FEATURE

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Research Area : Theoretical Physics

Quantum Gravity and Quantum Information

Understanding the nature of space and time has brought many revolutions to physics. In old days, the introduction of a coordinate system—which is often attributed to Descartes—led to Newtonian mechanics through contributions by many scientists, including Galileo. At the beginning of the 20th century, Albert Einstein proposed special relativity, which, in a sense, unifies space and time. According to this theory, space and time are closely related, causing phenomena such as a contraction of length and a delay of time when an object moves at a speed close to that of light. These effects are precisely measured in present-day experiments, leaving no room to doubt the theory. The resulting concept of *spacetime*, which treats space and time in a unified manner, has been playing a fundamental role in contemporary physics until today.

One of the most important properties of spacetime was revealed by Einstein himself about 10 years after the announcement of special relativity. According to this new theory called general relativity, spacetime is not a fixed entity, as one might imagine from the term “coordinate system”; rather, it is a dynamical object that can be bent or distorted. General relativity says that it is these bending and distortion of spacetime that we call gravity. For example, gravity between the Sun and the Earth arises because the existence of the Sun bends spacetime around it. The Earth moves around the Sun because of this bending of spacetime.

This revolutionary theory of spacetime makes an important prediction: because spacetime is dynamical, its distortions can propagate independently of the existence of matter. For example, when heavy bodies such as black holes move violently, they disturb spacetime greatly. This effect can then propagate to places far away from the location of the original bodies as wave of spacetime distortions. This wave is called gravitational wave, and it was first detected directly in 2016, almost 100 years after the formulation of general relativity. This was an extremely important experimental discovery, and for this achievement the 2017 Nobel Prize in Physics was awarded.

Theory of Quantum Gravity

As we have seen, our understanding of spacetime has dramatically evolved over the last few hundred years. However, we know that this is not the end. The reason is that our world obeys quantum mechanics.

Quantum mechanics was discovered at the beginning of the 20th century in the process of studying microscopic objects such as atoms, but later it was found to be a more fundamental principle governing the world. Quantum mechanics makes a number of strange predictions. For example, an object such as an electron cannot exist in one place; rather, its location spreads probabilistically. (Since this spread is usually so small that we cannot perceive it directly in our daily life.) It is also possible that two worlds exist parallelly in probability space, the state we call a superposition. While these phenomena sound extremely strange from the sense of everyday life, the correctness of the theory has been repeatedly demonstrated experimentally. In fact, the accuracy of these experiments has dramatically improved in recent years, so that predictions of quantum mechanics have now been confirmed precisely at a detailed level. Furthermore, scientists are advancing the development of a computer—quantum computer—which has capability completely different from the conventional one and which positively utilizes the principle of quantum mechanics.

Let us return the story to spacetime. General relativity explains all gravitational phenomena that have been observed until now. This includes minuscule deviations from the predictions of Newtonian gravity, the existence of gravitational wave, and the expansion of the universe. There is almost no doubt about the correctness of the theory at scales we can currently observe. General relativity, however, does not contain the effect of quantum mechanics, which is a basic principle of the world we live in. This is not an experimentally urgent problem. The scale at which the quantum effect of spacetime becomes important directly can be estimated theoretically, which is about 10^{-33} cm, the distance called the Planck length. This is about 16 orders of magnitude smaller than the smallest length probed so far by the world's most powerful accelerator. Does this mean that it is meaningless to investigate the quantum effect on spacetime?

There are a number of reasons, at least theoretically, why this is not the case. One is the problem of infinity. Quantum mechanics and general relativity do not get along very well. Assuming that the theory of gravity at our scales is general relativity, its simple extrapolation below the Planck length—where quantum effects on spacetime and gravity become important—leads to an innumerable amount of infinities, destroying the predictivity of the theory. This means that in order to obtain a complete theory including quantum effects on spacetime and gravity, it is not enough to simply consider general relativity and quantum mechanics together. The currently most promising theory to solve this problem is string theory, whose structure and consequences are vigorously studied by many researchers.

Another reason to study quantum mechanical theory of spacetime and gravity—commonly referred to as *quantum gravity*—is that it brings about a revolutionary picture of spacetime. As we will see in this article, it is not true that the effect of quantum gravity becomes important only at the Planck length; on the contrary, it has increasingly become clear that it also plays a decisive role in physics at long distances such as the physics of black holes and multiverse cosmology. Moreover, this relatively recent progress is creating a new research area that unifies two fields of quantum gravity and quantum information sciences, which have been developed separately so far. Below I will give a brief overview of this new development.

