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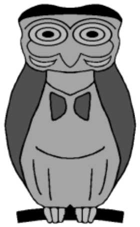
Concern on "ground state" of spacetimes with/without black holes

~ rigidity of spacetime ~

Tetsuya Shiromizu

Department of Physics, Kyoto University



Shiromizu and Ohashi, PRD87, 087501(2013)
Shiromizu and Tanabe, arXiv:1303.6056

Content

1. Simple question
2. GR
3. Einstein-Gauss-Bonnet
4. Dynamical Chern-Simons gravity

1. Simple question

Questions here

In massive gravity or so,

Strictly stationary and vacuum spacetime is Minkowski?

In BH spacetimes, staticity(stationarity) implies spherical (axi) symmetry ?

At first glance, not…because of the mass

GR

- When total mass vanishes

  Minkowski spacetime

- What is equilibrium state with/without BH

  equilibrium state ~ stationary or static

  static(stationary) black hole is Schwarzschild(Kerr)

  strictly stationary spacetime is Minkowski

2. GR

Positive mass theorem I

Schoen & Yau(1979)

- Asymptotically flat $g_{ij} = \delta_{ij}\left[1 + \frac{2}{n-3}\frac{M}{r^{n-3}}\right] + O\left(\frac{1}{r^{n-2}}\right)$
- ${}^{(n-1)}R \geq 0$ (4≦n≦8, spin for n≧ 4)

⇨ (i) Mass *M* is non-negative.
(ii) *M=0* iff spacetime is Minkowski

Positive mass theorem II

Schoen & Yau(1981), Witten(1981)

-Asymptotically flat $$g_{ij} = \delta_{ij}\left[1 + \frac{2}{n-3}\frac{M}{r^{n-3}}\right] + O\left(\frac{1}{r^{n-2}}\right)$$

-Einstein equation, energy condition

- 4≦n≦8(4≦n for spin)

⇨ (i) Mass *M* is non-negative.
(ii) *M=0* iff spacetime is Minkowski

Strictly stationary spacetimes

- Configurations of static/stationary spacetimes

vacuum spacetime is Minkowski

Lichnerowicz 1955 [for 4 dimensions]

Proof

Komar mass formula for stationary and asymptotically spacetimes

$$M = -\frac{1}{8\pi}\int_{S_\infty} \nabla_\mu k_\nu dS^{\mu\nu}$$

$$= \frac{1}{4\pi}\int_\Sigma R_{\mu\nu} k^\mu n^\nu d\Sigma$$

$$= \frac{1}{4\pi}\int_\Sigma \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) k^\mu n^\nu d\Sigma = 0$$

⇨ spacetime is flat

Positive mass theorem

4D BH

- Static vacuum black hole is unique to be Schwarzschild spacetime

Israel 1967, Bunting & Masood-ul -Alam1987

- Stationary vacuum black hole is unique to be Kerr spacetime

Carter 1971, 73

Higher dimensional BH

- Static black holes is unique to be Schwarzschid.

  Gibbons, Ida & Shiromizu 2002

- Stationary black holes is axisymmetric.

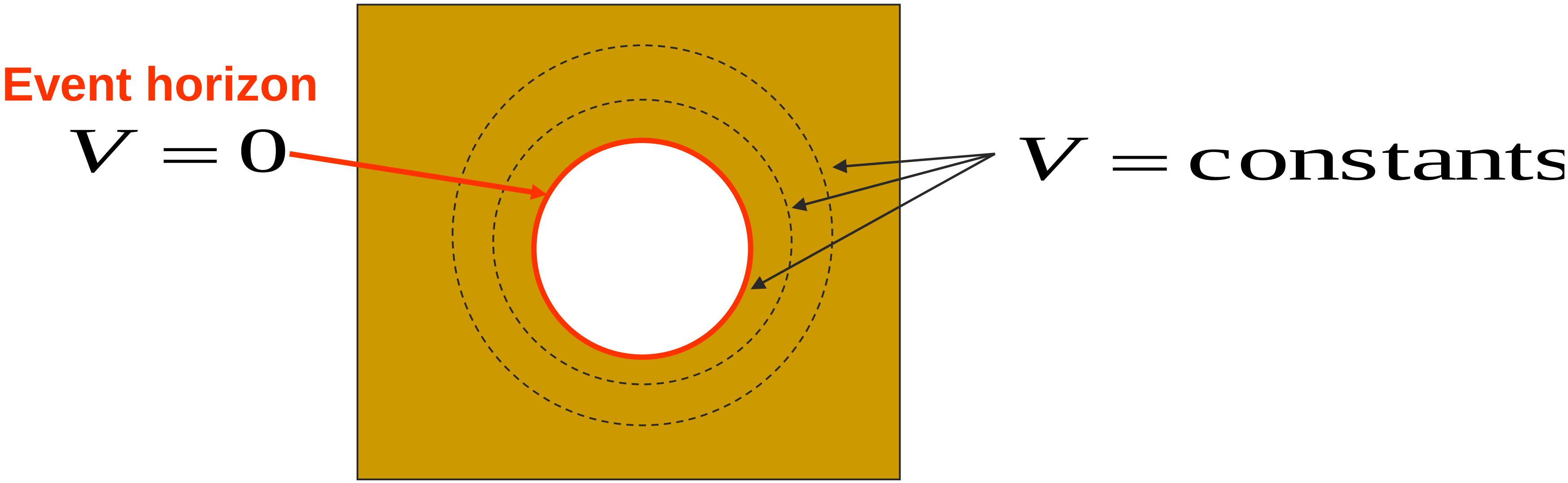
  Hollands, Ishibashi & Wald 2007

- **5-dim stationary** and **2-rotational symmetric** black holes are determined by mass, angular momentum and rod structure(~quasi local b.c. at horizon, axes).

