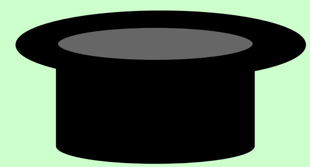
NLO QCD Predictions for *W* + 3 Jet Production at Hadron Colliders

Lance Dixon (SLAC)
for the **BlackHat** collaboration



C.F.Berger, Z. Bern, LD, D. Forde, F. Febres Cordero,
H. Ita, D. Kosower, D. Maître, 0803.4180
+ T. Gleisberg → 0902.2760, 0907.1984

IPMU Focus Week on Jet Physics
November 11, 2009

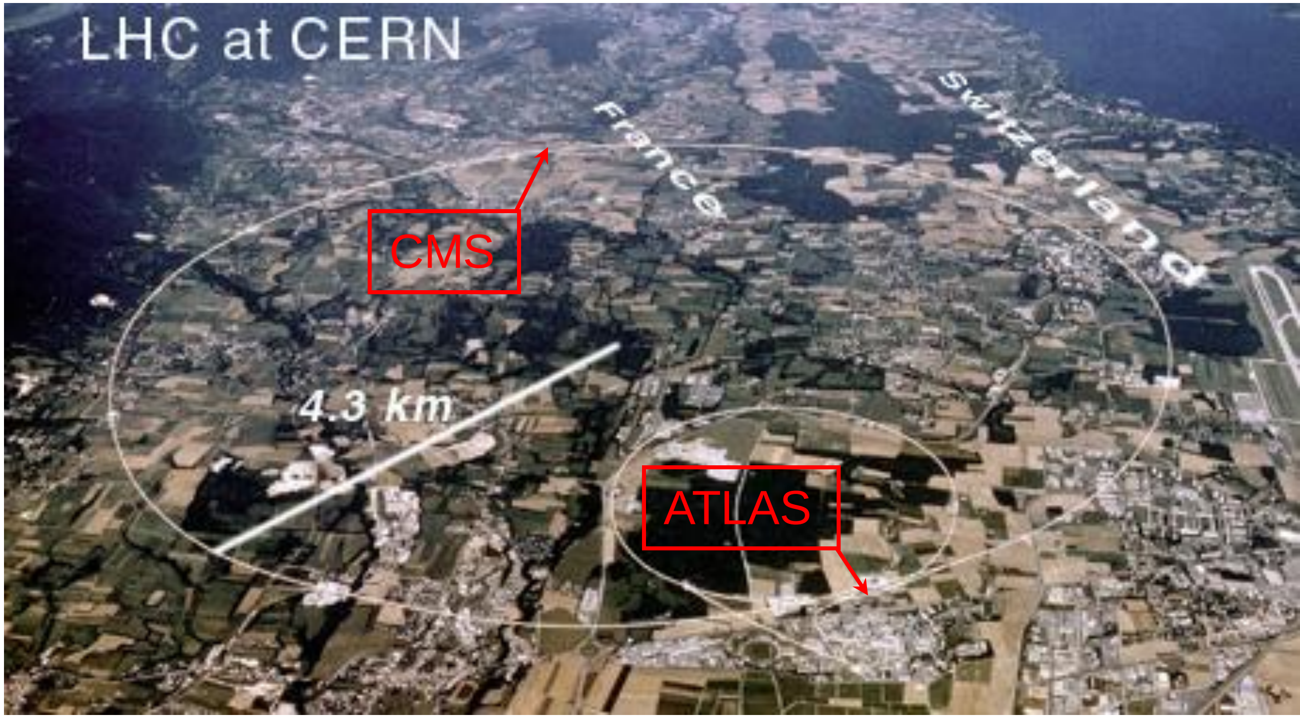
Outline

- Motivation
- *W* + 3 jets production at hadron colliders at NLO in QCD
- Lessons about choice of scales
- Strong and stable *W* polarization effects
- Preliminary [leading-color + N_f] NLO *Z* + 3 jets
- Conclusions

