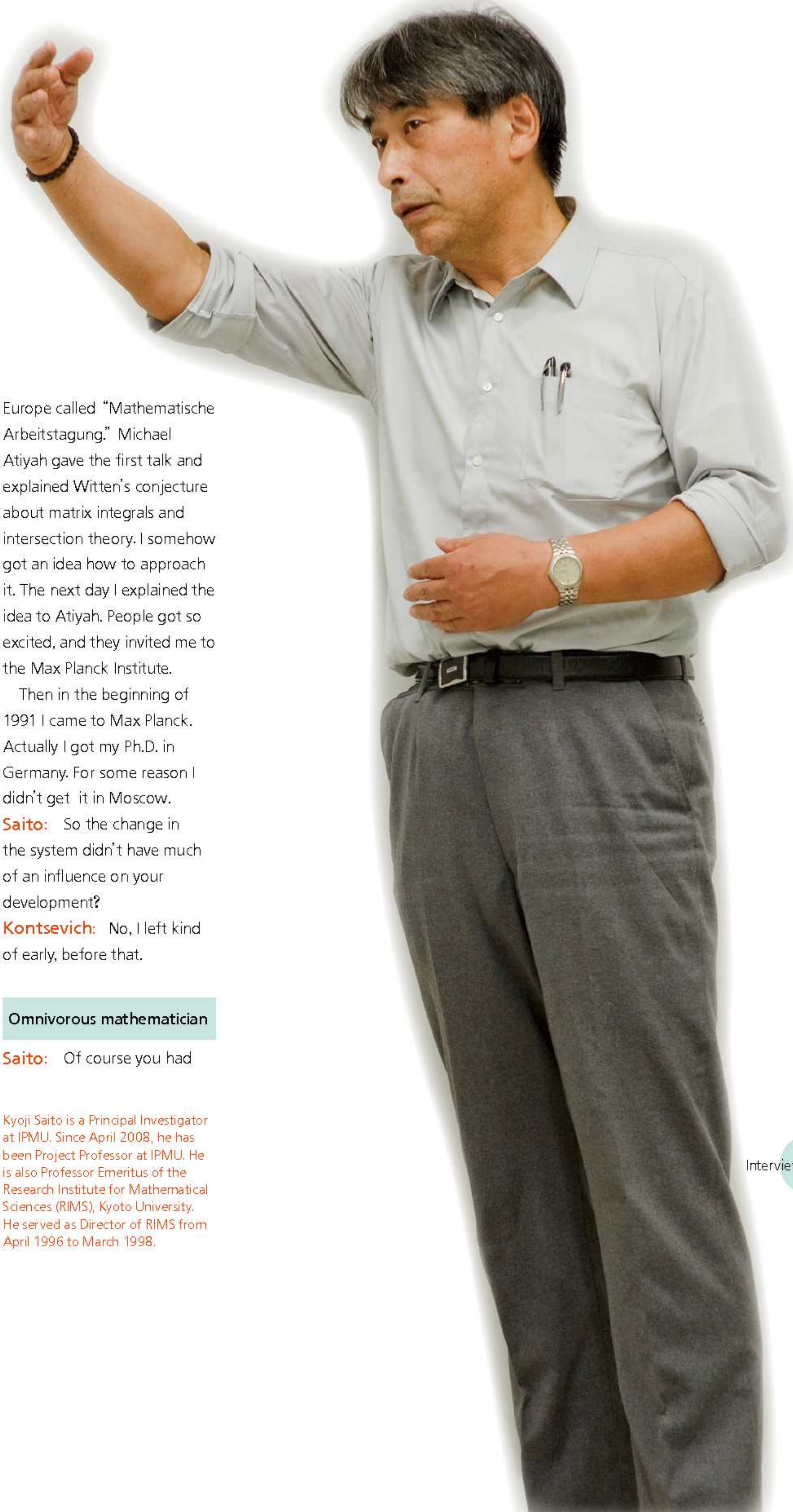

an interest in the Witten conjecture earlier. How did you get into this branch of mathematics? Obviously Kolmogorov and Gelfand were really big names in mathematics.

Kontsevich: Actually, I was a student of Gelfand.

Saito: Gelfand! You could claim they were doing mathematics, but there was a very strong interaction with mathematical physics. What do you think about this?

Kontsevich: Yes. Mathematical physics made a very strong impression on me. In my last year at university, in about 1984 or 1985, there was a big discovery in theoretical physics - conformal field theory, initiated by Belavin, Polyakov, and Zamolodchikov in Moscow. It was a discovery of critical behavior in two-dimensional systems. It was influenced also by the work of mathematicians, I have to say, because Feigin and Fuchs worked out the characters of Virasoro algebra by really formal reasons. It was not related to physics at the time.

Saito: You started on Gelfand-Fuchs cohomology and then came to more...

Kontsevich: Yes, I was studying this as well. But the Gelfand Seminar covered a very wide subject. All of mathematics was covered. Two hundred participants came every Monday. The Gelfand Seminar started in 1942 or 1943. It was in World War II, during the evacuation, when he started the seminar. It continued for more than 50 years. It was the major seminar in Moscow. There were 200 or 300 participants. It was also very long. It started at 7:00 in the evening and went on till midnight, almost till the last train on the metro. Great participants. Unpredictable.

