

# Higher Residue Week, 2016

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The “Higher Residue Week, 2016” at Kavli IPMU took place from June 6 to 10, 2016. The event started with a two-day mini-workshop and it continued as a series of lectures by Dmytro Shklyarov (TU Chemnitz). The main goal of the workshop was to present the recent work of Shklyarov who has introduced Hodge-like structures of dg-categories motivated by Kyoji Saito’s theory of primitive forms. The notion of a primitive form was invented by K. Saito during his visit to Harvard in 1980. The main motivation is to provide a generalization of the classical period map for Riemann surfaces in the settings of Singularity Theory, i.e., the study of isolated critical points of holomorphic functions.

The importance of K. Saito’s work increased significantly in the early 90s when Alexander Givental and Maxim Kontsevich noticed that the theory of primitive forms provides the key

concepts to state mirror symmetry and it can be used in symplectic geometry to compute Gromov-Witten invariants of compact Kahler manifolds. Mirror symmetry consists of finding a triple of an affine manifold  $Y$ , a holomorphic function  $f$  on  $Y$ , and a primitive form  $\omega$ . The key ingredient in Saito’s theory is the so-called *Higher Residue Pairing*. This is a certain non-degenerate bi-linear pairing defined on the twisted de Rham cohomology of  $Y$  via a sequence of residues. The primitive form is a special cohomology class satisfying an infinite system of bi-linear relations. The key observation of Givental is that the oscillatory integral built from  $f$  and  $\omega$  coincides with what he called the  $J$ -function, i.e., a certain generating series of genus-0 Gromov-Witten invariants.

The main contribution of Shklyarov is that by studying the category of matrix factorizations he found a categorical interpretation of the

twisted de Rham cohomology and the Higher Residue Pairing. In particular, the ideas and the concepts of K. Saito’s theory could be extended in much more general settings. As an application, Shklyarov also gave a talk during the mini-workshop based on his solution of a conjecture of Anton Kapustin and Yi Li, which implies the existence of an interesting  $A$ -infinity structure.

During the workshop, Hiroshi Ohta explained his joint work with Kenji Fukaya, Kaoru Ono and Yong-Geun Oh in open Gromov-Witten theory, which provides yet another tool to construct  $A$ -infinity structures and primitive forms. The workshop made an interesting contribution to understanding the big puzzle of mirror symmetry and primitive forms. Many ideas were exchanged and most importantly several possible directions for further investigations were clearly outlined.