The Energy Frontier Is at Hadron Colliders

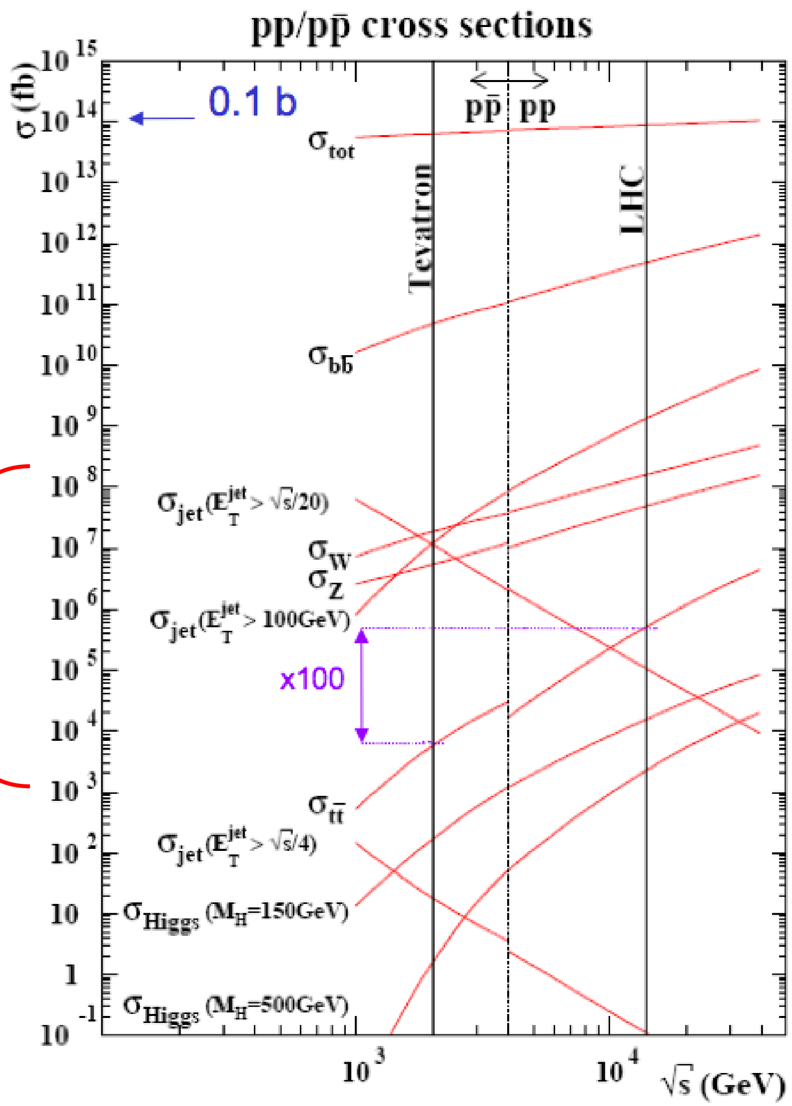
Tevatron Run II:
2001 → 2011?

LHC:
2009 → ???



Tevatron & LHC Are QCD Machines

Need precise understanding of “old physics” that looks like new physics

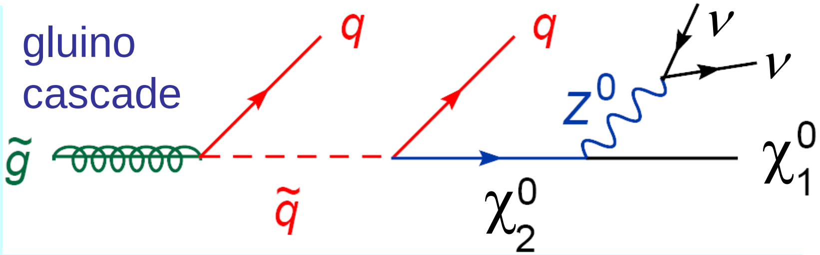


LHC is a multi-jet environment

← new physics?

