

Energy flow away from jets

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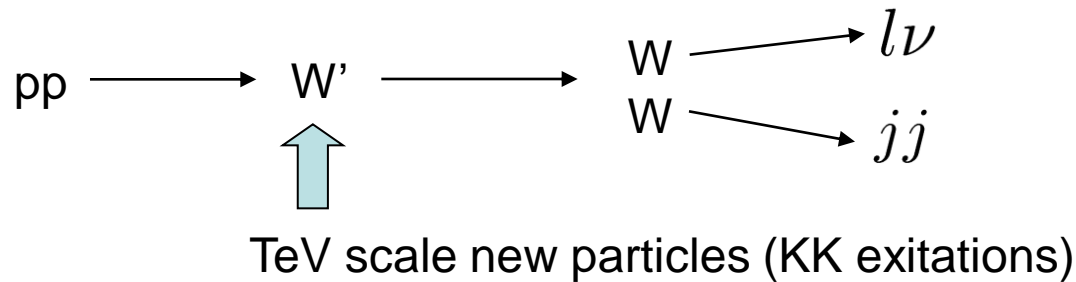
YH [arXiv:0810.0889](https://arxiv.org/abs/0810.0889) (JHEP);
Avsar, YH, Matsuo, [0903.4285](https://arxiv.org/abs/0903.4285) (JHEP);
YH, Ueda, [arXiv:0909.0056](https://arxiv.org/abs/0909.0056) (PRD)

Outline

- Motivation: High pt EW bosons at the LHC
- Sudakov vs. nonglobal logs
- Relation between jet physics and high energy scattering
- BMS equation and its application

High-pt electroweak bosons at the LHC

Highly boosted EW bosons (W,Z) might be important for the discovery of physics beyond the SM. e.g., Agashe et al. (2007)



$$\begin{array}{l} p_t^W \sim 1000 \text{ GeV} \\ m_J \sim 100 \text{ GeV} \end{array} \longrightarrow \text{QCD background from } W+1\text{-jet events}$$

Find a method to distinguish them event-by-event

- Jet substructure Almeida, Lee, Perez, Serman, Sung, Virzi (2008)
Butterworth, Davison, Rubin, Salam (2008) (Higgs)
- Energy flow

Away-from-jets region

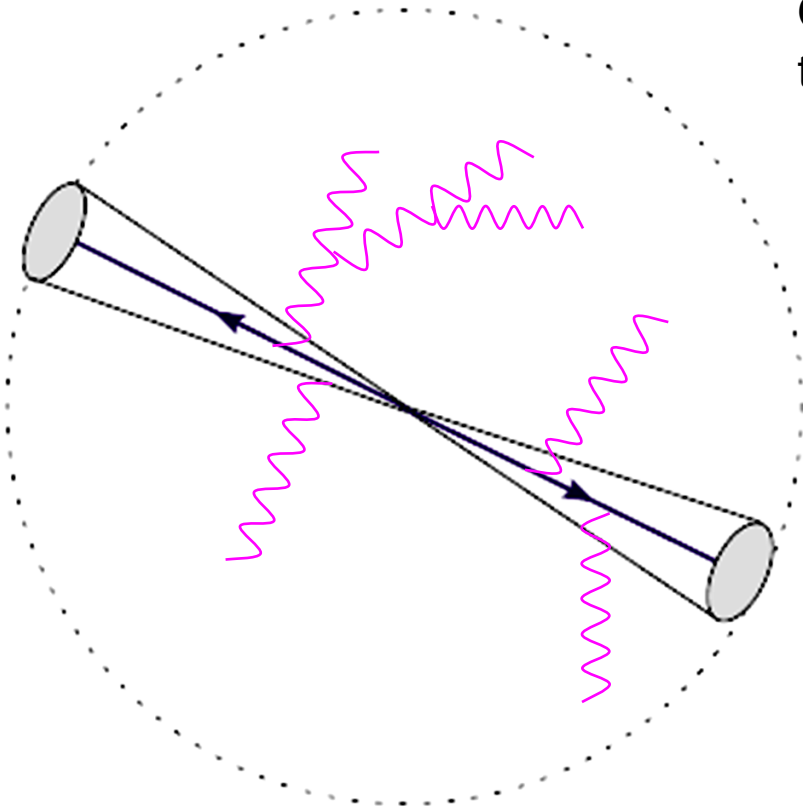
Observables related to the interjet energy flow typically involve two hard scales.

$$p_t \gg E_{out} \gg \Lambda_{QCD}$$

Gluons emitted at large angle,
insensitive to the collinear singularity

Resum only the soft logarithms

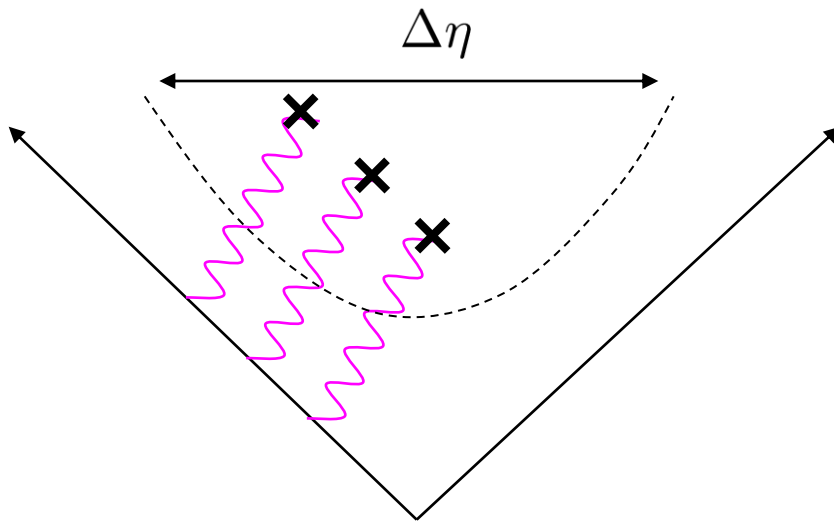
$$(\ln p_t / E_{out})^n$$



There are **two** types of logarithms.

Sudakov vs. non-global logs

Sudakov logarithm e.g., Oderda and Sterman (1998)



Real emission forbidden, $k_i \leq E_{out}$
 virtual emission allowed $k_i \leq p_t$

→ Miscancellation between the real and virtual contributions.