Quantum Mechanics of Spacetime and the Holographic Principle

The beginning was the physics of black holes. In a theory with gravity, when the density of matter becomes larger than a certain limit, it produces a region around it from which nothing can escape due to its strong gravity. The boundary of this region is called a horizon, and the inside is a black hole.

Black holes had been known to have the following intriguing property: in general relativity, the total area of horizons does not decrease. For example, when a black hole swallows an object, its area increases. (Here and below, when we refer to the area of a black hole, it means the area of the horizon of the black hole.) Also, when two black

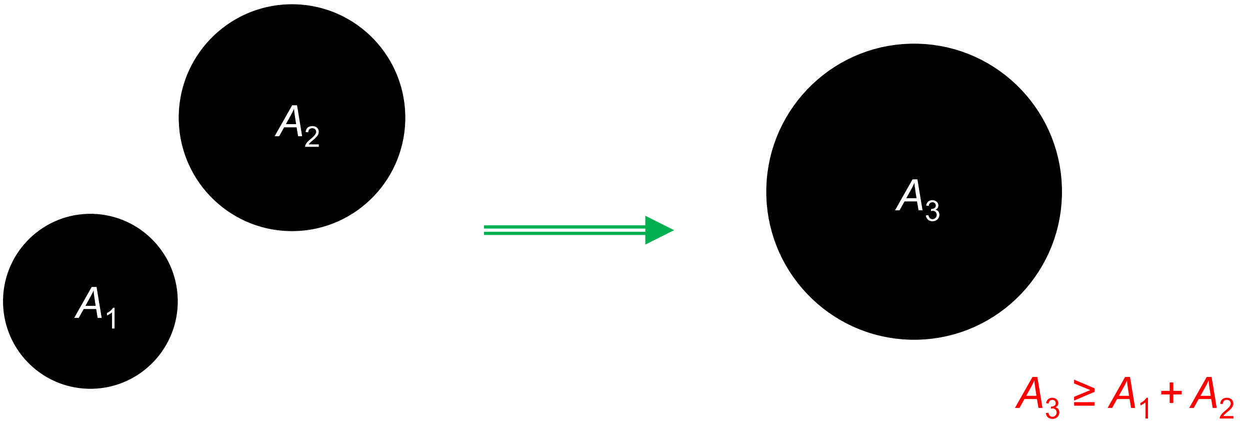

Figure 1: The area, A_3, of the black hole formed after the merging of two black holes is equal to or larger than the sum of the areas, A_1 and A_2, of the original black holes.

holes merge into one— this phenomenon was actually observed by the detection of gravity wave in 2016!—the area of the produced black hole is equal to or larger than the sum of the areas of the initial two black holes; see Figure 1.

The fundamental importance of this fact was realized by Jacob Bekenstein in his revolutionary paper in 1973. Bekenstein asked what happens to physical quantities when an object is thrown into a black hole. The answer is clear for energy. The energy of the thrown object becomes (a part of) the mass of the black hole, so that the total energy is conserved. But what about entropy? According to the second law of thermodynamics, entropy should not decrease. However, the entropy of the thrown object seems to simply disappear when it is absorbed into the black hole. Does this mean that the second law of thermodynamics is violated?

What Bekenstein suggested in his 1973 paper is that the entropy of the thrown object is converted into the entropy of the black hole. What is the entropy of the black hole? He proposed that the area of the horizon is the one. Recall that the total sum of horizon areas does not decrease in general relativity. This is exactly the property that entropy must have. Indeed, by defining the entropy of a black hole in this way, it can be shown that the entropy of the black hole after the object is absorbed is always equal to or larger than the sum of the entropies of the initial black hole and the thrown object.

The idea that a black hole has entropy was initially received with skepticism. If a black hole has entropy, the first law of thermodynamics requires it to also have temperature. However, if it has temperature, then it must be radiating. Isn't it the case that nothing can come out of a black hole? This apparent contradiction, however, was resolved in 1974 when Stephen Hawking performed a calculation which included the effect of quantum mechanics into general relativity. While his calculation was not based on a complete theory of quantum gravity, he could still show that a black hole indeed emits radiation because of quantum effects. This established the thermodynamics of black holes.

A strange thing here is that the entropy of a black hole is given by its area. Since a black hole is the final state of the evolution of any initial state, the entropy of the black hole must be indicating "the largest possible entropy that the region can have." In modern days, we know that entropy is given by the logarithm of the number of possible quantum states that the system can take. If space is composed of some simple constituents as in ordinary materials (e.g., if space can be approximated by a lattice with a spacing of order the Planck length), then the largest entropy of a region must be proportional to its volume. However, the discovery of Bekenstein and Hawking says that it is proportional to the surface area.