Signals and Backgrounds

- New particles – whether from
  - supersymmetry
  - extra dimensions
  - new forces
  - Higgs boson(s)



typically decay into old particles:

quarks, gluons, charged leptons, neutrinos, photons, *W*s & *Z*s (which in turn decay to leptons, …)

- Kinematic signatures not always clean (*e.g.* mass bumps) if neutrinos, or other escaping particles present

- Need **precise Standard Model backgrounds** for a variety of multi-particle – and especially multi-jet –processes, to maximize potential for new physics discoveries

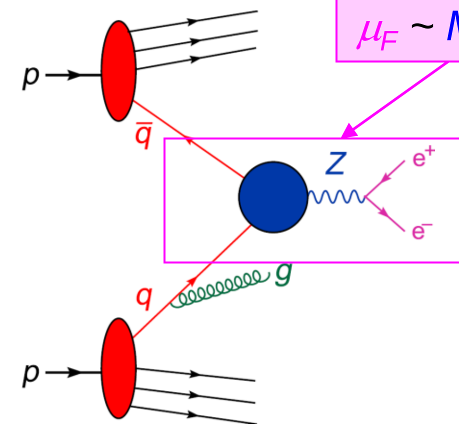
QCD Factorization & Parton Model

Collins, Soper, Sterman 1985

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- Sampled "one at a time" in hard collisions.

→ QCD-improved parton model

$\mu_F \sim M_Z$



infrared safe final state

Parton distribution functions – known to ~ 5% for x ~ 0.01-0.1

factorization scale

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Partonic cross section, computable in perturbative QCD

partonic CM energy^2

renormalization scale

Partonic Cross Section in Perturbation Theory

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO

Problem: Leading-order, tree-level predictions only qualitative due to **poor convergence** of expansion in $\alpha_s(\mu)$ (setting $\mu_R = \mu_F = \mu$)

Lack of One-Loop Amplitudes

At NLO, the bottleneck for more complex processes is the lack of availability of one-loop amplitudes.

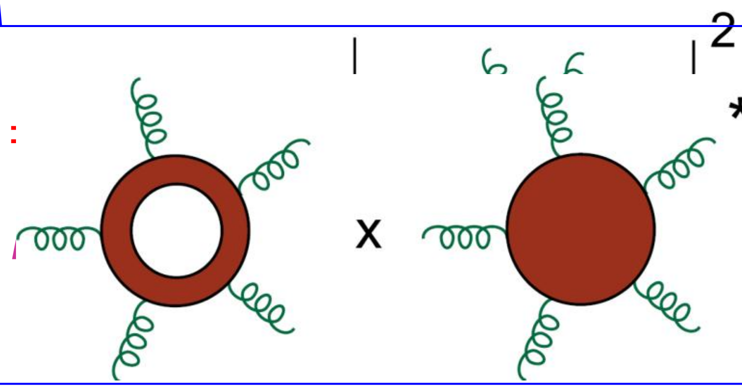
$$\sigma(n\ \text{jets}) = [\alpha_s(\mu)]^n\{A+\alpha_s(\mu)B+\alpha_s^2(\mu)C+\cdots\}$$

state of the art:

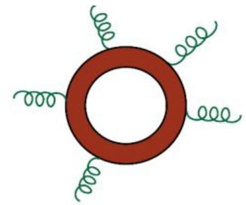
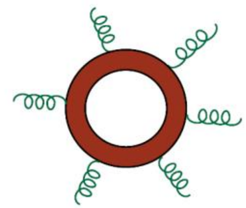
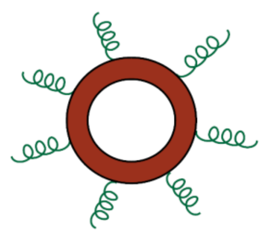
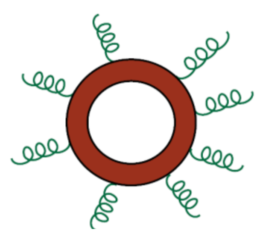
LO :