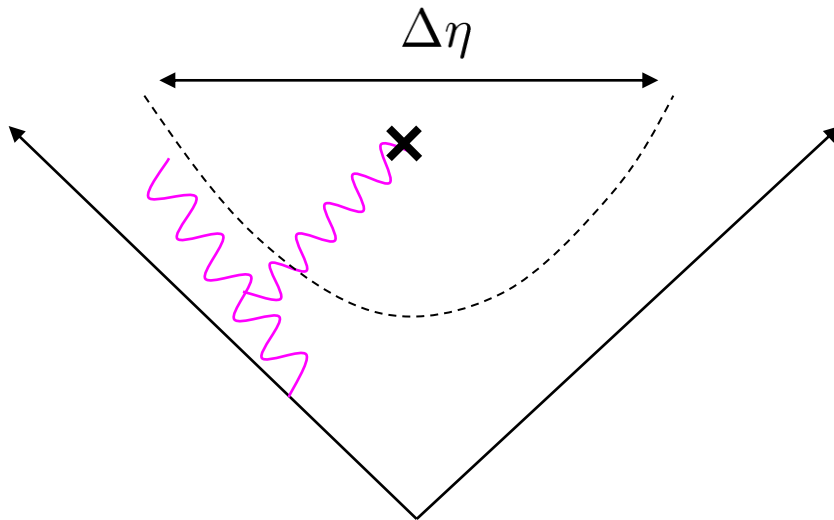
→ large logs $\left(\bar{\alpha}_s \ln \frac{p_t}{E_{out}}\right)^n$

$\xrightarrow{\text{exponentiate}}$

$$P\left(\sum_{i \in \text{gap}} E_i \leq E_{out}\right) \sim \exp\left(-\bar{\alpha}_s \Delta\eta \ln \frac{p_t}{E_{out}}\right)$$

Sudakov vs. non-global logs

Nonglobal logarithm Dasgupta and Salam (2001)



One should also forbid secondary emissions into the interjet region

Parametrically of the same order as the Sudakov logs.
Not easy to resum (does not exponentiate...)

Sensitive to the complicated multi-gluon configuration in the interjet region.

→ Monte Carlo simulation

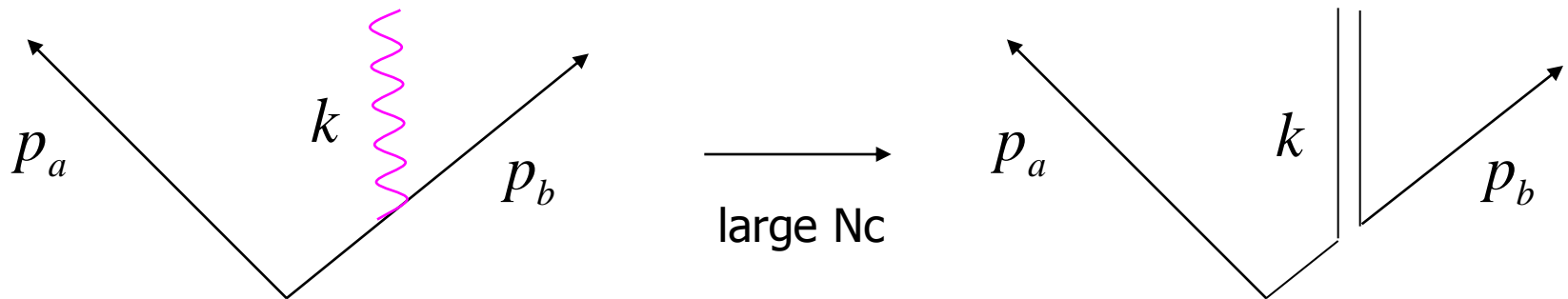
@Large-Nc

Marchesini-Mueller equation

Marchesini, Mueller (2003)

Differential probability for the soft gluon emission

$$dP = \bar{\alpha}_s \omega d\omega \frac{d\Omega_k}{4\pi} \frac{p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} \approx \bar{\alpha}_s \frac{d\omega}{\omega} \frac{d\Omega_k}{4\pi} \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{ak})(1 - \cos \theta_{bk})}$$



Evolution of the **interjet gluon number**. Non-global logs included.

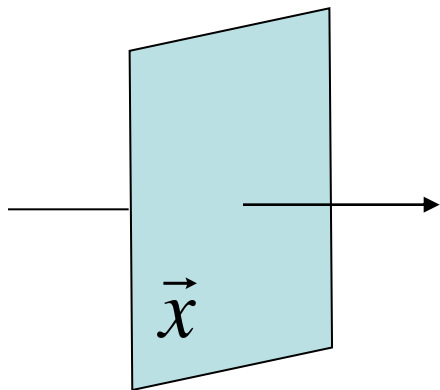
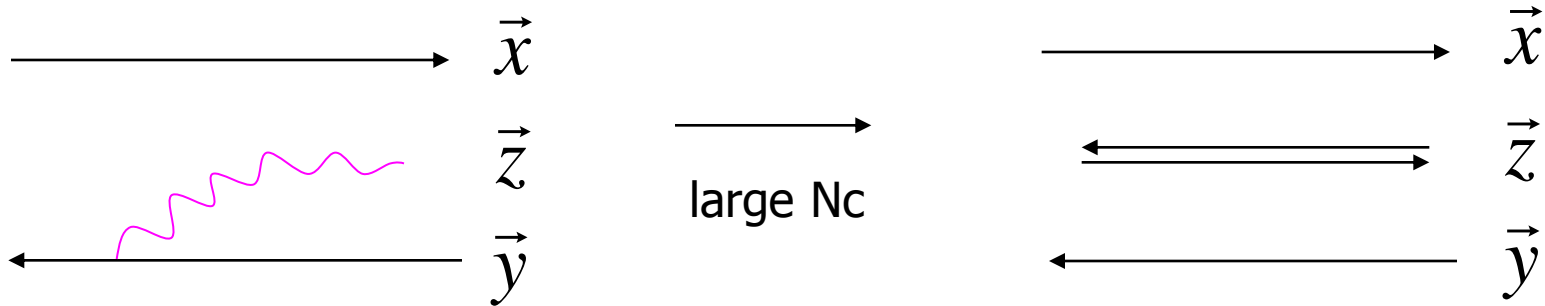
$$\partial_Y n(\theta_{ab}, \theta_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\Omega_k}{4\pi} \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{ak})(1 - \cos \theta_{bk})} \times (n(\theta_{ak}, \theta_{cd}, Y) + n(\theta_{bk}, \theta_{cd}, Y) - n(\theta_{ab}, \theta_{cd}, Y)) .$$

$Y = \ln p_t / E_{out}$
"rapidity"

BFKL equation (dipole version)