This implies that in a quantum theory with gravity, the number of fundamental degrees of freedom is given by that of spacetime with one dimension less than that of the original, dynamical spacetime. For example, while spacetime we live in seems to have three spatial dimensions (ignoring possible small extra dimensions), the "true theory" describing it must be formulated in spacetime with two spatial dimensions and one time dimension. This is possible because if you try to fill matter at each point in space (e.g., at each site of the Planck-size lattice), black holes form long before it is completely filled, and putting further matter only increases the size of the black holes. Namely, it is merely a fiction that the space we live in has full three dimensional degrees of freedom.

The idea that the theory of quantum gravity is formulated in spacetime that has lower dimensions than the apparent, dynamical spacetime is called the *holographic principle*. The lower dimensional theory formulated in this way—called the holographic theory—does not have the discrepancy between the number of true and apparent degrees of freedom, so it does not have gravity: the dynamical spacetime in higher dimensions and its associated gravity are only emergent. The fact that the holographic theory does not contain gravity at the fundamental level means that it can be treated as a regular quantum system, and hence can give a rigorous definition of quantum gravity. This seems to be a pretty crazy conclusion. It would be natural if one cannot believe such a thing.

AdS/CFT Correspondence

However, it has been shown—though only in some special cases—that quantum gravity indeed satisfies the holographic principle! This discovery was made by studying structures of string theory. In 1997, Juan Maldacena proposed, based on numerous evidences, that quantum gravity describing physics in spacetimes that asymptotically approach a certain space called Anti-de Sitter (AdS) space is equivalent to conformal field theory (CFT) formulated in non-gravitational spacetime that has dimensions one less than the gravitational asymptotically AdS space. This relationship is called the *AdS/CFT correspondence*.

The AdS/CFT correspondence is an extremely powerful mechanism despite the fact that it applies only to special spacetimes. First, CFT is a class of quantum field theory and is mathematically well defined. This implies that quantum gravity in spacetimes that are not so distant from the one we live in (asymptotically AdS space) is defined for the first time without relying on perturbation theory. Also, many theories describing the nature (e.g., QCD describing nuclear force and theories used in condensed matter physics) are well approximated by CFT at strong coupling. In general, it is extremely difficult to solve such a theory with strong coupling, but by the AdS/CFT correspondence we can solve it approximately using general relativity in one higher dimensions. And above all, this correspondence gives a concrete example of the holographic principle, showing how dynamical spacetime with gravity is generated from non-gravitational theory in lower dimensions.

The mechanism of generating dynamical spacetimes has been studied by many researchers, and in this process a very important discovery was made by Shinsei Ryu and Tadashi Takayanagi. What they found is the following relation between the geometry of AdS and quantum states in CFT; see Figure 2. Consider a quantum state on an equal time slice in CFT. Let us divide this slice into a subregion A and its complement $\bar{A}$. Then, the entanglement entropy* between A and $\bar{A}$ is given by the area of the surface in AdS that is anchored to

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*See, *Kavli IPMU News* No. 31, p. 28.

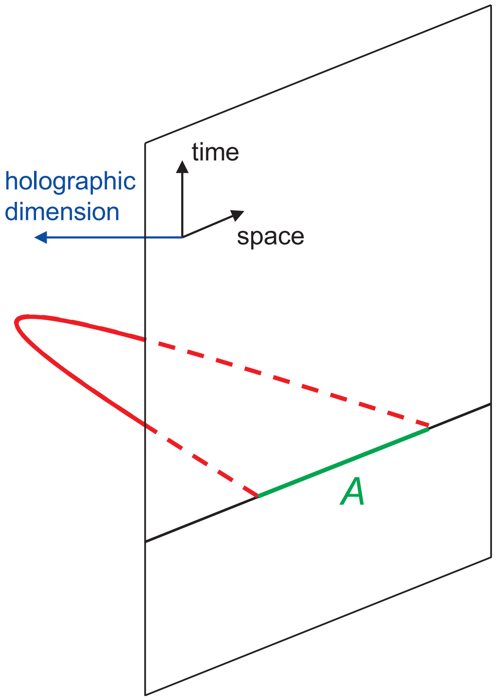

Figure 2: Entanglement entropy of a state in CFT between a spatial region A and its complement $\bar{A}$ is given by the area of the surface in AdS which is anchored to the boundary of A and whose area takes an extremal value.

the boundary of A and has the minimal area (more generally the surface which extremizes the area in AdS). Here, the entanglement entropy between A and $\bar{A}$ is a quantity representing the amount of quantum entanglement between the degrees of freedom in A and $\bar{A}$.