NLO = loop x tree* + …

n=3

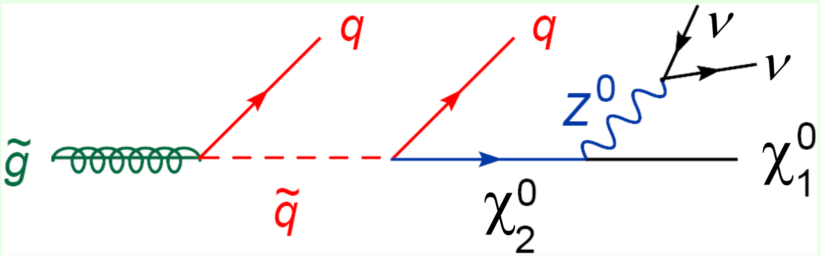


Strong growth in difficulty at one loop (NLO) with number of final-state objects

# of jets	# 1-loop Feynman diagrams (gluons only)
3	 810
4	 10,860
5	 168,925
6	 3,017,490

Background to Search for Supersymmetry

- **Cascade from gluino to neutralino** (dark matter, escapes detector)

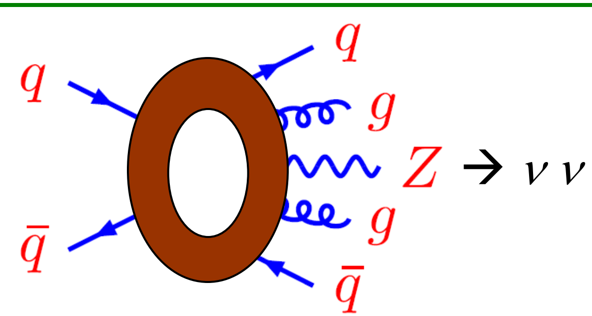


- **Signal: missing energy + 4 jets**
- SM background from Z + 4 jets, Z → neutrinos

Current state of art for Z + 4 jets: ALPGEN, based on **LO tree** amplitudes → normalization still quite uncertain

