NLO QCD Predictions for $W + 3$ Jet Production at Hadron Colliders

Lance Dixon (SLAC) for the BlackHat collaboration

+ T. Gleisberg → 0902.2760, 0907.1984

IPMU Focus Week on Jet Physics
November 11, 2009
Outline

• Motivation
• $W + 3$ jets production at hadron colliders at NLO in QCD
• Lessons about choice of scales
• Strong and stable $W$ polarization effects
• Preliminary [leading-color + $N_f$] NLO $Z + 3$ jets
• Conclusions
The Energy Frontier Is at Hadron Colliders

Tevatron Run II: 2001 → 2011?

LHC: 2009 → ???

LHC at CERN

CMS

ATLAS

4.3 km
Tevatron & LHC Are QCD Machines

LHC is a multi-jet environment

Need precise understanding of “old physics” that looks like new physics

← new physics?
Signals and Backgrounds

- **New** particles – whether from
  - supersymmetry
  - extra dimensions
  - new forces
  - Higgs boson(s)

  Typically decay into **old** particles:
  - quarks, gluons, charged leptons, neutrinos, photons,
  - Ws & Zs (which in turn decay to leptons, …)

- Kinematic signatures **not always clean** (e.g. mass bumps)
  if neutrinos, or other escaping particles present

- **Need precise Standard Model backgrounds** for a
  variety of **multi**-particle – and especially multi-jet – processes,
  to maximize potential for **new physics discoveries**
QCD Factorization & Parton Model

Collins, Soper, Sterman 1985

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- Sampled “one at a time” in hard collisions.
  → QCD-improved parton model

Partonic cross section, computable in perturbative QCD

\[ \sigma^{pp \rightarrow X}(s; \alpha_s, \mu_R, \mu_F) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \ f_a(x_1, \alpha_s, \mu_F) f_b(x_2, \alpha_s, \mu_F) \]

\[ \times \ \hat{\sigma}^{ab \rightarrow X}(s x_1 x_2; \alpha_s, \mu_R, \mu_F) \]

Parton distribution functions – known to ~ 5% for x ~ 0.01-0.1

factorization scale

renormalization scale

infrared safe final state

QCD-improved parton model
Partonic Cross Section in Perturbation Theory

\[ \hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[ \hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right] \]

LO  NLO  NNLO

**Problem:** Leading-order, tree-level predictions only qualitative due to poor convergence of expansion in \( \alpha_s(\mu) \)

(setting \( \mu_R = \mu_F = \mu \) )
Lack of One-Loop Amplitudes

At NLO, the bottleneck for more complex processes is the lack of availability of one-loop amplitudes.

\[ \sigma(n \text{ jets}) = [\alpha_s(\mu)]^n \{ A + \alpha_s(\mu) B + \alpha_s^2(\mu) C + \cdots \} \]

state of the art:

NLO = loop x tree* + …

n=3
Strong growth in difficulty at one loop (NLO) with number of final-state objects

<table>
<thead>
<tr>
<th># of jets</th>
<th>1-loop Feynman diagrams (gluons only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image" alt="Diagram" /> 810</td>
</tr>
<tr>
<td>4</td>
<td><img src="image" alt="Diagram" /> 10,860</td>
</tr>
<tr>
<td>5</td>
<td><img src="image" alt="Diagram" /> 168,925</td>
</tr>
<tr>
<td>6</td>
<td><img src="image" alt="Diagram" /> 3,017,490</td>
</tr>
</tbody>
</table>
Background to Search for Supersymmetry

- Cascade from gluino to neutralino (dark matter, escapes detector)
- Signal: missing energy + 4 jets
- SM background from $Z + 4$ jets, $Z \rightarrow$ neutrinos

Current state of art for $Z + 4$ jets:
ALPGEN, based on LO tree amplitudes → normalization still quite uncertain

- Motivates goal of

$$pp \rightarrow Z + 4 \text{ jets at NLO}$$

2 legs beyond state-of-art
Tevatron

W + n jets Data

CDF, 0711.4044 [hep-ex]

NLO (MCFM)

LO matched to parton shower MC with different schemes

<table>
<thead>
<tr>
<th>number of jets</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16%</td>
<td>7%</td>
</tr>
<tr>
<td>2</td>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>42%</td>
<td>11%</td>
</tr>
</tbody>
</table>

L. Dixon  NLO W + 3 Jets

% uncertainty

n = 1

n = 2

n = 3
only LO available – until this year
A Better Way to Compute?

