Pseudo transverse mass shining on buried new particles

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Motivation

• Pair-produced new particle $Y$ decaying into visible particles, $V$ plus invisible WIMPs, $\chi$.

$$ P + P \rightarrow Y_1 + Y_2 + etc \rightarrow V_1 + \chi_1 + V_2 + \chi_2 + etc $$

• Measuring new particle masses is not easy.

✓ Partonic CM frame ambiguity
✓ Several missing particles
✓ Complex event topologies

• When the event reconstruction is impossible, measuring kinematic boundaries is a good way to determine new particle masses.

✓ Transverse mass of W-boson / Invariant mass methods / $M_{T^2}$-kink methods ..
✓ Observing the endpoints (kinematic boundaries) of the variables is important.
• However, in general, identifying a meaningful endpoint is not a trivial task with many sources of systematic uncertainties.

- Various kinds of the signal endpoint shape
- Hard to estimate the backgrounds, especially for jets.
  - Jet combinatorics with hard ISR
  - In many cases, they are irreducible.
- Jet E resolution, Finite total decay widths effects.
- …
• Example of endpoint measurement (1)

- Invariant masses of visible particles in SUSY cascade decay chain
- Gaussian smeared linear signal + backgrounds function fits well with 1~10 GeV systematic uncertainty.
- Example of endpoint measurement (2)

\[ M_{T2}^{max} (\sim q_R \sim q_R \rightarrow q\chi^+ q\chi) = RH \text{ squark mass, } m_x \text{ known} \]

ATLAS Technical Design Report 2009

\[ m_{\tilde{q}_R} = 611 \text{ GeV} \quad m_{\tilde{q}_R} = 406 \text{ GeV} \]

Max of \( M_{T2} \) measurement usually has O(1\~10\%) systematic uncertainty in fitting process (fitting function, cuts, range ...).
• In the existence of large systematic uncertainty, extracting the mass parameter using template least chi-square methods or global fit would provide large uncertainty also.

• Anyway, even with the bulk distribution with large uncertainty, one can always define the signal endpoint as a breakpoint in the distribution with proper resolution/width effect.

• Can we reduce the systematic uncertainty in finding the signal endpoint?

• At least, for $M_{T2}$ endpoint measurement, it’s possible!
**$M_{\pi T}$ and $M_{\pi T2}$**

- $M_{T2}$ - An extension of the transverse mass, $M_T$ for the event with two missing particles

**Transverse mass of $Y \rightarrow V(p) + \chi(k)$**

$$M_T^2 = m_V^2 + m_\chi^2 + 2 \sqrt{m_V^2 + |p_T|^2} \sqrt{m_\chi^2 + |k_T|^2} - 2p_T \cdot k_T$$

⇒ Independent of the longitudinal momenta.
⇒ One may use an arbitrary trial WIMP mass $m_\chi$ to define $M_T$

(True WIMP mass = $m_\chi^{true}$)

**Stransverse mass of $Y_1 Y_2 \rightarrow (V_1(p_1) + \chi_1(k_1)) + (V_2(p_2) + \chi_2(k_2))$**

$$M_{T2}^2 = \min[\max\{M_T(Y_1), M_T(Y_2)\}]$$

⇒ Minimization over all possible WIMP transverse momenta

- For all events, $M_{T \& T2}(m_\chi = m_\chi^{true}) \leq m_Y^{true}$
• The mass constraint from $M_{T2} : p_0$

\[ M_{T2}^{\text{max}}(x) = p^0 + \sqrt{p^0{}^2 + x^2}, \]
\[ x = \text{trial WIMP mass}, \text{ Visible particle mass } \sim 0 \]
\[ p^0 = \frac{m_\gamma^2 - m_x^2}{2m_\gamma}, \text{ the momentum of } v \& \chi \text{ in the rest frame of } Y \]

• It’s an interesting result of $M_{T2}$ kinematics because each of the two mother particles is not at rest in LAB frame!