- Motivates goal of

$$pp \to Z + 4 \text{ jets at NLO}$$



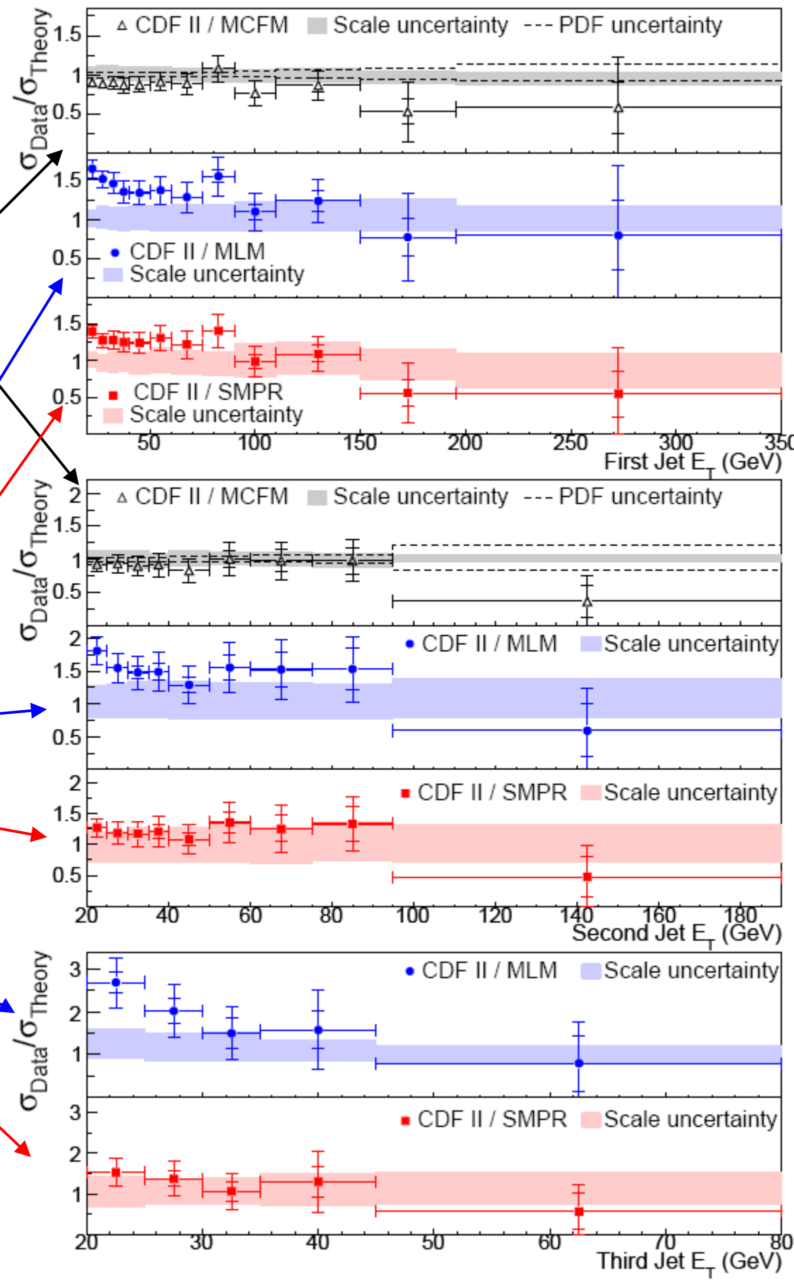
2 legs beyond state-of-art

Tevatron
W + n jets Data

CDF, 0711.4044 [hep-ex]

NLO (MCFM)

LO matched to parton shower MC with different schemes



n = 1

n = 2

n = 3 only LO available – until this year

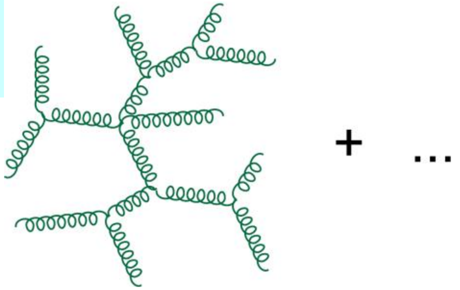
% uncertainty

number of jets	LO	NLO
1	16%	7%
2	30%	10%
3	42%	11%

A Better Way to Compute?

- Backgrounds (and many signals) require detailed understanding of scattering amplitudes for many ultra-relativistic ("massless") particles – especially quarks and gluons of QCD

- **Feynman diagrams** can be used – in principle



- However, **Feynman diagrams**, while

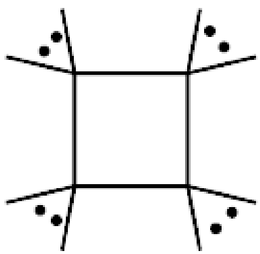
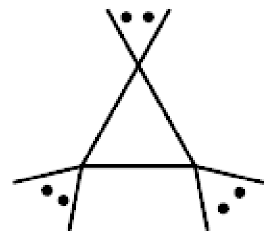
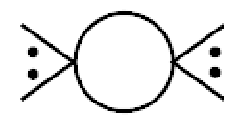
very general and **powerful**, are **not optimized** for these processes

- **On-shell** methods, exploiting **analyticity**, can be more efficient, especially for multi-gluon + quark processes!
- We have implemented these methods numerically in a **C++** program, **BlackHat**

One-Loop Amplitude Decomposition

When all external momenta are in *D=4*, loop momenta in *D=4-2ε* (dimensional regularization), one can write: BDDK (1994)

coefficients are all rational functions – determine algebraiclally from products of trees using (generalized) unitarity