- Backgrounds (and many signals) require detailed understanding of scattering amplitudes for many ultra-relativistic ("massless") particles – especially quarks and gluons of QCD.

- Feynman diagrams can be used – in principle.

- However, Feynman diagrams, while very general and powerful, are not optimized for these processes.
- On-shell methods, exploiting analyticity, can be more efficient, especially for multi-gluon + quark processes!
- We have implemented these methods numerically in a C++ program, BlackHat.
One-Loop Amplitude Decomposition

When all external momenta are in $D=4$, loop momenta in $D=4-2\epsilon$ (dimensional regularization), one can write: BDDK (1994)

coefficients are all rational functions – determine algebraically from products of trees using (generalized) unitarity

$$A^{1\text{-loop}} = \sum_i d_i \ + \ \sum_i c_i \ + \ \sum_i b_i$$

known scalar one-loop integrals, same for all amplitudes

rational part
Inside BlackHat
Several Related Implementations

**CutTools:**
- NLO \(W + 3\) jets
  - Ossola, Papadopolous, Pittau, 0711.3596
  - Binoth+OPP, 0804.0350
  - Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723

**Rocket:**
- One-loop \(n\)-gluon amplitudes for \(n\) up to 20;
  - \(W + 3\) jets amplitudes
  - Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
  - NLO \(W + 3\) jets in leading-color (large \(N_c\)) approximation
    - Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
    - Melnikov, Zanderighi, 0910.3671

**Blackhat:**
- Berger, Bern, LD, Febres Cordero, Forde, H. Ita, D. Kosower, D. Maître, 0803.4180, 0808.0941
- One-loop \(n\)-gluon amplitudes for \(n\) up to 7,…;
  - amplitudes needed for NLO production of \(W, Z + 3\) jets

D-dim’l unitarity
D-dim’l unitarity + on-shell recursion
• Background to SUSY searches in the “Jets + MET” channel, when the charged lepton in $W \rightarrow l \nu$ is lost
• Also closely related to $Z + 3$ jets, another SUSY background when $Z \rightarrow \nu \nu$
• Similar to top-quark pair production in semi-leptonic $W$ decay channel, $t \bar{t} \rightarrow l \nu + 4$ jets
• Many different kinematic configurations can appear in final state – have to be careful to choose scale $\mu$ correctly to avoid pathologies!
Color Sampling for Virtual Corrections

- Divide into **leading-color** terms, such as:

\[
\begin{align*}
\text{leading-color terms:} & \\
\text{subleading-color terms:} & 
\end{align*}
\]

The latter include **many more terms**, and are much more time-consuming for computer to evaluate. But they are much **smaller** (~ 1/30 of total cross section) so we evaluate them much less often.
Numerical Stability of Virtual Terms

• Nontrivial because there are many kinematic regions where there are large cancellations between terms in this expansion, leading to roundoff error:

\[ A^{1\text{-loop}} = \sum_i d_i + \sum_i c_i + \sum_i b_i + R + \mathcal{O}(\epsilon) \]

• BlackHat has a lot of tests for instability; if a piece of \( A^{1\text{-loop}} \) is unstable, it recomputes that piece with higher precision (~32 digits).

• Resulting distributions of \( \log(\text{relative error}) \) →

Real radiation handled using automated Catani-Seymour dipole subtraction

Gleisberg, Krauss 0709.2881
$W + 3$ jets at Tevatron at NLO

- Same cuts as CDF:
  - $E_T^{\text{jet}} > 20$ GeV, $|\eta^{\text{jet}}| < 2$
  - $E_T^e > 20$ GeV, $|\eta^e| < 1.1$
  - $E_T > 30$ GeV, $M_T^W > 20$ GeV
  - $M_T^W = \sqrt{2E_T^e E_T^\nu (1 - \cos(\Delta \phi_{e\nu}))}$

- Except: we use SISCone; CDF used IR unsafe JETCLU

- Much smaller uncertainties than at LO.
- Agrees well with data; more data coming soon!
$W + n \text{ jets cuts at } \text{LHC}$

$\sqrt{s} = 14 \text{ TeV}$

$|\eta^{\text{jet}}| < 3$, \hspace{1cm} $R = 0.4$, \hspace{1cm} $|\eta^{e}| < 2.5$, \hspace{1cm} $E_T^{e} > 20 \text{ GeV}$

$E_T^{\nu} > 30 \text{ GeV}$, \hspace{1cm} $M_T^{W} > 20 \text{ GeV}$.