(!) It is only valid when total transverse momentum of 2 mother particle system is zero. $(p_T(Y_1) + p_T(Y_2) = 0)$
\(M_\pi \) and \(M_{\pi T}\)

**Transverse mass**\((M_T)\) & **invariant mass**\((M)\) of \(Y \rightarrow V(p) + \chi(k)\)

\[
M_T^2 = m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |p_T|^2} \sqrt{m_\chi^2 + |k_T|^2} - 2p_T \cdot k_T \leq M^2
\]

\(\Rightarrow\) Defined in **any frame** with fixed endpoint, \(M\)

**Pseudo transverse mass**\((M_{\pi T})\) & **pseudo invariant mass**\((M_\pi)\)

\[
M_{\pi T}^2 = m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |p_T^0|^2} \sqrt{m_\chi^2 + |p_T^0|^2} - 2(R(\pi)p_T^0) \cdot (-)p_T^0
\]

\[
\leq M_\pi^2 (= m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |p_T^0|^2} \sqrt{m_\chi^2 + |p_T^0|^2} Cosh\Delta \eta - 2(R(\pi)p_T^0) \cdot (-)p_T^0
\]

\# \(R(\pi)\) is the rotation in transverse plain by angle, \(\pi\)

\# \(\Delta \eta = \) rapidity difference between the visible and invisible particles

\(\Rightarrow\) Defined in **the rest frame of \(Y\)** with fixed \(M_{\pi T}\) endpoint, \(M_\pi\)

\(\Rightarrow\) Endpoint useful only for a **mother particle with** \(P_T = 0\)

\(^*\) The way of combining momenta constituents for \(M_{\pi T}\) is exactly same with the collider variable, \(M_{CT}\) [Tovey, arXiv:0802.2879]
If it is possible, then the pseudo transverse mass endpoint will also provide us the $P^0$

$$M^\text{max}^2_{\pi T} (x) = m_V^2 + x^2 + 2 \sqrt{m_V^2 + |p^0|^2} \sqrt{\frac{x^2}{2}} + |p^0|^2 - |p^0|^2$$

$x =$ trial WIMP mass

→ How about the new mother particle pair, each with nonzero $P_T$?
Pseudo-stransverse mass ($M_{\pi T 2}$) for

$$Y_1 Y_2 \rightarrow (V_1 (p_1 ) + \chi_1 (k_1 )) + (V_2 (p_2 ) + \chi_2 (k_2 ))$$

$$M_{\pi T 2}^2 \equiv \min[\max \{ M_{\pi T} (Y_1 ) , M_{\pi T} (Y_2 ) \}]$$

$$M_{\pi T} \equiv m_{\nu}^2 + m_{\chi}^2 + 2 \sqrt{m_{\nu}^2 + |p_T|^2} \sqrt{m_{\chi}^2 + |k_T|^2} - 2 (R(\pi)p_T) \cdot k_T ,$$

# $p_T$ s' are visible transverse momenta in the LAB frame

# $\min&\max$ over all possible invisible momentum $k_T$

- $M_{\pi T}$ endpoint can be realized using $M_{\pi T 2}$ (pseudo-stransverse mass) variable defined in the LAB frame for the pair of mother particles with total $P_T=0$!
• Then, the endpoint behavior in trial WIMP mass, \( x \), also provides the \( P^0 \)

\[
M_{\pi T_2}^{\text{max}} (x) = m_{\nu}^2 + x^2 + 2\sqrt{m_{\nu}^2 + |p^0|^2} \sqrt{x^2 + |p^0|^2} - |p^0|^2
\]

\( x = \) trial WIMP mass

• Condition for PST endpoint :

\[
\delta_T \equiv |P_T (Y_1 + Y_2)| = 0
\]

**\( M_{\pi T_2} \) solution for an event with \( \delta_T = 0 \)**

*(For single visible particle in each decay chain)*

\[
M_{\pi T_2}^2 (m_{\nu}^{(i)}, \vec{P}_T^{v(i)}, \chi) = \chi^2 - A_T
\]

\[
+ \sqrt{1 + \frac{4 \chi^2}{(2A_T - m_{\nu}^{(1)^2} - m_{\nu}^{(2)^2})^2}} [A_T^2 - (m_{\nu}^{(1)}m_{\nu}^{(2)})^2]
\]