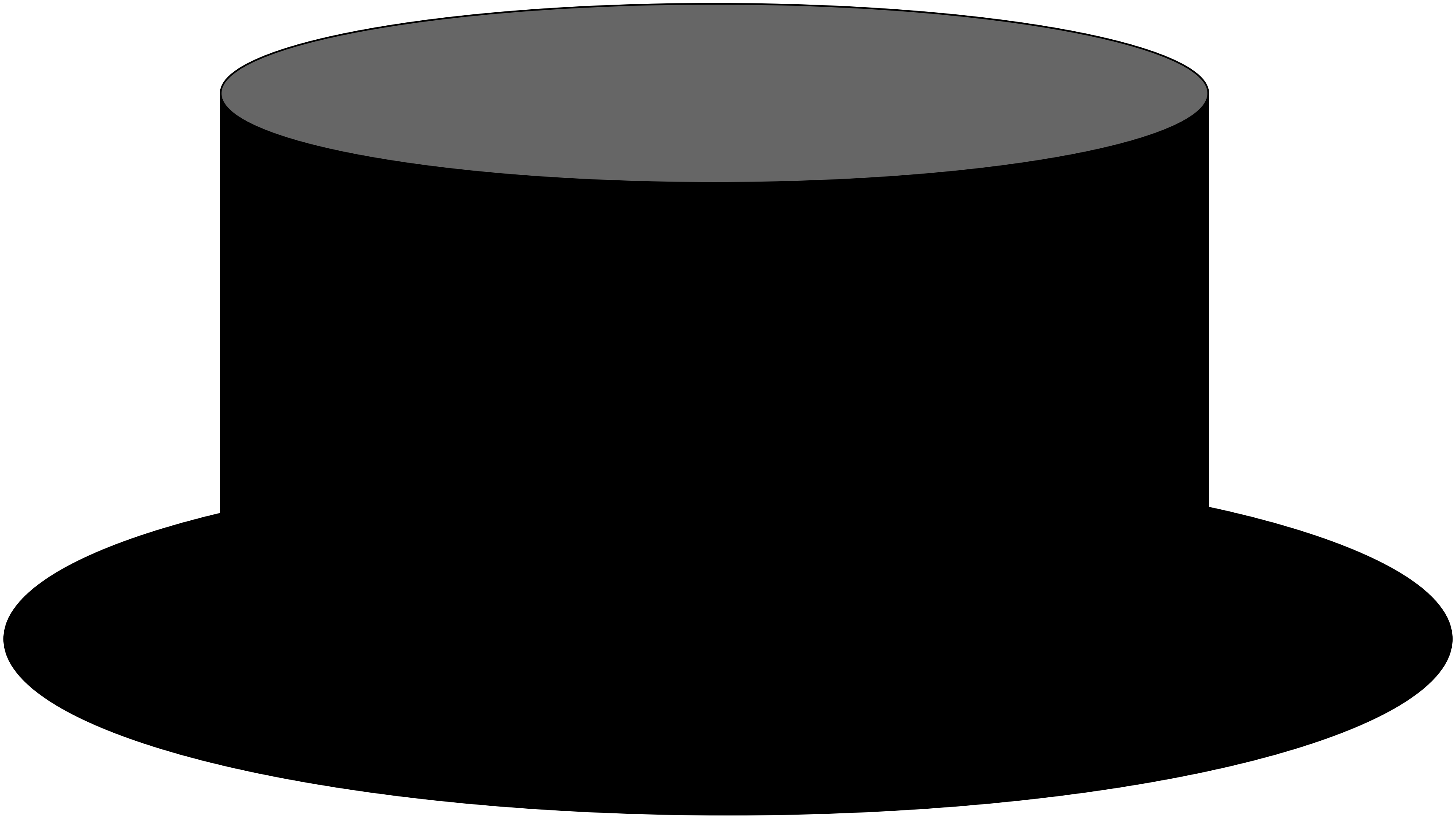
$A^{1-\text{loop}} = \sum_i d_i$  $+ \sum_i c_i$  $+ \sum_i b_i$ 

$+ \; R \; + \mathcal{O}(\epsilon)$

rational part

known **scalar** one-loop integrals, same for all amplitudes

Inside BlackHat



Several Related Implementations

CutTools: Ossola, Papadopolous, Pittau, 0711.3596

NLO *WWW* Binoth+OPP, 0804.0350

NLO *ttbb* Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723

Rocket: Giele, Zanderighi, 0805.2152

One-loop n-gluon amplitudes for n up to 20;

W + 3 jets amplitudes

Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762

NLO *W* + 3 jets in leading-color (large N_c) approximation

Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445

Melnikov, Zanderighi, 0910.3671

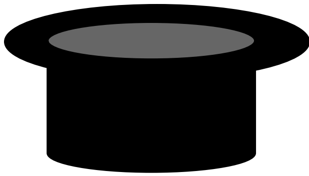
D-dim'l unitarity

Blackhat: Berger, Bern, LD, Febres Cordero, Forde, H. Ita, D. Kosower, D. Maître, 0803.4180, 0808.0941