  Hollands & Yazadjiev 2007

Static black hole spacetimes

metric $$ds^2 = -V^2(x^i)dt^2 + g_{ij}(x^i)dx^i dx^j$$



$t =$ constant spacelike surface

Vacuum, Einstein, higher dimensions

Sketch of proof

Gibbons, Ida & Shiromizu 2002

conformal transformation $\tilde{g}_{ij} = \left(\frac{1 \pm V}{2}\right)^{4/(n-3)} g_{ij}$, such that $\tilde{M} = 0, {}^{(n-1)}\tilde{R} = 0$

- Positive mass theorem $\Rightarrow \tilde{g}_{ij}$: flat space

- Regularity on event horizon

  ⇒ V=0 surface in conformally transformed space is spherical symmetric

- $\Delta\left(\frac{1}{1+V}\right) = 0, \quad \Delta = \partial_{x_1}^2 + \partial_{x_2}^2 + \cdots + \partial_{x_{n-1}}^2$

⇨ electrostatic potential with spherical boundary

typical examination for undergraduate students

3. Einstein-Gauss-Bonnet

Einstein-Gauss-Bonnet

String theory predicts the presence of the Gauss-Bonnet term

Zwieback 1985, Gross & Sloan 1987, Metsaev & Tseytlin 1987

Einstein-Gauss-Bonnet(EGB)

$$S=\int d^n x\sqrt{-g}\left[R+\alpha(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2)\right] \qquad \alpha \sim M_{string}^2$$

Field equation

$$R_{\mu\nu}=-\alpha\tilde{H}_{\mu\nu}$$

$$\tilde{H}_{\mu\nu}=2R_{\mu\alpha\beta\rho}R_{\nu}{}^{\alpha\beta\rho}-4R_{\mu\alpha\nu\beta}R^{\alpha\beta}-4R_{\mu}^{\ \alpha}R_{\nu\alpha}+2RR_{\mu\nu} -\frac{1}{n-2}g_{\mu\nu}(R_{\alpha\beta\rho\sigma}R^{\alpha\beta\rho\sigma}-4R_{\alpha\beta}R^{\alpha\beta}+R^2)$$

Various noble theorems?

Singularity theorem, positive mass theorem, area theorem, features of apparent horizons, …

Convergence condition

$$R_{\mu\nu} k^\mu k^\nu \geq 0$$

$$\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) k^\mu k^\nu \geq 0$$

k^a : timelike or null

⇔ non-negativity of energy density in Einstein

It is not natural to assume the above in EGB

But, if the corrections are tiny,…

$$R_{\mu\nu} = -\alpha \tilde{H}_{\mu\nu} \qquad \tilde{H}_{\mu\nu} \approx 2 R_{\mu\alpha\beta\rho} R_\nu{}^{\alpha\beta\rho}$$

$$\tilde{H}_{00} \approx 2 R_{0ijk} R_0{}^{ijk} + 2 R_{0i0j} R_0{}^{i0j} \ ?$$

As expected, no definite sign

Note： static BH uniqueness dose not depend on the above condition directly.
The structure of the field equations is important.

Check list for BH uniqueness

-Positive mass theorem

-finding a conformal transformation to show conformal flatness

-finding the harmonic function in the flat space

-spherical symmetry on “the horizon” in the flat space

Looks hard !

Yet there is no reason that uniqueness holds in full order of α!

Lichnerowicz in EGB

Strictly stationary and vacuum spacetimes should be Minkowski in GR.

Lichnerowicz 1955

??? The above holds in EGB ???

There is no reason that it holds in full order of α.
But, if the corrections are tiny,…

For simplicity, we focus on static cases

Static spacetimes

metric

$$ds^2 = -V^2(x^i)dt^2 + g_{ij}(x^i)dx^i dx^j$$

equations

$$\begin{cases} R_{00} = VD^2V = -\alpha \tilde{H}_{00} \\ {}^{(n-1)}R + \alpha\, {}^{(n-1)}L_{GB} = 0 \end{cases}$$

$$\begin{cases} \tilde{H}_{00} = -4\dfrac{n-4}{n-2} V D_i D_j V\, {}^{(n-1)}G^{ij} + \dfrac{1}{n-2} V^2\, {}^{(n-1)}L_{GB} \\ {}^{(n-1)}L_{GB} := {}^{(n-1)}R_{ijkl}\, {}^{(n-1)}R^{ijkl} - 4\, {}^{(n-1)}R_{ij}\, {}^{(n-1)}R^{ij} + {}^{(n-1)}R^2 \end{cases}$$

$$= {}^{(n-1)}C_{ijkl}\, {}^{(n-1)}C^{ijkl} - 4\frac{n-4}{n-3}\left({}^{(n-1)}R_{ij}\, {}^{(n-1)}R^{ij} - \frac{n-1}{4(n-2)}\, {}^{(n-1)}R^2 \right)$$