Differential probability for the dipole splitting

$$dP = \bar{\alpha}_s \frac{d\omega}{\omega} d^2\vec{z} \frac{(\vec{x} - \vec{y})^2}{(\vec{x} - \vec{z})^2 (\vec{z} - \vec{y})^2}$$



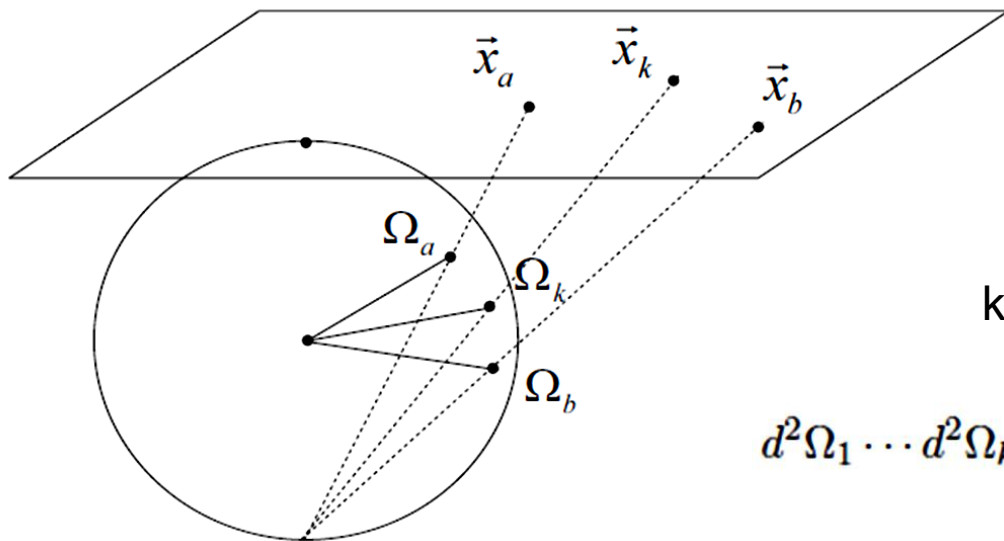
$$\partial_Y n(x_{ab}, x_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2 (\vec{x}_{bk})^2} \times (n(x_{ak}, x_{cd}, Y) + n(x_{bk}, x_{cd}, Y) - n(x_{ab}, x_{cd}, Y))$$

Stereographic projection

YH (2008)

Exact map between the BFKL and the Marchesini-Mueller equations

$$\frac{d^2\Omega_k}{4\pi} \frac{1 - \cos\theta_{ab}}{(1 - \cos\theta_{ak})(1 - \cos\theta_{bk})} = \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$



$$x^1 = \tan \frac{\theta}{2} \cos \phi$$

$$x^2 = \tan \frac{\theta}{2} \sin \phi$$

k-gluon emission probability

$$d^2\Omega_1 \cdots d^2\Omega_k \frac{1 - \cos\theta_{ab}}{(1 - \cos\theta_{a1})(1 - \cos\theta_{12}) \cdots (1 - \cos\theta_{kb})}$$

$$= d^2x_1 \cdots d^2x_k \frac{x_{ab}^2}{x_{a1}^2 x_{12}^2 \cdots x_{kb}^2}$$

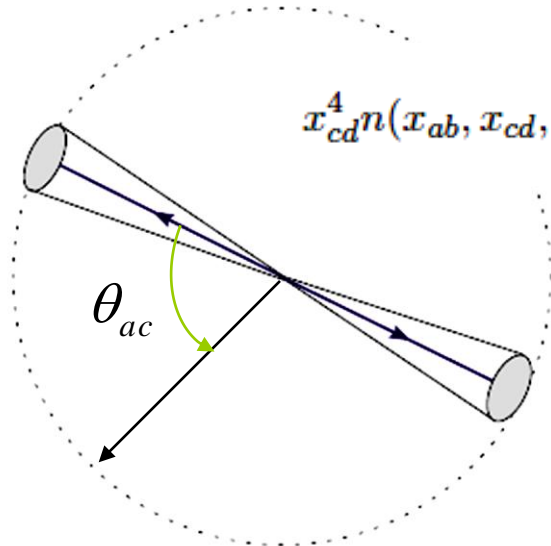
Solution to the Marchesini-Mueller equation

BFKL kernel invariant under the 2D **conformal group** $SL(2,C)$

$$\frac{d^2 \tilde{x}_\gamma}{2\pi} \frac{(\tilde{x}_\alpha - \tilde{x}_\beta)^2}{(\tilde{x}_\alpha - \tilde{x}_\gamma)^2 (\tilde{x}_\gamma - \tilde{x}_\beta)^2}, \quad z = x^1 + ix^2 \rightarrow z' = \frac{\alpha z + \beta}{\gamma z + \delta}$$

Exact solution to the BFKL equation known. Due to conformal symmetry, it is a function only of the **anharmonic ratio**.

$$|\rho|^2 = \frac{x_{ab}^2 x_{cd}^2}{x_{ac}^2 x_{bd}^2} = \frac{(1 - \cos \theta_{ab})(1 - \cos \theta_{cd})}{(1 - \cos \theta_{ac})(1 - \cos \theta_{bd})}$$



$$x_{cd}^4 n(x_{ab}, x_{cd}, Y) \sim \theta_{cd}^4 n(\theta_{ab}, \theta_{cd}, Y) \sim \frac{|\rho|}{(D\bar{\alpha}_s Y)^{3/2}} \ln \left(\frac{16}{|\rho|} \right) e^{4 \ln 2 \bar{\alpha}_s Y} e^{-\frac{2 \ln^2(|\rho|/16)}{D\bar{\alpha}_s Y}}$$