The importance of this relationship, called the Ryu-Takayanagi formula, is not just that it relates two quantities. First, the fact that a geometric object in the gravitational bulk—a minimal area surface in AdS—is related to a quantum information theoretic quantity in the holographic theory—an entanglement entropy—implies that the mechanism of generating dynamical spacetime is intrinsically quantum mechanical. Also, taking the view that the theory of quantum gravity is rigorously defined only holographically, we may say that what we perceive as gravitational spacetime is nothing other than quantum information theoretic properties of quantum states in a more fundamental, non-gravitational theory. In fact, by using the Ryu-Takayanagi formula, one can show that the Einstein equation in higher dimensional AdS background is an automatic consequence of the relation between the geometry of emergent spacetime and quantum information in the holographic theory.

The AdS/CFT correspondence gives us a basic picture of how dynamical spacetime may be generated. We could draw the following analogy. Consider liquid water in place of spacetime. We want to know its microstructure such as water molecules and forces acting between them. For this purpose, it is barely useful to study phenomena that occur as perturbation in the limit that liquid water is regarded as continuum, e.g., water waves. These are described by the Navier-Stokes equation of fluid dynamics, which corresponds to the Einstein equation in our context, regarding spacetime as continuum. In contrast, AdS/CFT can tell us the identity of water molecules and forces between them, i.e., CFT. A big difference between water and spacetime, however, is that in the case of spacetime its constituents somehow live in one lower dimensions.

Beyond AdS/CFT

As we have seen above, AdS/CFT is extremely powerful, but the actual spacetime we live in is not an asymptotically AdS space. On the other hand, the holographic principle is expected to apply more generally in quantum gravity as can be seen from the consideration of black holes. How then does holography work in general spacetimes? This question has been investigated by several research groups. Below, I will briefly describe the work we have done.

The starting point is to identify which of the properties of AdS/CFT are specific to AdS space and which are more generally applicable in quantum gravity. For example, we expect that, from various considerations, the fact that the holographic theory is local quantum field theory like CFT is due to a special feature of asymptotically AdS space. Then, which of the properties of AdS/CFT are manifestations of the more general holographic principle?

Based on a number of suggestions, we have hypothesized that the relationship between quantum information in a lower dimensional holographic theory and the geometry of higher dimensional quantum gravity, as represented by the Ryu-Takayanagi formula, is general. Since the holographic theory for general spacetimes is not yet known, this requires us to extract essential elements of the theory using entropy bounds obtained by extending the result of Bekenstein and Hawking. I omit the details of this procedure due to the limitation of pages. In any case, the idea is to examine the validity of the hypothesis by analyzing what consequences it leads to.

In a sense, this strategy corresponds, e.g., to the one Max Planck took when he examined the consequences of his quantum hypothesis upon the creation of quantum mechanics. At that time, there was an experimental result—the black body spectrum—to which he compared the consequences. In the case of quantum gravity, however, there is no corresponding direct experimental result to which we can compare theoretical results. Then how can we test the validity of the hypothesis?

Our idea is to use general relativity as "experiment." Specifically, under the assumption that the holographic relationship between quantum information and generated geometries is valid in general spacetimes, we have derived the properties that the dynamical spacetimes must satisfy in order for the quantum information theoretic quantities to be consistently interpreted holographically. The result was a series of monotonicity properties and inequalities, which were not known before. If the original assumption is correct, then these properties must be satisfied for general spacetimes, and indeed we were able to prove them using general relativity.

An important point is that if general relativity did not possess features that are needed to prove the properties we found, then the original assumption could have been rejected. In other words, the hypothesis that quantum information theoretic properties of states in the holographic theory are the origin of dynamical spacetime in cases beyond AdS/CFT has passed a nontrivial test. This increases our confidence about the holographic principle and relation between quantum information and quantum gravity.

Summary

This article has described the relationship based on the holographic principle between quantum information and dynamical spacetime, including AdS/CFT. This has been investigated not only by our group but also by other researchers, and important discoveries have been made day-to-day. These discoveries are providing fundamentally new answers to the question of what spacetime really is. Hopefully, this new research area emerging at the intersection of quantum gravity and quantum information will reveal a deep and fundamental picture of spacetime.