Lichnerowicz in EGB Shiromizu & Ohashi 2013

$$D^2V = 4\frac{n-4}{n-2}\alpha D_i D_j V\,{}^{(n-1)}G^{ij} - \frac{\alpha}{n-2}V\,{}^{(n-1)}L_{GB}$$

$${}^{(n-1)}R = O(\alpha),\ {}^{(n-1)}R_{ij} = \frac{1}{V}D_i D_j V + O(\alpha) \qquad + \quad {}^{(n-1)}C_{ijkl} = O(\alpha)\ [\text{assumption!}]$$

$$\Rightarrow {}^{(n-1)}L_{GB} = -4\frac{n-4}{n-3}\frac{1}{V}D_i D_j V\,{}^{(n-1)}R^{ij} + O(\alpha)$$

- $$D^i\left(D_i V - 4\frac{n-4}{n-3}\alpha\,{}^{(n-1)}R_{ij}D^j V\right) = O(\alpha^2)$$

- $$M = 0$$

$${}^{(n-1)}R = -\alpha\,{}^{(n-1)}L_{GB} = \frac{4(n-4)}{n-3}\alpha\,{}^{(n-1)}R_{ij}\,{}^{(n-1)}R^{ij} + O(\alpha^2) \ \Rightarrow \text{If } \alpha > 0,\ {}^{(n-1)}R \geq 0$$

- flat space ⇒ Minkowski spacetimes

4. Dynamical Chern-Simons gravity

Dynamical Chern-Simons gravity

dCSG Jackiw and Pi 2003,…

$$S=\int d^4x\sqrt{-g}\left[\kappa R+\frac{\alpha}{4}\theta^* R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-\frac{\beta}{2}(\nabla\theta)^2\right]$$

Field equation

$$\begin{cases} R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\frac{\alpha}{\kappa}C_{\mu\nu}=\frac{1}{2\kappa}T^{\theta}_{\mu\nu} \\ \nabla^2\theta=\frac{\alpha}{4\beta}R_{\mu\nu\alpha\beta}{}^*R^{\mu\nu\alpha\beta} \end{cases}$$

$$\begin{cases} C^{\mu\nu}=\nabla_\sigma\theta\varepsilon^{\sigma\delta\alpha(\mu}\nabla_\alpha R^{\nu)}_\delta+\nabla_\sigma\nabla_\delta\theta^* R^{\delta(\mu\nu)\sigma} \\ T^{\theta}_{\mu\nu}=\beta\nabla_\mu\theta\nabla_\nu\theta-\frac{\beta}{2}g_{\mu\nu}(\nabla\theta)^2 \end{cases}$$

Field equations in static spacetimes

Static $$ds^2 = -V^2(x^i)dt^2 + g_{ij}(x^k)dx^i dx^j$$

Trivial parts

$$R_{0i} = C_{00} = C_{ij} = T^{\theta}_{0i} = R_{\mu\nu\alpha\beta}\,{}^{*}R^{\mu\nu\alpha\beta} = 0$$

A part of field equations

$$\begin{cases} D^2 V = 0 \\ {}^{(3)}R = \dfrac{2\beta}{\kappa}(D\theta)^2 \\ D^2\theta + \dfrac{1}{V} D_i V D^i \theta = 0 \end{cases}$$

$$C_{0i} \neq 0$$

Strictly static spacetimes

$$\begin{cases} D^2 V = 0 \Rightarrow M = 0 \\ {}^{(3)}R \geq 0 \end{cases}$$

⇒ Minkowski spacetime

static BH uniqueness

conformal transformation $\tilde{g}_{ij} = \left(\frac{1 \pm V}{2}\right)^4 g_{ij}$, such that $\tilde{M} = 0, {}^{(3)}\tilde{R} \geq 0$

- Positive mass theorem $\Rightarrow \tilde{g}_{ij}$: flat space $\Rightarrow \theta = \text{constant}$
- Regularity on event horizon

⇒ V=0 surface in conformally transformed space is spherical symmetric

- $\Delta\left(\frac{1}{1+V}\right) = 0, \quad \Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$

electrostatic potential with spherical boundary

Spherical symmetric ⇒ Schwarzschild

Then, Massive gravity…

- Strictly static/stationary spacetime is Minkowski?
- In BH spacetime, staticity implies the presence of spherical symmetry?

In general, not. But, if specify the asymptotic behaviors or employ a perturbative argument, you may do something.

$$g = -V^2(x^i)dt^2 + 2\beta_i(x^j)dtdx^i + g_{ij}(x^k)dx^i dx^j$$

$$\eta = -dt^2 + \delta_{ij}dx^i dx^j$$

Unitary gauge : Stuckelberg $\phi^\alpha = x^\alpha$,………………….