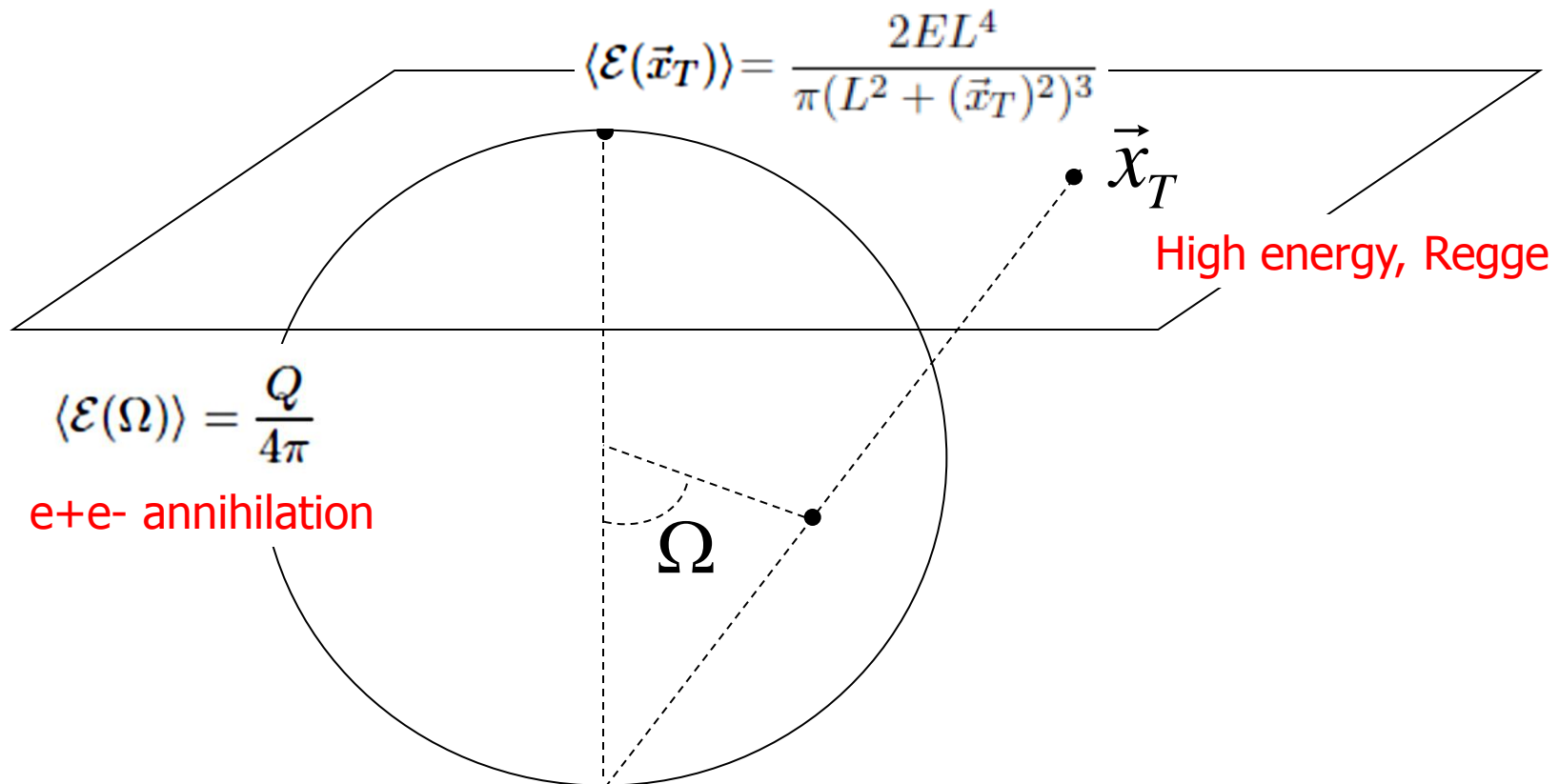
Analytical insight into the distribution and correlation of dipoles (gluons) in the interjet region.

YH (2008);
Avsar, YH, Matsuo (2009)

Exact map at strong coupling

YH (2008)

The **SAME** transformation works in the strong coupling limit of N=4 SYM !



The two processes are mathematically identical. The only difference is the choice of the coordinate system in AdS_5 in which to express its physics content.

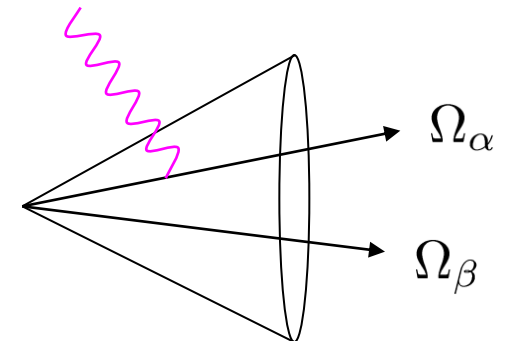
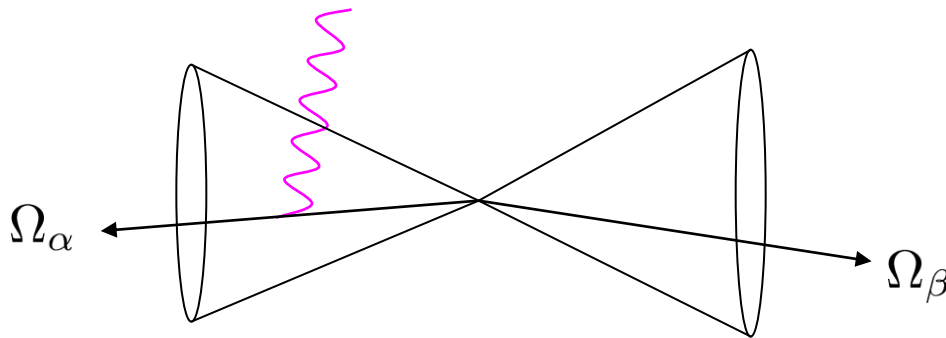
BMS equation

Banfi, Marchesini, Smye (2002)

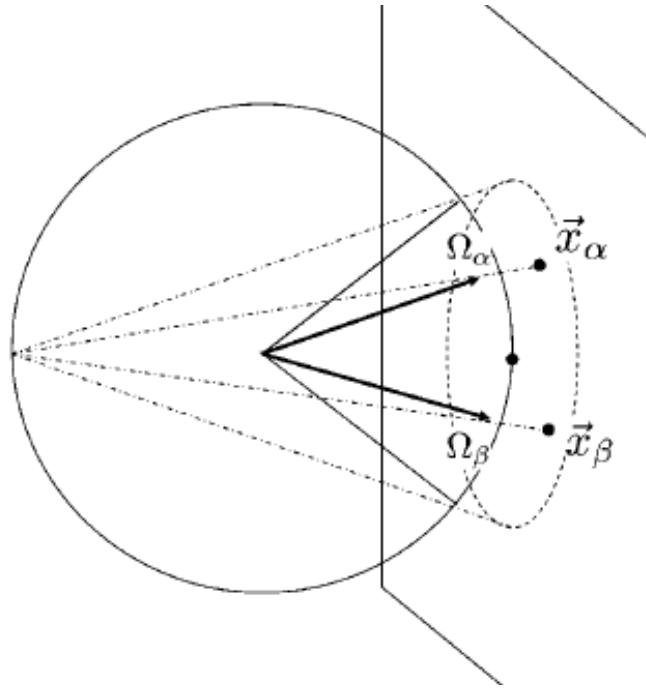
P_τ : Probability that the total energy emitted outside the jet cone(s) is **less** than E_{out}

