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# Inflation in Bimetric gravity

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K.Nomura, J.Soda, When is multimetric gravity ghost free?, Phys.Rev.D86 (2012) 084052.

Y.Sakakihara,J.Soda,T.Takahashi, On cosmic no-hair in bimetric gravity and the Higuchi bound, PTEP (2013) 033E02

# Mystery in contemporary physics

| | | |
|---|---|---|
| quarks + leptons + gauge bosons | 4% | |
| dark matter | 24% | supersymmetric partners? |
| dark energy | 72% | ?????????????????????? |

Standard logic

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_p^2} T_{\mu\nu}$$

**Modified gravity**

$f(R)$ gravity

DGP gravity

**Exotic matter**

quintessesnce

k-essence

ghost condensation

There exists a slightly different view point.

# graviton : the last piece of particle physics

No one doubt the existence of gravitons, but no one found gravitons!
It would be nice if gravitons account for accelerating universe.

Massive gravitons as dark energy

$$S \approx M_p^2 \int d^4x \left[ (\partial h)^2 - m_g^2 h^{\mu\alpha} h^{\nu\beta} \eta_{\mu\nu} \eta_{\alpha\beta} \right]$$

$$\rho = 3M_p^2 H^2 = 3M_p^2 \frac{\Lambda}{3} = M_p^2 m_g^2 = \left(10^{27}\,\mathrm{eV} \times 10^{-33}\mathrm{eV}\right)^2 = \left(1\ \mathrm{meV}\right)^4$$

$$m_g \approx 10^{-33}\mathrm{eV}$$

We need gravitational Higgs. $\eta_{\mu\nu} = \langle f_{\mu\nu} \rangle$

$$S \approx M_p^2 \int d^4x \left[ (\partial h)^2 - m_g^2 h^{\mu\alpha} h^{\nu\beta} f_{\mu\nu} f_{\alpha\beta} \right]$$

Cf. spin-2 meson f meson, a meson, etc. Isham, Sakam, Strathdee 1971

$g_{\mu\nu}$ : graviton $f_{\mu\nu}$ : spin-2 matter → bimetric gravity

# Why inflation?

Recently, a consistent theory has been found — Hassan & Rosen 2012

$$S=\frac{M_g^2}{2}\int d^4x\sqrt{-g}R(g)+\frac{M_f^2}{2}\int d^4x\sqrt{-f}R(f)+m^2M_{gf}^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

Apparently, there arises a cosmological constant if f is proportional to g, which may explain the dark energy.

However, this is not a big surprise because we introduced one dimensionful parameter.

Hence, we need to test this possibility with precise cosmological data such as CMB data.

Thus, we need to consider inflation more seriously.

Once we admit bimetric gravity, there is no reason to refuse multimetric gravity.

# Contents

1. Before studying inflation, we need to understand consistency of multimetric gravity.

2. In order to explain dark energy bare mass must be quite small,
   which may cause violation of the cosmic no-hair conjecture
   or the violation of the Higuchi bound

In the de Sitter background, the equation of graviton is the same as the scalar field

Now we may have $m < H_0$ If so, the shear will slow roll like the inflaton

and the cosmic no-hair will be violated.

That also implies the existence of a ghost
since the mass is below the Higuchi bound $m_{eff}^2 = 2H_0^2$

3. We numerically examine the inflationary dynamics. (In progress)

# Consistency of multimetric gravity

# Ghost free non-linear bimetric gravity

Hassan & Rosen 2012

$$S=\frac{M_g^2}{2}\int d^4x\sqrt{-g}R(g)+\frac{M_f^2}{2}\int d^4x\sqrt{-f}R(f)+m^2M_{gf}^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

$$\left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\alpha}\left(\sqrt{g^{-1}f}\right)^{\alpha}{}_{\nu}\equiv g^{\mu\alpha}f_{\alpha\nu}$$

$$e_0(X)=1$$

$$e_1(X)=\mathrm{tr}\,X$$

$$e_2(X)=\frac{1}{2}\left[\left(\mathrm{tr}\,X\right)^2-\mathrm{tr}\,X^2\right]$$

