

Focus Week: New Invariants and Wall Crossing

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This year marks the 80th anniversary of quantum field theory. Quantum field theory is a program to apply the principles of quantum mechanics to “fields” such as the electromagnetic field, and it is the fundamental language of elementary particle physics. However, it is incomplete as a mathematical theory. The mathematical formulation of the Yang-Mills theory (an important example of quantum field theory) has been posed as one of the 7 Millennium Problems by the Clay Mathematics Institute along with the Poincaré Conjecture, the Riemann Hypothesis and the P versus NP Problem.

Although quantum field theory has not yet become a part of mathematics, some approximate results by physicists have been made into precise mathematical conjectures, and they have inspired developments in modern mathematics. For example, roughly 40% of Fields Medalists in mathematics since 1990 have worked in areas closely related to quantum theory. In return, such developments have provided powerful new tools to study quantum field theory and superstring theory. Since one of the stated goals of IPMU is to generate new mathematics by enhancing collaborations between mathematicians

and physicists, the research opportunity in quantum field theory has not escaped our attention.

Though physicists use integrals over infinite dimensional spaces of field configurations to perform computations in quantum field theory, such integrals are not well-defined. Fortunately, for certain quantities in supersymmetric field theories, the infinite dimensional integrals reduce to finite dimensional integrals, for which precise mathematical approaches are possible. This Focus Week has gathered leading mathematicians and physicists working on “quantum invariants” defined by such integrals. The “wall crossing” in the title of the Focus Week refers to a phenomenon where the values of quantum invariants change by some kind of phase transition. It has important consequences both in mathematics and in physics. In mathematics, it has provided new insights into the classification of higher dimensional geometry. In physics, it has played fundamental roles in deriving low energy effective theories from superstring theory, in analyzing quantum states of black holes, and in studying strongly coupled effects in gauge theories.

In response to the rapid progress in the field,



"Conference in the Time of Influenza"

this Focus Week was quickly put together in three months by the organizational committee consisting of 1 mathematician and 3 physicists. It gathered 11 invited lecturers from all over the world, among whom 7 were mathematicians and 4 were physicists. The organizers made various arrangements to enhance interactions between mathematicians and physicists. For example, each invited lecturer was asked to prepare two hour lectures. The first hour of each lecture was to aim at a broad audience. After a 10 minute break, latest results were presented in the second hour. This format turned out to be very popular. Here is a typical quote from the comments sent by one of the invited lecturers after the meeting: "I really liked the idea of the format of the talk with two hours, one geared to the general audience and one more specialized. Personally I've learned a lot from the first hours of the lectures."

In addition to invited lectures, we solicited talks by young researchers. Among them, 4 mathematicians and 4 physicists were selected to give a 45 minute talk each. The total number of registered participants was 66, among whom 32 were mathematicians and 34 were physicists. It was genuinely an

interdisciplinary meeting, "crossing walls" between mathematics and physics.

Since the Focus Week was held in the midst of the H1N1 Influenza pandemic, every possible step was taken to ensure the health and well-being of the participants. We placed alcohol-based hand sanitizers at the 2 entrances of the conference building and asked everybody to use them. We bought 700 surgical masks for the conference, encouraged participants to get new masks every day and reminded them to wear them. In the opening address, we made a special presentation to demonstrate how to wear and dispose of the masks properly. In addition to asking all the participants to measure their body temperatures in every morning and evening, we set up a thermographic camera at the exit of the conference hall and measured the body temperatures of all the participants before lunch every day. Participants cooperated with us with a sense of humor; some called the meeting as "Conference in the Time of Influenza" making a parody of the famous novel* by Gabriel García Márquez.

* *Love in the Time of Cholera*