Entanglement Entropy

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Since in quantum mechanics we interpret a particle as a wave, we can take a linear combination of physical states. For example, let us consider a system with two spins of electrons. For the first example, we can think of a state ($|\Psi_1\rangle$ in the figure below) defined by the condition that one of the two spins is up (called spin A), while the other is down (called spin B). Such a direct product state is a classical state. On the other hand, we can consider another state ($|\Psi_2\rangle$) obtained by taking a linear combination of the previous state and its opposite state with equal weight, which is called an EPR pair. Such a state which cannot be written as a direct product state has a non-zero correlation between A and B and thus has quantum entanglement. Even though the total state is uniquely fixed, if we look at its subsystem, there is ambiguity on which state is realized. A quantity which measures the amount of quantum entanglement is entanglement entropy S_A, which is defined as the von Neumann entropy for the reduced density matrix. This estimates how many EPR pairs can be extracted from the entanglement between A and B.

$$S_A = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

(i) $|\Psi_1\rangle = |\uparrow\rangle_A |\downarrow\rangle_B \sim \left[\, \overset{\uparrow}{A}\ \ \underset{\downarrow}{B} \,\right]$

$S_A = 0$

(ii) $|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B + |\downarrow\rangle_A |\uparrow\rangle_B\right)$

$S_A = \log 2 \qquad \sim \left[\, \overset{\uparrow}{A}\ \ \underset{\downarrow}{B}\ \text{ or }\ \underset{\downarrow}{A}\ \ \overset{\uparrow}{B} \,\right]$