$$e_3(X)=\frac{1}{6}\left[\left(\mathrm{tr}\,X\right)^3-3\mathrm{tr}\,X\,\mathrm{tr}\,X^2+2\mathrm{tr}\,X^3\right]$$

$$e_4(X)=\det X$$

$\beta_n$ : parameters

$$M_{gf}^2=\left(\frac{1}{M_g^2}+\frac{1}{M_f^2}\right)^{-1}$$

Massive theory of gravity

$$f_{\mu\nu}=\eta_{ab}\,\partial_\mu\phi^a\,\partial_\nu\phi^b$$

de Rham & Gabadadze 2010

de Rham, Gabadadze, Tolley 2011

# Why are these ghost free?

Nomura & Soda 2012

$$g_{\mu\nu} = \begin{pmatrix} -N^2(t) & 0 \\ 0 & \gamma_{ij}(t) \end{pmatrix} \qquad f_{\mu\nu} = \begin{pmatrix} -L^2(t) & 0 \\ 0 & \omega_{ij}(t) \end{pmatrix} \qquad \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu} = \begin{pmatrix} -\dfrac{L}{N} & 0 \\ 0 & \sqrt{\gamma^{-1}\omega} \end{pmatrix}$$

$$\sqrt{-g}e_0\left(\sqrt{g^{-1}f}\right) = \sqrt{-g} \times 1 = N\sqrt{\gamma}$$

$$\sqrt{-g}e_1\left(\sqrt{g^{-1}f}\right) = \sqrt{-g}\,\mathrm{tr}\sqrt{g^{-1}f} = \sqrt{\gamma}N\left(\frac{L}{N} + \mathrm{tr}\sqrt{\gamma^{-1}\omega}\right) = \sqrt{\gamma}\left(L + N\,\mathrm{tr}\sqrt{\gamma^{-1}\omega}\right)$$

$$\begin{aligned}\sqrt{-g}e_2\left(\sqrt{g^{-1}f}\right) &= \sqrt{-g}\,\frac{1}{2}\left[\left(\mathrm{tr}\sqrt{g^{-1}f}\right)^2 - \mathrm{tr}\,g^{-1}f\right] \\ &= \sqrt{\gamma}N\frac{1}{2}\left[\left(\frac{L}{N} + \mathrm{tr}\sqrt{\gamma^{-1}\omega}\right)^2 - \frac{L^2}{N^2} - \mathrm{tr}\,\gamma^{-1}\omega\right] = \sqrt{\gamma}\left(L\,\mathrm{tr}\sqrt{\gamma^{-1}\omega} + \frac{1}{2}N\left\{\left(\mathrm{tr}\sqrt{\gamma^{-1}\omega}\right)^2 - \mathrm{tr}\,\gamma^{-1}\omega\right\}\right)\end{aligned}$$

$$\begin{aligned}\sqrt{-g}e_3\left(\sqrt{g^{-1}f}\right) &= \sqrt{-g}\,\frac{1}{6}\left[\left(\mathrm{tr}\sqrt{g^{-1}f}\right)^3 - 3\,\mathrm{tr}\sqrt{g^{-1}f}\,\mathrm{tr}\,g^{-1}f + 2\,\mathrm{tr}\left(g^{-1}f\right)^{\frac{3}{2}}\right] \\ &= \frac{1}{6}\sqrt{\gamma}N\left[\left(\mathrm{tr}\sqrt{\gamma^{-1}\omega}\right)^3 - 3\,\mathrm{tr}\sqrt{\gamma^{-1}\omega}\,\mathrm{tr}\,\gamma^{-1}\omega + 2\,\mathrm{tr}\left(\gamma^{-1}\omega\right)^{\frac{3}{2}}\right] + \frac{1}{2}\sqrt{\gamma}L\left\{\left(\mathrm{tr}\sqrt{\gamma^{-1}\omega}\right)^2 - \mathrm{tr}\,\gamma^{-1}\omega\right\}\end{aligned}$$

$$\sqrt{-g}e_4\left(\sqrt{g^{-1}f}\right) = \sqrt{-g}\,\det\sqrt{g^{-1}f} = \det\sqrt{-f} = L\det\sqrt{\omega}$$