$$\partial_\tau P_\tau(\Omega_\alpha, \Omega_\beta) = - \int_{\mathcal{C}_{out}} \frac{d^2\Omega_\gamma}{4\pi} \frac{1 - \cos\theta_{\alpha\beta}}{(1 - \cos\theta_{\alpha\gamma})(1 - \cos\theta_{\gamma\beta})} P_\tau(\Omega_\alpha, \Omega_\beta) \\ + \int_{\mathcal{C}_{in}} \frac{d^2\Omega_\gamma}{4\pi} \frac{1 - \cos\theta_{\alpha\beta}}{(1 - \cos\theta_{\alpha\gamma})(1 - \cos\theta_{\gamma\beta})} \left(P_\tau(\Omega_\alpha, \Omega_\gamma) P_\tau(\Omega_\gamma, \Omega_\beta) - P_\tau(\Omega_\alpha, \Omega_\beta) \right)$$

$$\tau = \bar{\alpha}_s \ln \frac{p_t}{E_{out}} \quad (\text{fixed coupling})$$



Mapping the BMS equation onto a plane



$$\partial_\tau P_\tau(\vec{x}_\alpha, \vec{x}_\beta) = -f_{\alpha\beta} P_\tau(\vec{x}_\alpha, \vec{x}_\beta) + \int_{|\vec{x}_\gamma| < r_{in}} \frac{d^2 \vec{x}_\gamma}{2\pi} \frac{(\vec{x}_\alpha - \vec{x}_\beta)^2}{(\vec{x}_\alpha - \vec{x}_\gamma)^2 (\vec{x}_\gamma - \vec{x}_\beta)^2} (P_\tau(\vec{x}_\alpha, \vec{x}_\gamma) P_\tau(\vec{x}_\gamma, \vec{x}_\beta) - P_\tau(\vec{x}_\alpha, \vec{x}_\beta))$$

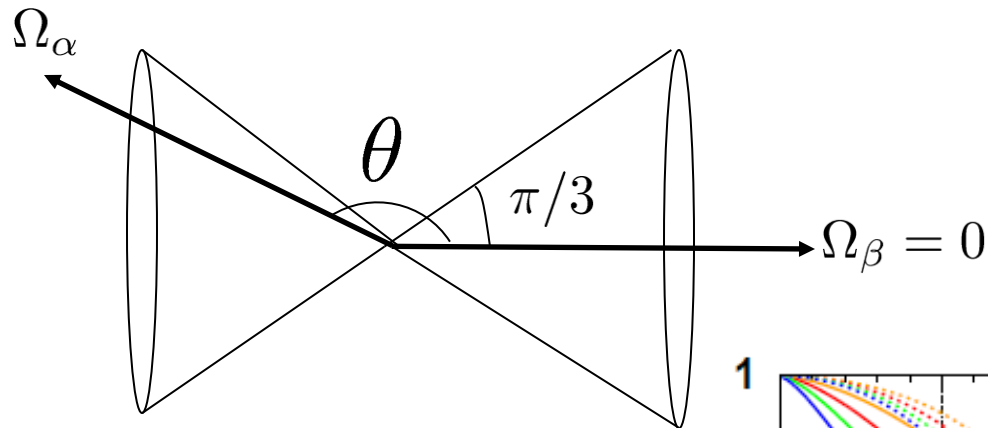
Identical to the **Balitsky-Kovchegov equation** of saturation physics.

SL(2,C) broken down to the subgroup SU(1,1) \rightarrow **Poincare disk**.

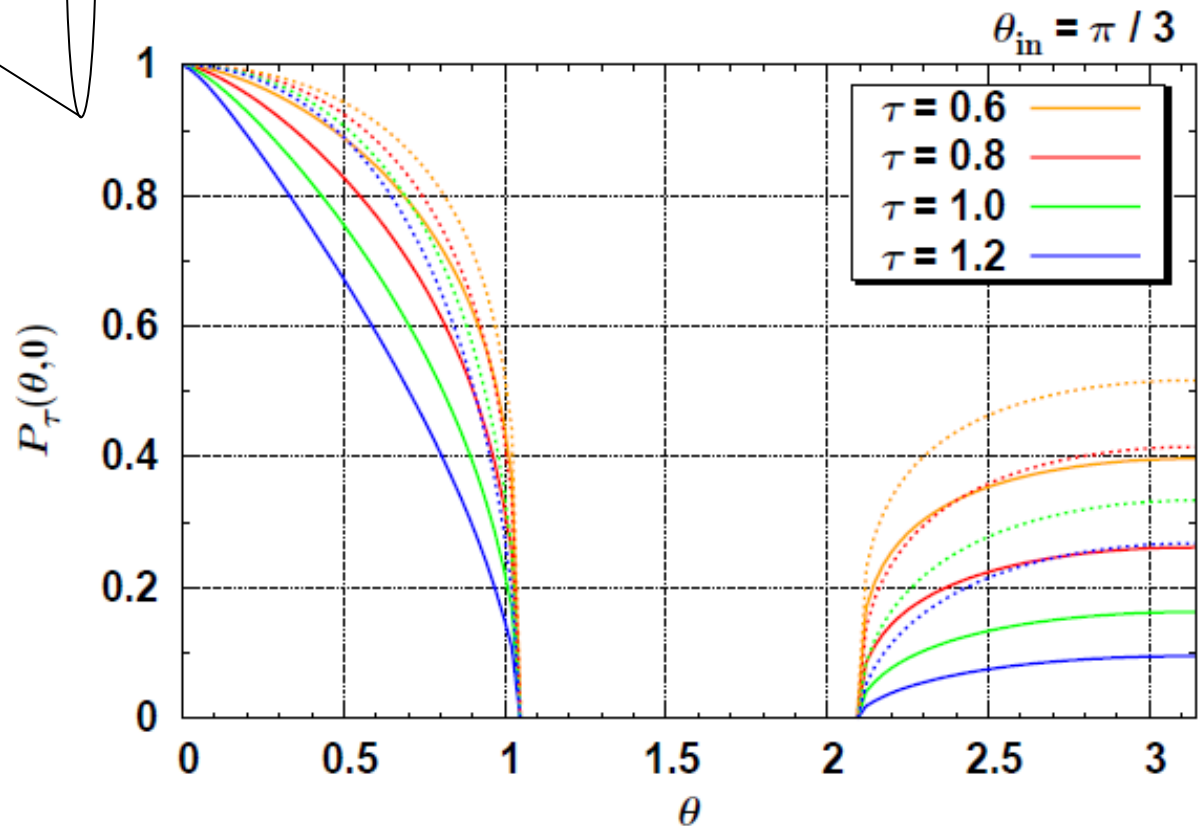
$P_\tau(\Omega_\alpha, \Omega_\beta)$ depends only of the **chordal distance**

$$d^2(\vec{x}_\alpha, \vec{x}_\beta) = \frac{(\vec{x}_\alpha - \vec{x}_\beta)^2}{(1 - \vec{x}_\alpha^2)(1 - \vec{x}_\beta^2)} = \frac{\sin^2 \theta_{in} (1 - \cos \theta_{\alpha\beta})}{2(\cos \theta_\alpha - \cos \theta_{in})(\cos \theta_\beta - \cos \theta_{in})}$$