\( A_T = E_T^{v(1)}E_T^{v(2)} + \vec{P}_T^{v(1)} \cdot \vec{P}_T^{v(2)} \)
Properties of $M_{\pi T^2}(x)$ distribution

- If $m_{\text{vis}} \sim 0$, $M_{\pi T^2}(x)$ projection of events has amplified endpoint structure with proper value of trial WIMP mass, $x$ originated from Jacobian factor between $M_T$ and $M_{\pi T}$

\[
\sigma^{-1} \frac{d\sigma}{dM_{\pi T}(x)} \sim J(M_{\pi T}(x), M_{\pi T}(x)) \sigma^{-1} \frac{d\sigma}{dM_T(x)}
\]

\[
J = \frac{M_\pi(x)}{M(x)} \left( \frac{E_x + P_o}{E_x - P_0} \right)^2
\]

\[
\rightarrow \begin{cases} 
M_T \text{ max region, } J_{\text{max}} \to \infty \text{, when } x \text{ is small} \\
M_T \text{ min region, } J_{\text{min}} \to 1
\end{cases}
\]
In result of very different compression rate, most of the large $M_{T2}$ events are accumulated in narrow $M_{\pi T2}$ endpoint region.
• A faint breakpoint (e.g. signal endpoint) with small slope difference, $\Delta a \rightarrow \Delta a` = J^2 \Delta a$ by the amplification in $M_{\pi T2}$ projection.

With the salient breakpoint structure, the fitting scheme (function/range) can be elaborated, and it reduces the systematic uncertainties in extracting the position of the breakpoint.
Error analysis with histogram : \( (x_i, y_i \pm \sigma_i) \)

\( \sigma_i = \) statistical error of the i-th bin

**Statistical error for breakpoint(BP)**

(using Least Square methods)

\[
\delta_{BP}^2 \sim \frac{\sigma^2}{\Delta a^2} \rightarrow \frac{J^2 \sigma^2}{J^4 \Delta a^2} \sim \frac{1}{J^2} \delta_{BP}^2
\]

\[
\therefore \delta_{BP}^{\text{(stat)}}(M_{\pi T 2}) \sim \frac{1}{J} \delta_{BP}^{\text{(stat)}}(M_{T 2})
\]

However, the error propagation factor \( \sim J \) for getting \( p^0 \),

\[
\delta_{p^0}^{\text{(stat)}}(M_{\pi T 2}) \sim \delta_{p^0}^{\text{(stat)}}(M_{T 2}) : \text{ No advantage for statistical errors.}
\]
Systematic error for BP using Segmented Linear Regression:

\((x_i, y_i) \rightarrow \text{Find the BP with maximal "Coefficient of Explanation"}\)

\[
\delta_{BP}^2 \sim \frac{\sum \epsilon^2}{\Delta a^2} \rightarrow \frac{\sum \epsilon'^2}{J^4 \Delta a^2} \sim \frac{1}{J^4} \delta_{BP}^2
\]

(\(\sum \epsilon'^2 \sim \sum \epsilon^2\), similar square sum of residuals after maximization with elaborated fitting functions)

\[
\therefore \delta_{BP}^{(sys)}(M_{\pi T2}) \sim \frac{1}{J^2} \delta_{BP}^{(sys)}(M_{T2})
\]

Taking into account the error propagation factor,

\[
\delta_{p^0}^{(sys)}(M_{\pi T2}) \sim \frac{1}{J} \delta_{p^0}^{(sys)}(M_{T2}) : O(1/J) \text{ reduction is expected!}
\]
• Shining on buried new particle endpoints (1)
  - 2 signal endpoints from same signature (2lepton+ MET)
  - Measurement of mass differences precisely with small systematic fit errors
  - Example (1) LH or RH slepton pair production $\rightarrow 2l + 2\chi_{10}$
• **Shining on buried new particle endpoints (2)**
  - Amplifying & identifying the correct 2 jet signal endpoint from squark decays to gluino.

  \[ \sim q \sim q \rightarrow j_1 \sim g \ j_2 \sim g \rightarrow j_1 \ j_3 j_5 \chi + j_2 \ j_4 j_6 \chi \]

  \( \text{using } > 6 \text{ jets events.} \)

  - SUSY spectrum

\[
m_{\tilde{q}} = 1036.6 \text{ GeV} \\
m_{\tilde{g}} = 649.4 \text{ GeV} \\
m_{\tilde{\chi}^0_1} = 98.6 \text{ GeV}
\]

\[
\sigma(\tilde{q}\tilde{q}) = 0.5 \text{ pb}, \ \sigma(\tilde{g}\tilde{q}) \sim \sigma(\tilde{g}\tilde{g}) = 3 \text{ pb}
\]

The spectrum is properly separated so that the jets from squark decay and gluino decay are hard to be distinguished by any cuts.