# Constraint algebra

$$\beta_0 = 3 \quad \beta_1 = -1 \qquad \beta_4 = 1 \qquad \beta_2 = \beta_3 = 0$$

$$H = NC_N + LC_L$$

$$C_N = C_0(g,\pi) + 2m^2 M_{gf}^2 \sqrt{\det\gamma}\left(tr\sqrt{\gamma^{-1}\omega} - 3\right)$$

$$C_L = C_0(f,p) + 2m^2 M_{gf}^2 \left(\sqrt{\det\gamma} - \sqrt{\det\omega}\right)$$

$$\left\{C_N, C_L\right\} = 2m^2 M_{gf}^2 \left[\frac{1}{2} M_g^2 \pi_i^i - M_f^2 \sqrt{\frac{\det\gamma^{-1}}{\det\omega}} \left(\frac{1}{2} p_i^i tr\sqrt{\gamma^{-1}\omega} - tr\left(\sqrt{\gamma^{-1}\omega}\, p\omega\right)\right)\right]$$

# Bimetric gravity is ghost free

Nomura & Soda 2012

Mini-superspace

$$g_{\mu\nu} = \begin{pmatrix} -N^2 & 0 \\ 0 & \gamma_{ij} \end{pmatrix} \qquad f_{\mu\nu} = \begin{pmatrix} -L^2 & 0 \\ 0 & \omega_{ij} \end{pmatrix}$$

hamiltonian

$$H = NC_N + LC_L$$

primary constraint

$$C_N = 0 \qquad C_L = 0$$

secondary constraint

$$\dot{C}_N = \{C_N, H\} = L\{C_N, C_L\} \simeq 0$$

$$\dot{C}_L = \{C_L, H\} = -N\{C_N, C_L\} \simeq 0$$

diagonalize a metric　　primary　　secondary　　gauge fixing

$$\frac{24-6-2-1-1}{2} = 7 = 2+5$$

# Multimetric gravity (1)

Nomura & Soda 2012

trimetric gravity

$$S=\frac{M_g^2}{2}\int d^4x\sqrt{-g}R(g)+\frac{M_f^2}{2}\int d^4x\sqrt{-f}R(f)+\frac{M_h^2}{2}\int d^4x\sqrt{-h}R(h)$$

$$+m_1^2M_{gf}^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n^1e_n\left(\sqrt{g^{-1}f}\right)+m_2^2M_{fh}^2\int d^4x\sqrt{-f}\sum_{n=0}^{4}\beta_n^2e_n\left(\sqrt{f^{-1}h}\right)+m_3^2M_{hg}^2\int d^4x\sqrt{-h}\sum_{n=0}^{4}\beta_n^3e_n\left(\sqrt{h^{-1}g}\right)$$

Mini-superspace approach

$$H=NC_N+LC_L+QC_Q\approx 0$$

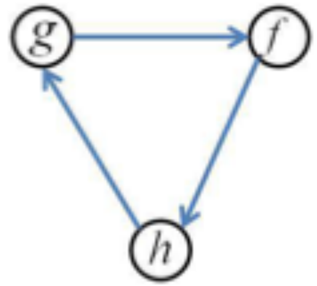
$$h_{\mu\nu}=\begin{pmatrix}-Q^2 & 0\\ 0 & \rho_{ij}\end{pmatrix}$$

$$\dot{C}_N=L\{C_N,C_L\}+Q\{C_N,C_Q\}\approx 0$$

$$\dot{C}_L=N\{C_L,C_N\}+Q\{C_L,C_Q\}\approx 0$$

$$\dot{C}_Q=N\{C_Q,C_N\}+L\{C_Q,C_L\}\approx 0$$

$\frac{L}{N}$ and $\frac{Q}{N}$ are determined.