Numerical results

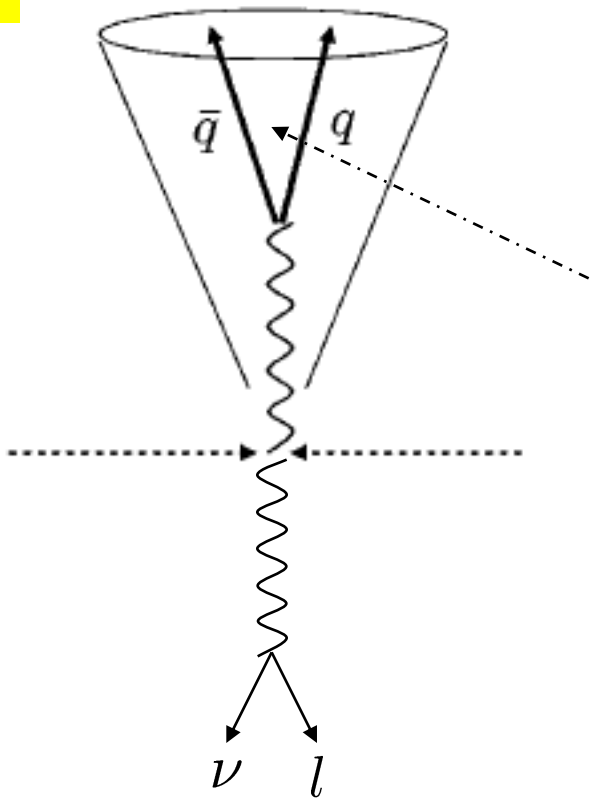


Two-cone case



Application: boosted W boson at the LHC

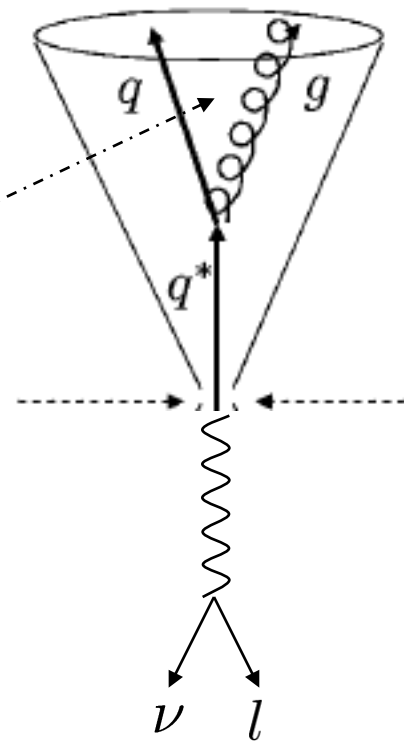
Signal



Very little radiation
(typically $\ll \alpha_s p_t$)
due to the **QCD coherence**

Background

$$\theta_{\alpha\beta} \gtrsim 2 \frac{M}{p_t}$$



Sizable radiation $\lesssim \alpha_s p_t$ expected

Weak boson jet

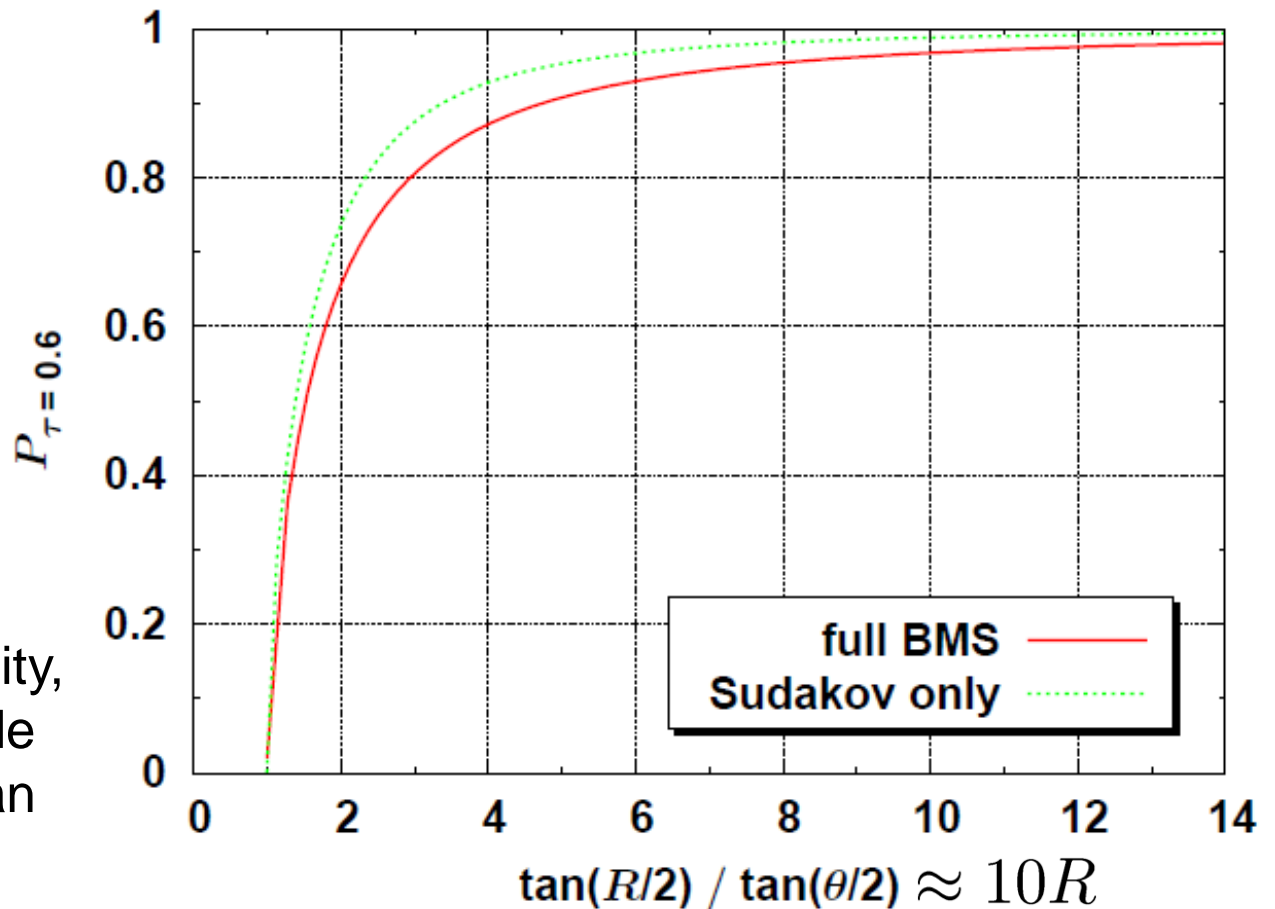
Take $p_t \sim 1 \text{ TeV}$
 $m_J \sim 100 \text{ GeV}$

