Workshop "Geometry of lattices and infinite dimensional Lie algebras"

Period: March 17 - 19, 2010
Place: Balcony A, IPMU, Kashiwa campus, the university of Tokyo
Organizer: K. Saito (IPMU)

PROGRAM

17 March	
10:30 - 12:00	H. Aoki:
	On the convergence of Fourier-Jacobi expansion
13:30 - 15:00	N. Scheithauer:
	Moonshine for Conway's group
15:30 - 17:00	T. Basak:
	The diagrams for finite unitary reflection groups
17:15 - 18:45	T. Ishibe and K. Saito:
	Monoids in the fundamental groups of compliment of discriminants
19:00	Dinner
18 March	
10:30 - 12:00	S. Kondo:
	The Leech lattice and K3 surfaces
13:30 - 15:00	B. Venkov:
	Dence Euclidean lattices and energy minimizing
15:30 - 17:00	J. Sekiguchi:
	Holonomic systems and fundamental groups — The case of Saito free divisors in $\mathbf{C}^3$ —
19 March	
10:30-12:00	Discussions

Title and Abstract of Talks

- Boris Venkov (Steklov St. Ptersburg)
  Title: Dence Euclidean lattices and energy minimization.
  Abstract: I start with one simple inequality:

$$\frac{1}{|X|^2}\sum_{x_1,x_2\in X}(x_1,x_2)^4 \geq \frac{3}{n(n+2)}$$

  which hold for an arbitrary finite , antipodal ($X=-X$) subset X of the unit sphere $X\subset S^{n-1}\subset \mathbf{R}^N$. Subsets which have = , that is those who minimize foruth energy are very interesting and they are spherical 5-design. If $X+\Lambda_{mini}$ is the set of short vectors in a lattice Λ then Λ is called 5-design lattice. I discuss classical Voronoi theory of perfect lattices and its 5-design counter part.

- Nils Scheithauer (TU Darmstadt)
  Ttitle: Moonshine for Conway's group
  Abstract: The fake monster algebra is a generalized Kac-Moody algebra describing the physical states of a bosonic string moving on a 26-dimensional torus. The automorphism group of the Leech lattice acts by diagram automorphisms on this Lie algebra. Borcherds conjectured that the corresponding

twisted denominator identities are automorphic forms of singular weight on orthogonal groups. We prove this conjecture for elements whose order is equal to the level.

- Tathagata Basak (IPMU)
  Title: The diagrams for finite unitary reflection groups
  Abstract: An unitary reflection group is a finite subgroup of $U(n)$ generated by a set of elements of finite order each of which fix a hyperplane in complex n-space. Coxeter defined diagramatic presentations for many Unitary reflection groups which was extended for all the Unitary reflection groups by Broue et.al. Like the classical Dynkin diagrams, these diagrams encode a lot of information. For example, one can find the invariant degrees of the reflection group from the diagram and the diagrams encode a presentation of the fundamental group of the corresponding "braid space". But unlike the classical case there is no uniform definition for these diagrams. We shall try to briefly survey what is known and describe an attempt to characterize these diagrams. In some exceptional cases, in which the known diagrams did not have some desirable properties, our method yields new diagrams.

- Shigeyuki Kondo (Nagoya univ.)
  Title: The Leech lattice and K3 surfaces
  Abstract: Conway found a fundamental domain of the reflection group of the even unimodular lattice of signature (1,25) whose simple roots bijectively correspond to points on the Leech lattice. In this talk, I discuss an application of this theorem to Algebraic Geometry. I will take a supersingular K3 surface with the Artin invariant 1 in characteristic 2. I remark that the Picard lattice of this K3 surface is isomorphic to a reflective hyperbolic lattice with the biggest rank 22.

- Hiroki Aoki (Tokyo univ. of science)
  Title: On the convergence of Fourier-Jacobi expansion
  Abstract: By Fourier-Jacobi expansion, an automorphic form on O(2,s+2) gives a family of Jacobi forms of index 0,1,2,3,... . In my talk, we discuss the image of Fourier-Jacobi expansion. Mainly we treat the simplest case s=1, that is a Siegel modular form of degree 2. In this case, we have determined the image of Fourier-Jacobi expansion. This result is useful to show the convergence of Maass lifts and Borcherds products.

- Jiro Sekiguchi (Tokyo Univ. Agriculture and Tech.):
  Title: Holonomic systems and fundamental groups — The case of Saito free divisors in $\mathbf{C}^3$ —
  Abstract: Taking examples of Saito free divisors in $\mathbf{C}^3$ which are defined as 1-parameter deformations of simple curve singularities of types E_6, E_7, E_8, I discuss the relationship between two or three dimensional representations of the fundaments group of the complement of a Saito free divisor and holonomic systems including systems of uniformzation equations along such a divisor.

- Tadashi Ishibe (Hiroshima university), Kyoji Saito (IPMU)
  Title: Monoids in the fundamental groups of complements of free divisors
  Abstract: We study monoids generated by Zariski-van Kampen generators in the fundamental groups of complements of divisors. In the classical case of discriminant divisors, the monoids are braid monoids or, more generally, Artin monoids. We calculate the monoids for the case of 17 Sekiguchi-divisors. They have positive homogeneous presentation, but 4 of them are no longer Artin nor Garside monoids. We introduce a concept of fundamental elements, generalizing that for the Artin and braid monoids, and show the existence of them for all 17 monoids.