Diagonalization eliminate

primary

secondary

gauge fixing

$$\frac{36-6-3-0-1}{2}=13=2+5+5+1$$

**Ghost!**

# Multimetric gravity (2)

Nomura & Soda 2012

trimetric gravity

$$S=\frac{M_g^2}{2}\int d^4x\sqrt{-g}R(g)+\frac{M_f^2}{2}\int d^4x\sqrt{-f}R(f)+\frac{M_h^2}{2}\int d^4x\sqrt{-h}R(h)$$

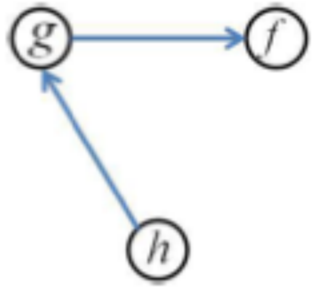
$$+m_1^2M_{gf}^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n^1 e_n\left(\sqrt{g^{-1}f}\right)+m_2^2M_{fh}^2\int d^4x\sqrt{-f}\cancel{\sum_{n=0}^{4}\beta_n^2 e_n}\left(\sqrt{f^{-1}h}\right)+m_3^2M_{hg}^2\int d^4x\sqrt{-h}\sum_{n=0}^{4}\beta_n^3 e_n\left(\sqrt{h^{-1}g}\right)$$

$$\dot{C}_N=L\{C_N,C_L\}+Q\{C_N,C_Q\}\approx 0$$

$$\dot{C}_L=N\{C_L,C_N\}+Q\{C_L,C_Q\}\approx 0$$

$$\dot{C}_Q=N\{C_Q,C_N\}+L\{C_Q,C_L\}\approx 0$$

two secondary




Diagonalization eliminate

primary

secondary

gauge fixing

$$\frac{36-6-3-2-1}{2}=12=2+5+5$$

Tree type interaction is ghost free

# Violation of cosmic no-hair?

# Simple Bimetric gravity

First , we study the slow roll limit of inflation, namely, de Sitter solution.

## Model

$$S=\frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R(g)-2\Lambda_g\right]+\frac{M_f^2}{2}\int d^4x\sqrt{-f}\left[R(f)-2\Lambda_f\right]+m^2M_e^2\int d^4x\sqrt{-g}\,e_2\left(1-\sqrt{g^{-1}f}\right)$$

$$e_2(X)=\frac{1}{2}\left[\left(\mathrm{tr}\,X\right)^2-\mathrm{tr}\,X^2\right] \qquad \frac{1}{M_e^2}=\frac{1}{M_g^2}+\frac{1}{M_f^2}$$

## Homogeneous and isotropic ansatz

$$ds_g^2=-N^2(t)dt^2+e^{2\alpha(t)}\left[dx^2+dy^2+dz^2\right] \qquad ds_f^2=-M^2(t)dt^2+e^{2\beta(t)}\left[dx^2+dy^2+dz^2\right]$$

## The reduced action

$$L=M_g^2e^{3\alpha}\left[-3\frac{\dot{\alpha}^2}{N}-N\Lambda_g\right]+M_f^2e^{3\beta}\left[-3\frac{\dot{\beta}^2}{M}-M\Lambda_f\right]+m^2M_e^2e^{3\alpha}\left[N\left(6-9\varepsilon+3\varepsilon^2\right)+M\left(-3+3\varepsilon\right)\right]$$

# Consistency condition

Defining dimensionless variables

$$\varepsilon = e^{\beta-\alpha} \qquad a_g = \frac{M_e^2}{M_g^2} \qquad \xi = \frac{m^2}{M_e^2} \qquad \lambda_g = \frac{\Lambda_g}{3M_e^2} \qquad \lambda_f = \frac{\Lambda_f}{3M_e^2} \qquad {}' \equiv \frac{1}{M_e}\frac{d}{dt}$$

$$0 < a_g < 1$$

We obtatn

$$\left(\frac{\alpha'}{N}\right)' - \xi a_g (M - N\varepsilon)\left(\frac{3}{2} - \varepsilon\right) = 0$$

$$\left(\frac{\alpha'}{N}\right)^2 = \lambda_g + \xi a_g (2-\varepsilon)(\varepsilon - 1)$$

$$\left(\frac{\beta'}{M}\right)' + \xi(1-a_g)\varepsilon^{-3}(M - N\varepsilon)\left(\frac{3}{2} - \varepsilon\right) = 0$$

$$\left(\frac{\beta'}{M}\right)^2 = \lambda_f + \xi(1-a_g)\varepsilon^{-3}(1-\varepsilon)$$

Bianchi identity

$$\xi\left(\frac{3}{2} - \varepsilon\right)\left(\frac{\beta' e^{\beta}}{M} - \frac{\alpha' e^{\alpha}}{N}\right) = 0$$

Thus, we have two branches.