$E_T^{\text{jet}} > 30 \text{ GeV}$ \hspace{1cm} or \hspace{1cm} $E_T^{\text{jet}} > 40 \text{ GeV}$

SISConc, $f = 0.5$ \hspace{1cm} or \hspace{1cm} $k_T$
Better Scale Choices

What’s going on?
Consider these 2 configurations:

- If (a) dominates, then
  \[ \mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)} \]
  is OK

- But if (b) dominates, then the scale \( E_T^W \) is way too low.

- Looking at large \( E_T \) for the 2\(^{nd}\) jet forces configuration (b).

- The total (partonic) transverse energy is a **better variable**; gets large properly for both (a) and (b)

- Another reasonable scale is in**variant mass** of the \( n \) jets

Bauer, Lange
0905.4739
Compare the Two Scale Choices

\[ \mu = E_{T}^{W} \]  
very poor

\[ \mu = \hat{H}_{T} \]  
elegant!

– LO/NLO quite flat, and also for many other observables
Berends observed that
\[ \frac{\sigma_{n+2 \text{ jets}}}{\sigma_{n+1 \text{ jets}}} \approx \frac{\sigma_{n+1 \text{ jets}}}{\sigma_n \text{ jets}} \]

We can compute
\[ r_{B,1} \equiv \frac{\sigma_3 \text{ jets} \sigma_1 \text{ jet}}{\sigma_2 \text{ jets}} \]
at LO, NLO

For $W + n$ jets, and for SISCones and kT jet algorithms

<table>
<thead>
<tr>
<th>$r_{B,1}$</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T^{\text{jet}} &gt; 30 \text{ GeV}$ SISCones</td>
<td>0.788</td>
<td>0.841</td>
</tr>
<tr>
<td>$E_T^{\text{jet}} &gt; 40 \text{ GeV}$ SISCones</td>
<td>0.713</td>
<td>0.805</td>
</tr>
<tr>
<td>$E_T^{\text{jet}} &gt; 30 \text{ GeV}$ kT</td>
<td>0.858</td>
<td>0.910</td>
</tr>
<tr>
<td>$E_T^{\text{jet}} &gt; 40 \text{ GeV}$ kT</td>
<td>0.787</td>
<td>0.873</td>
</tr>
</tbody>
</table>
Total Transverse Energy $H_T$ at LHC

$$H_T = \sum_j E_{T,j}^{\text{jet}} + E_T^e + E_T^{\nu}$$

often used in supersymmetry searches

$W^- + 3 \text{ jets} + X$

$\sqrt{s} = 14 \text{ TeV}$

$\mu_\kappa = \mu_\tau = \hat{H}_T$

$E_T^{\mu} > 30 \text{ GeV}, \ |\eta^{\mu}| < 3$

$E_T^e > 20 \text{ GeV}, \ |\eta^e| < 2.5$

$E_T > 30 \text{ GeV}, \ M_T^W > 20 \text{ GeV}$

$R = 0.4$ [sismecon]

BlackHat+Sherpa

flat LO/NLO ratio due to good choice of scale $\mu = H_T$
Jet Separations \[ \Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} \]

Pretty well modeled by LO, except for first 2 jets
Lepton Rapidity in $W + 3$ jets at LHC

Rapidity distributions “remember” that $u(x)/d(x)$ gets very large as $x \to 1$
$W^+ / W^-$ transverse lepton ratios trace a remarkably large and stable left-handed $W$ polarization at large $p_T(W)$ – independent of number of jets – will be useful to separate $W + n$ jets from top, maybe also from new physics.
Transverse spin can be confusing