and $\tau = 0.6$

corresponding to

$$E_{out} = 10 \text{ GeV}$$

(running coupling)



With 80~90% probability,
energy radiated outside
the jet cone is less than
10 GeV (only 1%)

QCD jet

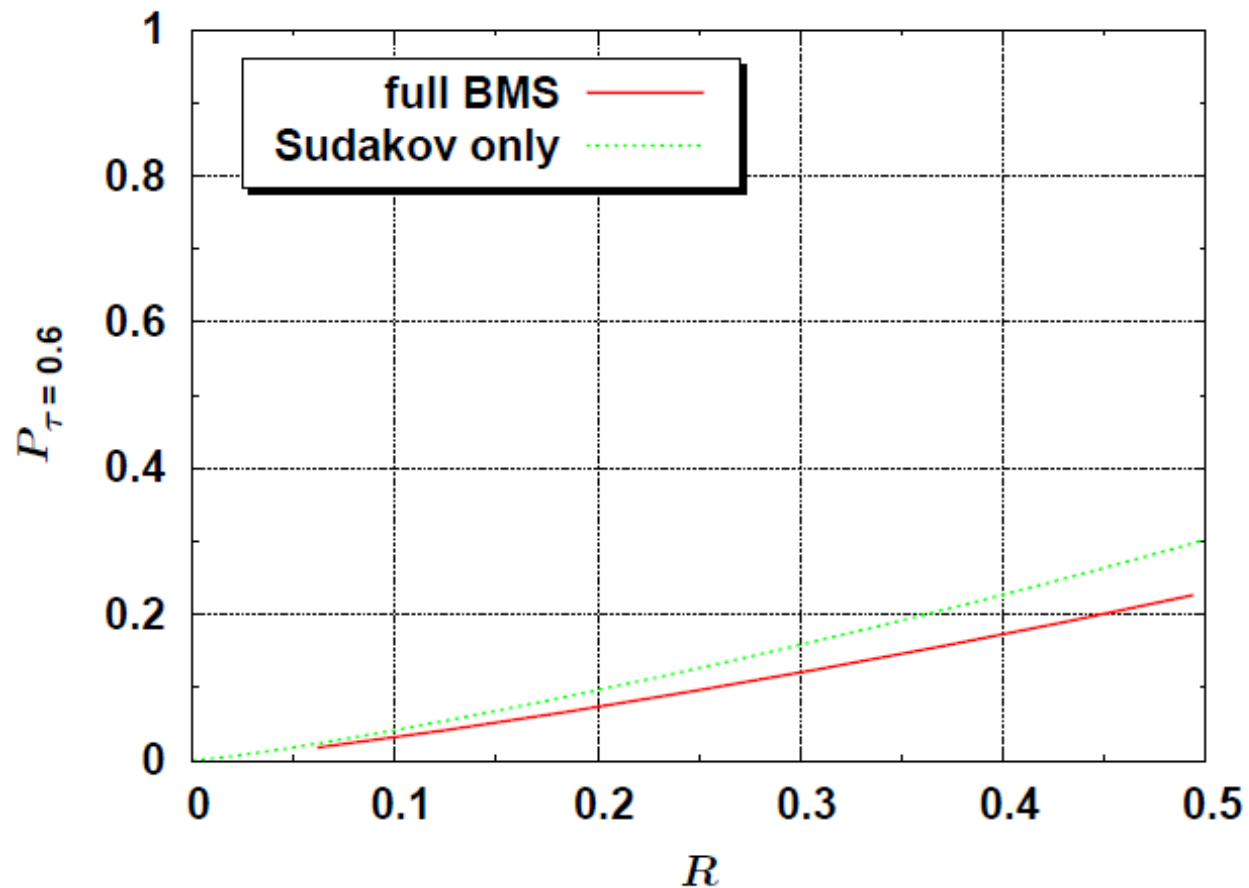
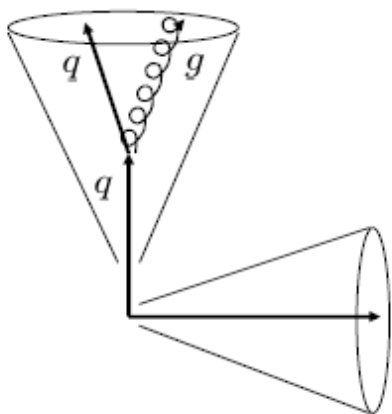
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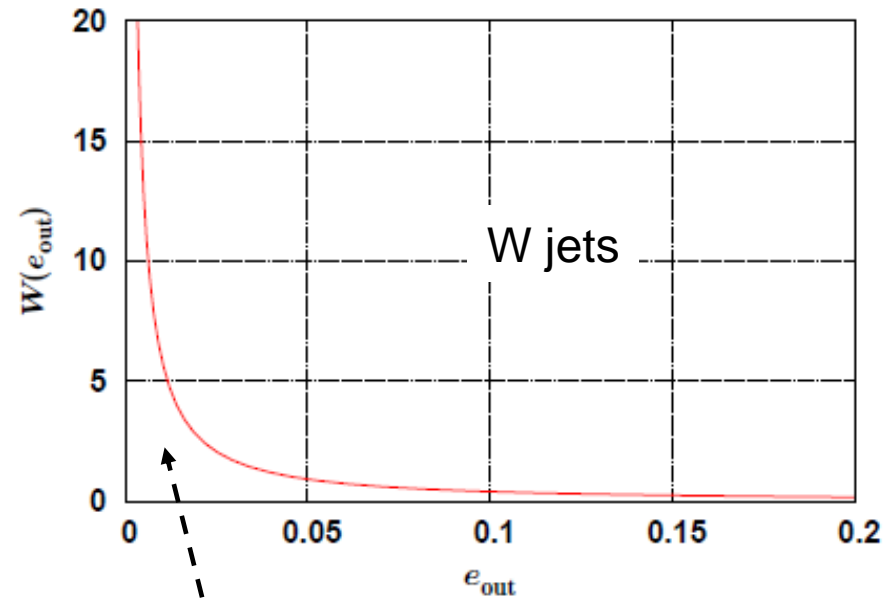
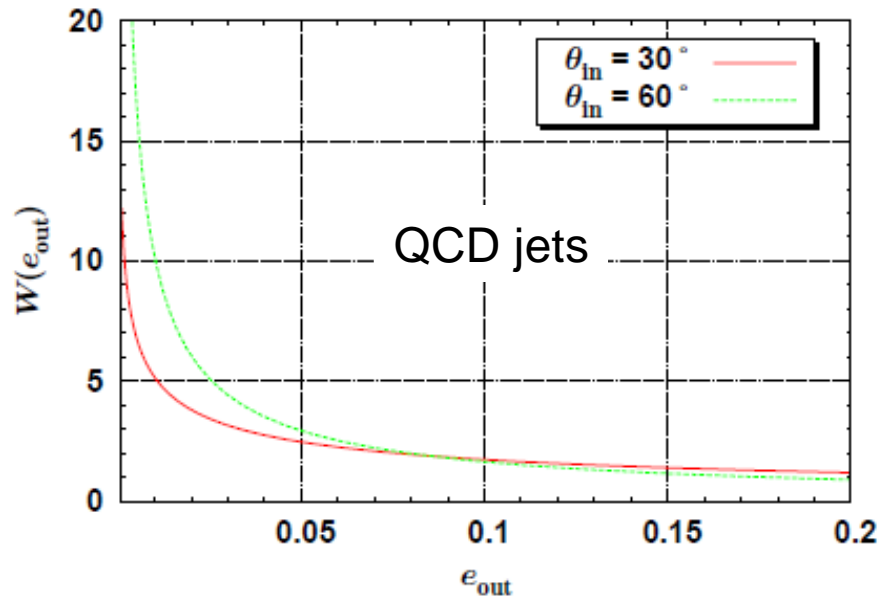
$$E_{out} = 10 \text{ GeV}$$