# Self-accelerating branch

$$\varepsilon = \frac{3}{2} \qquad \left(\frac{\alpha'}{N}\right)^2 = \lambda_g + \xi a_g (2-\varepsilon)(\varepsilon-1) \qquad \left(\frac{\beta'}{M}\right)^2 = \lambda_f + \xi (1-a_g)\varepsilon^{-3}(1-\varepsilon)$$

Even for $\lambda_g = 0$ we have accelerating solutions $\left(\frac{\alpha'}{N}\right)^2 = \frac{1}{4}\xi a_g$

This is true for massive gravity, while $\lambda_f > 0$ is necessary for bimetric gravity

The instability exists for these solutions

Tasinato, Koyama, Niz 2012

De Felice, Gumrukcuoglu, Mukohyama 2012

It is related to the absence of kinetic terms for extra modes

Gumrukcuoglu, Lin, Mukohyama 2012

# Normal branch and Cosmic no-hair

Volkov 2012
Strauss et al. 2012
Comelli et al. 2012

secondary constraint

$$M = \frac{\beta'}{\alpha'} N\varepsilon \longrightarrow g(\varepsilon) = \left(\lambda_f + \xi a_g\right)\varepsilon^3 - 3\xi a_g \varepsilon^2 + \left[-\lambda_g + 2\xi a_g - \xi\left(1-a_g\right)\right]\varepsilon + \xi\left(1-a_g\right) = 0$$

Non-trivial equation

$$\longrightarrow \varepsilon = const. \equiv \varepsilon_0$$

Taking a derivative with respect a time $\varepsilon = e^{\beta-\alpha}$ $\longrightarrow$ $\alpha' = \beta'$

gauge condition

$N = 1$ $\quad M = \varepsilon_0 \longrightarrow f_{\mu\nu} = \varepsilon_0 g_{\mu\nu}$

$$H_0^2 = \lambda_g + \xi a_g (2-\varepsilon_0)(\varepsilon_0 - 1)$$
$$= \lambda_f \varepsilon_0^2 + \xi\left(1-a_g\right)\frac{1-\varepsilon_0}{\varepsilon_0}$$

We need to check if the mass satisfies the Higuchi bound in this branch $m_{eff}^2 \geq 2H_0^2$

# Effective graviton mass

ansatz

$$ds_g^2 = -N^2(t)dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2 + dz^2\right)\right]$$

$$ds_f^2 = -M^2(t)dt^2 + e^{2\beta(t)}\left[e^{-4\lambda(t)}dx^2 + e^{2\lambda(t)}\left(dy^2 + dz^2\right)\right]$$

linear perturbations

$$\sigma'' + 3H_0\sigma' - \xi a_g \varepsilon_0 \left(3 - 2\varepsilon_0\right) q = 0 \qquad q = \lambda - \sigma$$

$$\lambda'' + 3H_0\lambda' + \xi\left(1 - a_g\right)\frac{1}{\varepsilon_0}\left(3 - 2\varepsilon_0\right) q = 0$$

Diagonalizing the above equations, we get

$$\frac{\sigma'}{a_g} + \varepsilon_0^2 \frac{\lambda'}{1 - a_g} \propto e^{-3H_0 t}$$

$$q'' + 3H_0 q' + m_{eff}^2 q = 0 \qquad m_{eff}^2 = \xi\left[a_g\varepsilon_0 + (1 - a_g)\frac{1}{\varepsilon_0}\right]\left(3 - 2\varepsilon_0\right) \qquad \xi = \frac{m^2}{M_e^2}$$

We need to know the ratio $\frac{m_{eff}^2}{H_0^2}$ to examine the stability and the cosmic no-hair.