We want to get the mass constraint, \( p^0 \)

\[
p^0 = \frac{m_{\tilde{q}}^2 - m_{\tilde{g}}^2}{2m_{\tilde{q}}}
\]

by construction of subsystem \( M_{\pi T2} \) using \( j_1, j_2 \) with gluino pair as effective missing particles.
Event and jet selection scheme for subsystem $M_{\pi T2}$

1. At least 6 jets with $P_T \geq 30$ GeV
2. No leptons, no b-jets
3. $\delta_T (\equiv |\not{E}_T + \sum_{i=1..6} p_T^i|) \leq 30$ GeV

- No particular 2-jet selection scheme.
- For signal processes, there exist 15 jet-paring combinations.
- Also there exists many background processes with gluino+squark / gluino+gluino production with hard ISR jets / ...
- We just consider all the hardest 6 jets and constructed all possible subsystem $M_{\pi T2}^{(n=1..15)}$ as follows

$$M_{\pi T2}^{(n)}(\chi) = \min_{k_1T + k_2T = e_T^{(n)}} \left[ \max \{ M_{\pi T}^{(n,1)}, M_{\pi T}^{(n,2)} \} \right]$$

$$M_{\pi T}^{(n,i)}(\chi)^2 = \chi^2 + m_i^{(n)2} + 2(e_i^{(n)}e_{\chi_i} + p_i^{(n)'}(\pi) \cdot k_iT)$$

$$e_T^{(n)} \equiv \not{E}_T + q_T^{(n)}.$$
• Histogram of all the subsystem $M_{\pi T2}$ and $M_{T2}$
• Expecting the correct tagged values ($<1/15$) consistently contribute to a slight slope discontinuity in $M_{T2}$
• Then, see the breakpoint enhancement in $M_{\pi T2}$ projection!
- Trial gluino mass = 1.24 p⁰ = 389.7GeV → J = 12.2
- Expected endpoint: \( M_{\pi T2} = 519.5, M_{T2} = 814.8 \text{ GeV} \)
- Bin size in selected as best one among 10 × (1, 2, 2.5) GeV
- Model fitting function: Gaussian smeared step func / G. S. linear functions
- Mean values of measured endpoint & Systematic uncertainty in fitting (varying ranges, widths, while keeping \( \chi^2/n < 2 \).)

\[
M_{\pi T2}^{exp} = 519.4 \pm 0.2 \text{ GeV}, \quad M_{T2}^{exp} = 797 \pm 20 \text{ GeV} \quad \Rightarrow \quad \frac{\delta M_{\pi T2}}{\delta M_{\pi T2}} \sim \frac{1}{J^2}
\]
• Simulation:
PYTHIA(\sim q\sim q, \sim g\sim g, \sim q\sim g \text{ production})(\text{fully showered and hadronized})
→ PGS 4.0
- \Delta E/E = 0.6/E \text{ in hadronic calorimeter}
- Jets were reconstructed using cone algorithm, \Delta R = 0.5
- We ignored the jet invariant masses in constructing \( M_{T2} \) and \( M_{\pi T2} \) (It was effective for reducing the jet energy res. effects in identifying the endpoint at the expected position.)
Conclusion

• $M_{\pi T2}$ distribution has very impressive **endpoint structure enhancement** with respect to varying trial WIMP mass, $x$

• Small slope discontinuities are amplified by $J(x)^2$, enlightening the breakpoint structures clearly

• It might give us a chance to measure the **mass constraints** with reduced systematic uncertainties, even in the case with irreducible heavy jet combinatoric backgrounds.