(running coupling)



Probability of having a definite value of $e_{out} = \frac{1}{p_t} \sum_{i \in \mathcal{C}_{out}} E_i$

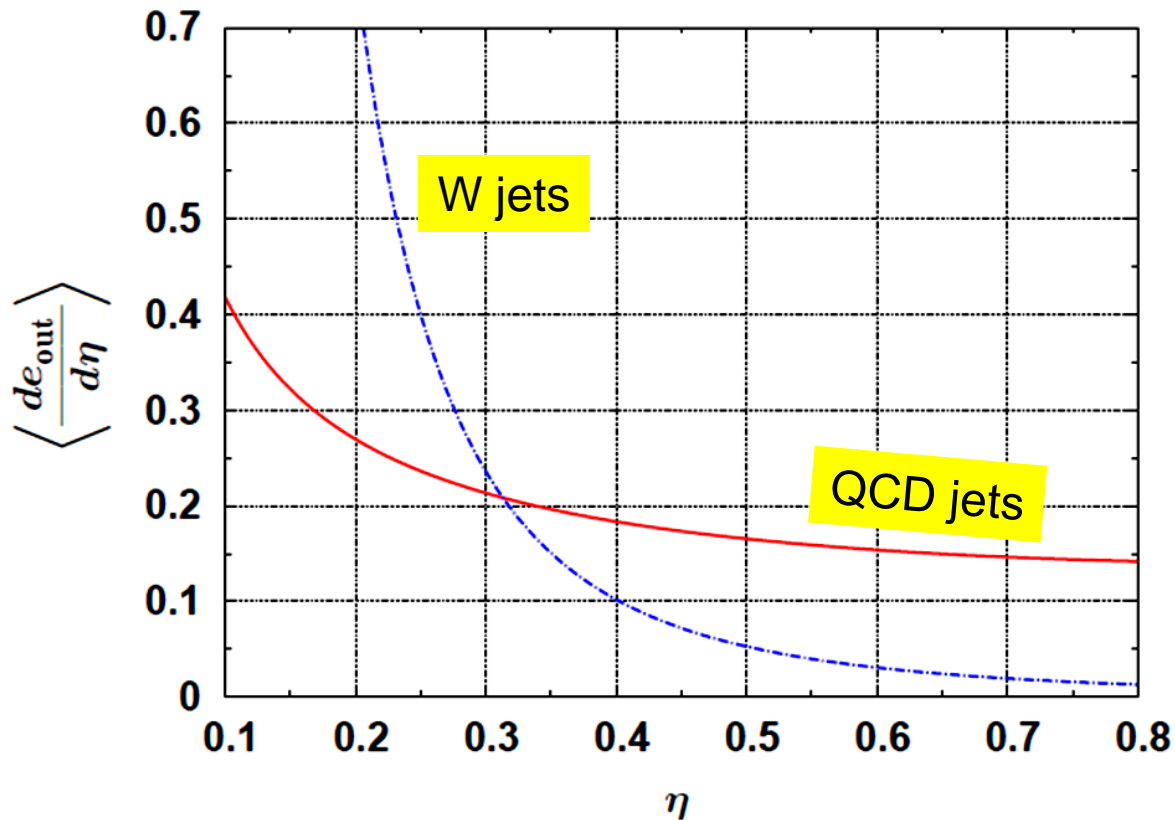
$$W(e_{out}) = \frac{\partial}{\partial e_{out}} P_\tau$$



Typically much less than $\frac{\langle E_{out} \rangle}{p_t} \sim \alpha_s \gtrsim 0.1$

Rapidity distribution of energy

“Average energy” $\langle e_{\text{out}} \rangle = \int_0^1 de_{\text{out}} W(e_{\text{out}}) e_{\text{out}}$



Warning: This is only a rough guide. The average energy cannot be reliably computed in the soft approximation.

Conclusions

- The large angle soft emission is deeply related to BFKL. Many analytical results obtainable.
- Fully included in Ariadne, partially in Herwig & Pythia.
[Banfi, Corcella, Dasgupta \(2007\)](#)
- Pattern and amount of energy flow different between EW and QCD jets.
- Take into account the initial state soft radiation and the underlying events. Cross-check with other proposals.

[Almeida et al. \(2008\)](#)