# Fate of anisotropy: Cases of $\lambda_f = 0$

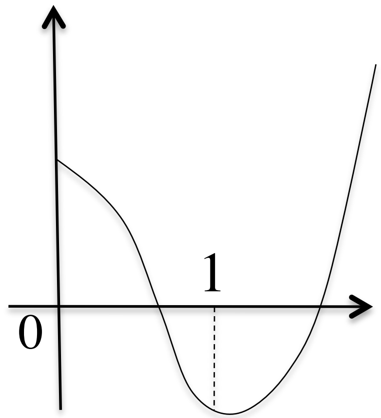
Sakakihara, Soda, Takahashi 2012

The existence of de Sitter

$$H_0^2 = \xi\left(1-a_g\right)\frac{1-\varepsilon_0}{\varepsilon_0} \longrightarrow 0<\varepsilon_0<1$$

$$g(\varepsilon) = \xi a_g \varepsilon^3 - 3\xi a_g \varepsilon^2 + \left[-\lambda_g + 2\xi a_g - \xi\left(1-a_g\right)\right]\varepsilon + \xi\left(1-a_g\right) = 0$$

$$g(0) = \xi(1-a_g) > 0, \quad g(1) = -\lambda_g < 0, \quad g(\infty) = \infty$$




The solution does exist.

Now we examine the ratio
$$\frac{m_{eff}^2}{H_0^2} = \frac{\left[a_g \varepsilon_0^2 + (1-a_g)\right]\left(3-2\varepsilon_0\right)}{(1-a_g)(1-\varepsilon_0)}$$

$$\frac{d}{d\varepsilon_0}\frac{m_{eff}^2}{H_0^2} > 0 \longrightarrow m_{eff}^2 > 3H_0^2$$

above the Higuchi bound!

The de Sitter is stable and consistent with the cosmic no-hair conjecture.

# What happens if $\lambda_f \neq 0$

We have to solve the equation

$$g(\varepsilon) = \left(\lambda_f + \xi a_g\right)\varepsilon^3 - 3\xi a_g \varepsilon^2 + \left[-\lambda_g + 2\xi a_g - \xi\left(1 - a_g\right)\right]\varepsilon + \xi\left(1 - a_g\right) = 0$$

We are interested in the situation $\lambda_f + \xi a_g > 0$

Hence,

$$g(\varepsilon) \to -\infty \quad as \quad \varepsilon \to -\infty \qquad\qquad g(\varepsilon) \to +\infty \quad as \quad \varepsilon \to +\infty$$

Note that $g(0) = \xi(1 - a_g) > 0 \qquad g''(0) = -6\xi a_g < 0$

Hence, the shape at the point $\varepsilon = 0$ is convex.

By calculating the discriminant, we found solutions exist in the range

$$\lambda_f\left(< -\xi a_g\right) \le \lambda_f \le \lambda_+ \qquad\qquad \frac{\lambda_+}{\xi} \approx \frac{4}{27(1 - a_g)^2}\left(\frac{\lambda_g}{\xi}\right)^3$$

Interestingly, there exists an upper bound.

# Higuchi bound appears!

The multiple solution is determined by the conditions $g(\varepsilon_*) = g'(\varepsilon_*) = 0$

They give rise to

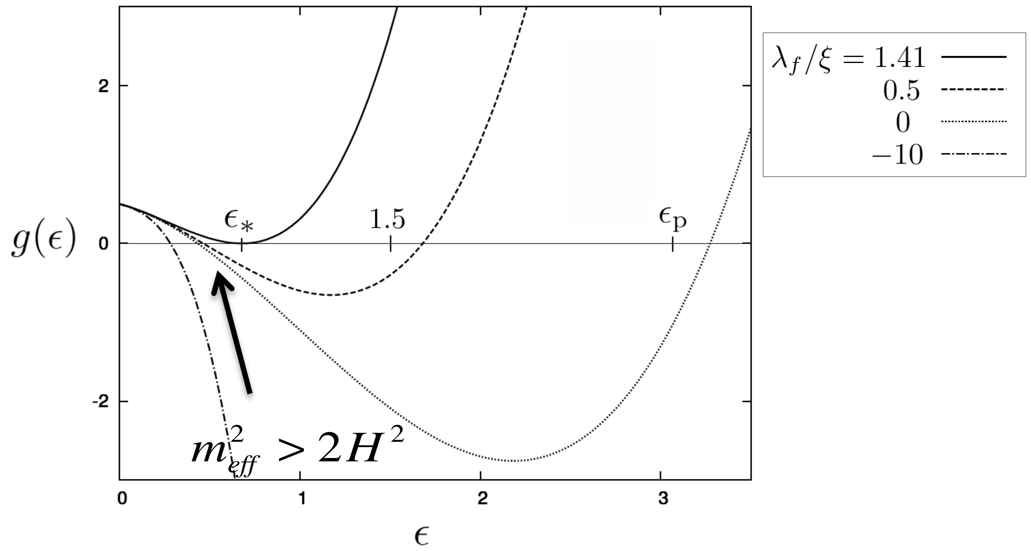
$$a_g \varepsilon_*^2 + \frac{2}{3}\left( \frac{\lambda_g}{\xi} - 3a_g + 1 \right)\varepsilon_* - 1 + a_g = 0$$