(stolen from recent talk by W. Vogelsang)
Origin of $W$ polarization in LO $W + 1$ jet

SU(2)$_L$ + valence quark dominance

$A_{\text{tree}} \propto \frac{\langle d \nu \rangle^2}{\langle u g \rangle \langle g d \rangle}$

$d\sigma \propto (k_d \cdot k_\nu)^2$

$A_{\text{tree}} \propto \frac{[u e]^2}{[u g][g d]}$

$d\sigma \propto (k_u \cdot k_e)^2$
$W^{+/−} + n \text{ jets: } e^+/e^− \ E_\text{T} \text{ ratio}$

- LO $\rightarrow$ NLO hardly affects ratios

L. Dixon           NLO W + 3 Jets
IPMU         Nov. 11, 2009
$W^{+/−} + n \text{ jets: Neutrino } E_T$
Actual $W$ polarization – LO $W + 2$ jets
Top quark pairs very different

Main production channels are $C$ invariant:

$$g\bar{g} \rightarrow t\bar{t} \hspace{1cm} q\bar{q} \rightarrow t\bar{t}$$

Semi-leptonic decay involves (partially) left-handed $W^+$

$$t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow be^+\nu \bar{b}jj$$

But charge conjugate decay involves (same degree) right-handed $W^-$

$$t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow bjj\bar{b}e^-\bar{\nu}$$

$\rightarrow$ electron and positron have almost identical $p_T$ distributions

$\rightarrow$ A nice handle on separating $W + \text{jets}$ from top

**Supersymmetry** may be like top – or not – depends on $qg \rightarrow \tilde{q}\tilde{g}$
First NLO $Z + 3$ jets Results

K-factor at $\mu = M_V$ is 20% larger than in $W + 3$ jets, but this was for $E_T > 20$ GeV, and SISConE with $f = 0.5$, $R = 0.4$
Algorithm Dependence of $Z + n$ jets

<table>
<thead>
<tr>
<th># of jets</th>
<th>LO parton</th>
<th>NLO parton</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4205.67(1.91)^{+800.69}_{-615.94}$</td>
<td>$6062.55(7.85)^{+495.80}_{-462.51}$</td>
</tr>
<tr>
<td>2</td>
<td>$422.22(0.33)^{+168.22}_{-108.99}$</td>
<td>$575.17(1.30)^{+71.22}_{-75.88}$</td>
</tr>
<tr>
<td>3</td>
<td>$28.66(0.03)^{+17.87}_{-9.97}$</td>
<td>$39.46(0.19)^{+8.29}_{-8.32}$</td>
</tr>
</tbody>
</table>

Berends ratio:

- SISConed $f = 0.75$
- Anti-kT $R = 0.7$
- kT $N_f$

Preliminary leading-color $+ N_f$
Conclusions

- **New and efficient** computational approaches to one-loop QCD amplitudes needed for important Tevatron and LHC backgrounds:
  - exploit **analyticity**: build loop amplitudes up out of trees
  - implemented numerically in C++ program **BlackHat**, as well as **CutTools** and **Rocket**
- NLO $W + 3$ jets agrees well with Tevatron data
- LHC kinematics and pp initial state $\Rightarrow$ different effects
- Valuable lessons already learned about scales and $W$ polarization
- Preliminary [leading-color + $N_f$] NLO $Z + 3$ jets results too
- $W/Z + 4$ jets also now feasible
- Other groups have produced NLO results for several other processes using similar methods ($V V V$, $t \bar{t} b \bar{b}$, …)
- Will aid in optimal exploitation of LHC data!
Extra slides
Infrared safety

Cones tricky to get right. Seeds can cause problems.
• JETCLU (CDF) + D0 cone algorithms were IR unsafe for NLO W + 2 jets
• Midpoint OK for W + 2 jets, but (probably) fails for W + 3 jets

Figure 1: Configuration illustrating one of the IR unsafety problems of the midpoint jet algorithm ($R = 1$); (a) the stable cones (ellipses) found in the midpoint algorithm; (b) with the addition of an arbitrarily soft seed particle (red wavy line) an extra stable cone is found.

SIScone is a practical (fast enough) seedless cone algorithm that avoids these problems

Salam, Soyez