Saito: You have described your history up until you came to the Witten conjecture. Since then you have been involved in so many very big subjects.

Kontsevich: Even before I was covering many subjects and doing many projects which are not yet written. Omnivorous mathematician, I have to say.

Saito: Yes, I understand these tendencies. But how do you choose these subjects? Do you have some global picture of what you want to do, perhaps unconsciously? Or are you just attacking the problems you find in front of you?

Kontsevich: I'm not attacking problems. I'm just trying to formulate for myself what is going on. The Witten conjecture was one of the few things which I really solved as a problem.

Saito: I understand very well. At least in this seminar today, you described a new general framework to understand many aspects. From my side, it looks like the study of periods over some vanishing cycles, but of course there are so many other aspects to this you have described. In my case, I have a goal of describing a period map for a certain primitive form, but in your case...

Kontsevich: No, no. I do not have any particular goal. Just to understand the mathematics of physics of quantum field theory. It has been a great source of inspiration for the last twenty years.

The interaction between mathematics and physics: from Witten to the future

Saito: That's very nice. Now we are coming to a more central topic in our discussion, the Institute for the Physics and Mathematics of the Universe. In this sense, there is an interaction between physics and mathematics. How do you describe this interaction?

Kontsevich: It was very successful. During the forties, fifties, and sixties, there was not much interaction between theoretical physics and mathematics. But then various ideas started to flow in both directions. Gauge theory in fundamental particles, quarks, is related to bundles with connections in mathematics. Then there were super-symmetry and integrable systems. There were different periods and different directions. Then there came the Witten era. Before that it had been quantum groups, conformal field theories, and the beginning of topological theories. It's a very fruitful relationship. But many things go not only in one way. It's not just from physics to mathematics; it's also from mathematics to physics.

Saito: It is influential for both sides. And this relationship is very fruitful. I agree. But could you describe, as you see the prospects, how it should go further? At least for me, we can't yet see the end.

Kontsevich: It seems that at the end of the day, this great structure that was discovered from string theory, M-theory, is kind of like a huge analytic function. If you know in detail one point you know all points.

All theories in physics of various dimensions seem to be related to limiting cases of this big universal object, which will keep mathematicians occupied maybe for several hundred years. [Laughs] Maybe less; I don't know. But it's really one of the major things which will happen to mathematics.

Grasping the essence of problems from a single sign

Saito: Could you describe a little how you would like to work further in this interaction?

Kontsevich: Yes, it's very hard to predict. I don't really make any plans, I have to say.

Saito: You don't make any plans.

Kontsevich: Yeah. At work I usually have many, many unfinished projects, so I try to think of them as they are probably related.

Saito: That shows that you are really in the middle of working hard and actively where things are still moving.

Kontsevich: Oh yes. There are plenty of interesting directions.

Saito: Could you give some more explicit examples?

Kontsevich: One very broad theory which constantly appears in my work is the relationship with noncommutative geometry, noncommutative algebra, and string theory. I have many projects related to this relation between, say, the multiplication of matrices (it is associative but non-commutative) and the geometry of surfaces. There are really an amazing number of relations between geometric and algebraic intuition. From the past I remember that from 1992 and 1993 I proposed homological mirror symmetry by formal algebraic analogies. This was a few years before string theorists had D-branes. So they reinvented it several years later in physical terms. But as a result of this discovery of homological mirror symmetry, which was described with a language of very abstract algebraic theory of triangulated categories, now it's actually used by physicists. That was completely unexpected. Yes, it's one of the most abstract mathematical theories.

Saito: You didn't expect that it could be used more in physics?

Kontsevich: No. I came to this theory because it looked like an ultimate formulation of the phenomenon of mirror symmetry. But physicists really put it into a different framework. It's something which potentially can calculate physical quantities in string theory models.

Saito: This is one typical point of your work. But I find that in much of your work, by hearing one symptom you capture the central point of the problem and then give some general big framework. That's my general impression of what you are doing.

Kontsevich: Yeah, I really don't work on examples at such a level.

Saito: How can you work in that way?

Kontsevich: For myself sometimes I work on one or two examples, but...

Saito: You already keep some examples in mind, but still you construct theory.

Kontsevich: Yes. And generally I find examples sometimes to be misleading. [Laughter]. Because often the properties of examples are too special, you cannot see general properties if you constantly work too much on concrete examples.

Saito: I know one very famous example, Grothendieck. He's a person who can make a big, big framework without any examples. Actually this framework is not general nonsense, but profoundly captures the mathematics. I find what you are doing similar. You give a big framework which captures the core of the subject. It's really an amazing ability; not many mathematicians are doing this. So I repeat the question again: How do you do it?

Kontsevich: I don't know. I think it is just an experience, nothing special. A friend of mine and I kind of jokingly call ourselves "specialists in general questions."

Saito: Today in the seminar I have already seen that you are currently working on this new framework. I hope that you continue working in this direction and that you are successful.

Kontsevich: I would like to thank you for the invitation. It was a great pleasure to give talk at IPMU with a very active audience and a relaxed atmosphere. In fact it is my first visit there, and I see that you have an excellent research group. I wish the brightest future to the Institute, and hope to come to Kashiwa again, maybe next year.