Surprisingly, we obtain

$$m_{eff}^2(\varepsilon_0) - 2H_0^2(\varepsilon_0)$$

$$= -\frac{3\xi}{\varepsilon_0}\left[ a_g \varepsilon_*^2 + \frac{2}{3}\left( \frac{\lambda_g}{\xi} - 3a_g + 1 \right)\varepsilon_* - 1 + a_g \right]$$

$$= \frac{3\xi a_g}{\varepsilon_0}(\varepsilon_* - \varepsilon_0)(\varepsilon_0 - \varepsilon_2) \qquad \varepsilon_2 < 0$$




There exists solutions for which Higuchi bound is satisfied and the cosmic no-hair holds.

# Inflation in bimetric gravity

# Numerical approach?

$$\alpha'^2 = \xi a_g (2-\varepsilon)(\varepsilon-1) + \frac{a_g}{3}\left(\frac{1}{2}\phi'^2 + \frac{1}{2}\mu^2\phi^2\right) \qquad \varepsilon = e^{\beta-\alpha}$$

$$\alpha'' = \xi a_g (M-\varepsilon)\left(\frac{3}{2}-\varepsilon\right) - \frac{a_g}{2}\phi'^2$$

$$\left(\frac{\beta'}{M}\right)^2 = \xi\left(1-a_g\right)\varepsilon^{-3}(1-\varepsilon) + \frac{(1-a_g)}{2}\left(\frac{1}{2}\psi'^2 + \frac{1}{2}\kappa^2\psi^2\right)$$

$$\beta'' - \frac{M'}{M}\beta' = -\xi\left(1-a_g\right)M\varepsilon^{-3}(M-\varepsilon)\left(\frac{3}{2}-\varepsilon\right) - \frac{(1-a_g)}{2}\psi'^2$$

$$\phi'' + 3\alpha'\phi' + \mu^2\phi = 0$$

$$\psi'' + 3\alpha'\phi' - \frac{M'}{M}\psi' + \kappa^2 M^2\psi = 0$$

# Simple cases $\psi = 0$

Evolution equations

$$\alpha'' = -\frac{1}{2a_g}\phi'^2\left[1+\frac{2(1-a_g)}{a_g}\varepsilon^2\left(\frac{3}{2}-\varepsilon\right)\right]^{-1}$$

$$\beta' = \alpha' + \frac{1-a_g}{a_g\xi}\phi'^2\,\varepsilon\left[1+\frac{2(1-a_g)}{a_g}\varepsilon^2\left(\frac{3}{2}-\varepsilon\right)\right]^{-1}\alpha'$$

$$\phi'' + 3\alpha'\phi' + \mu^2\phi = 0$$

Initial conditions

$\alpha(0) = 0$ scaling degree is used.

$$\alpha'^2 = \frac{\xi}{1-a_g}\left(\frac{1}{\varepsilon}-1\right) \longrightarrow \beta(0)$$

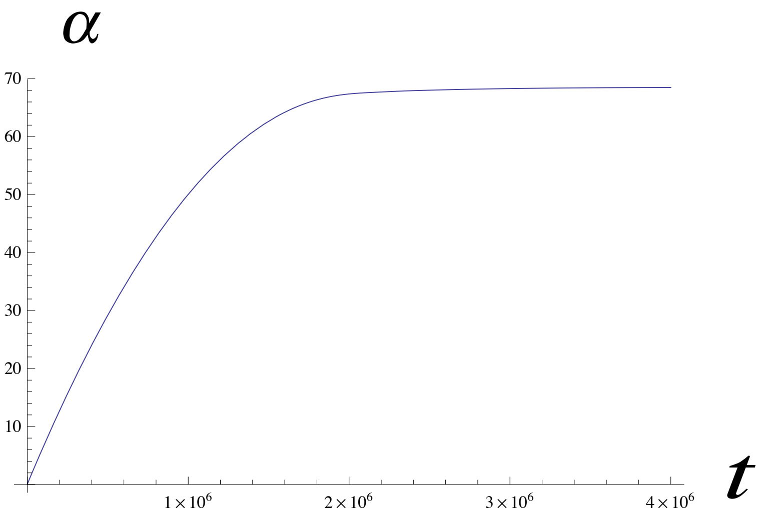
$$\alpha'^2 = \xi a_g(2-\varepsilon)(\varepsilon-1) + \frac{a_g}{3}\left(\frac{1}{2}\phi'^2 + \frac{1}{2}\mu^2\phi^2\right) \longrightarrow \phi'(0)$$

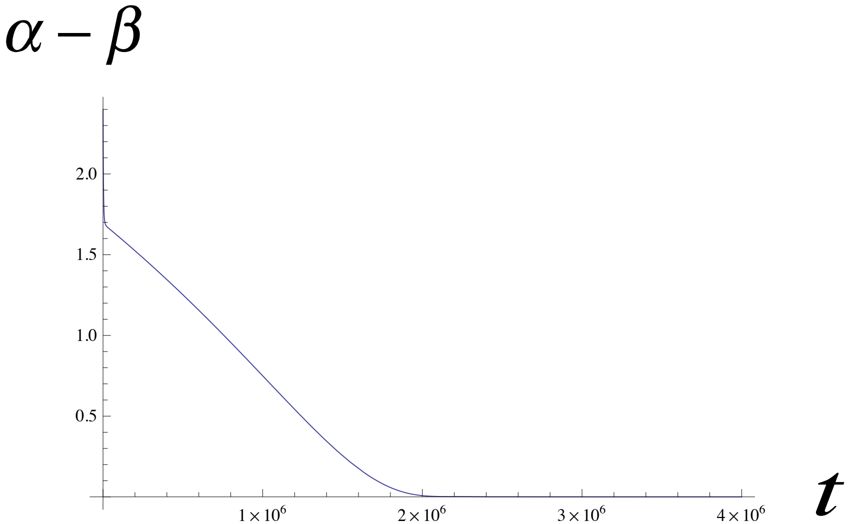
$\phi(0), \quad \alpha'(0)$ are given freely.

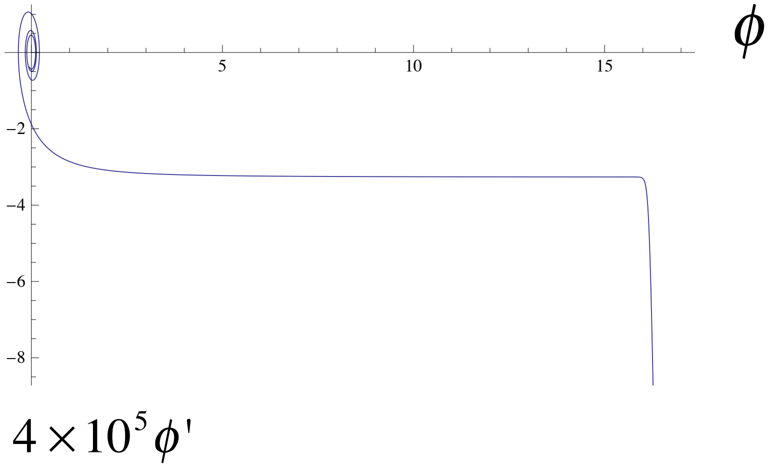
# Results

$a_g = 0.99999 \qquad \mu = 10^{-5} \qquad \xi = 10^{-14}$

$\phi(0) = 17$



# Plan of future work

- Complete background analysis
- Calculate primordial fluctuations
- Calculate CMB spectrum
- Make a unified picture from inflation to dark energy

# Conclusion

- Multimetric gravity with tree type interaction seems to be consistent

- The Higuchi bound is dynamically satisfied,

  hence no violation of the cosmic no-hair occur.

- Inflation tends to drive two metrics to the same one.