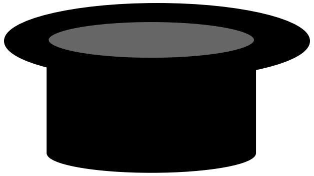
One-loop n-gluon amplitudes for n up to 7,…;

amplitudes needed for NLO production of *W,Z* + 3 jets

D-dim'l unitarity + on-shell recursion



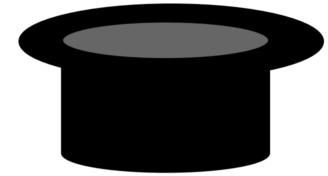
W + 3 jets at NLO



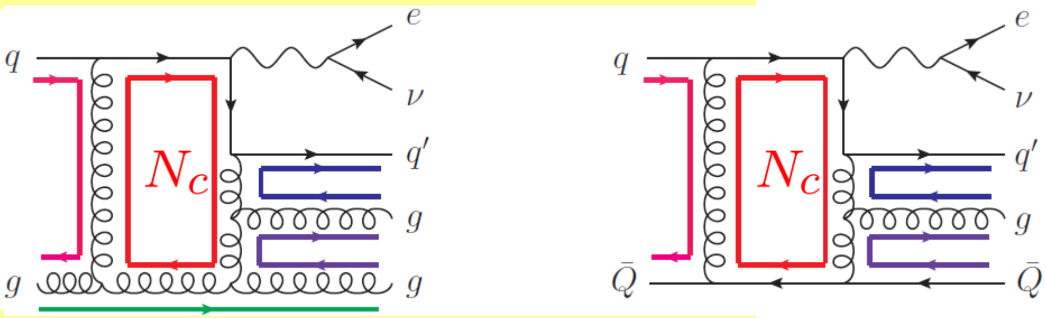
C.F.Berger, Z. Bern, LD, D. Forde, F. Febres Cordero, T. Gleisberg, H. Ita, D. Kosower, D. Maître, 0902.2760, 0907.1984

- Background to SUSY searches in the "Jets + MET" channel, when the charged lepton in $W \to l\ \nu_l$ is lost
- Also closely related to Z + 3 jets, another SUSY background when $Z \to \nu\,\nu$
- Similar to top-quark pair production in semi-leptonic W decay channel, $t\,\bar{t} \to l\ \nu_l + 4$ jets
- Many different kinematic configurations can appear in final state – have to be careful to choose scale μ correctly to avoid pathologies!

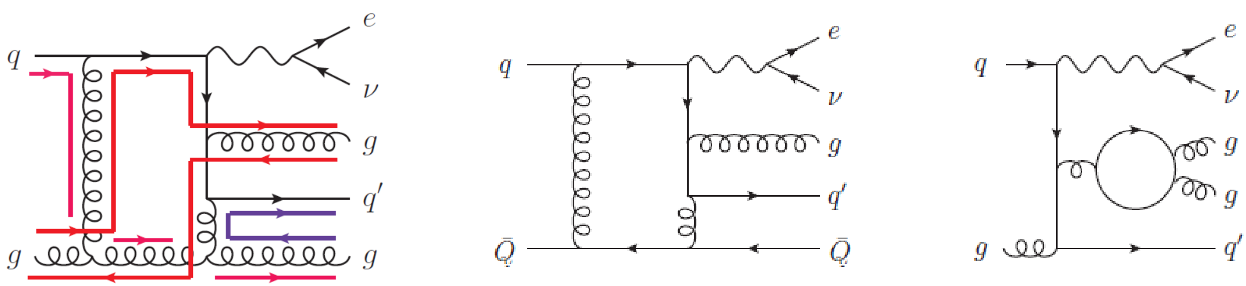
Color Sampling for Virtual Corrections



• Divide into leading-color terms, such as:



and subleading-color terms, such as:



The latter include many more terms, and are much more time-consuming for computer to evaluate. But they are much smaller (~ 1/30 of total cross section) so we evaluate them much less often.

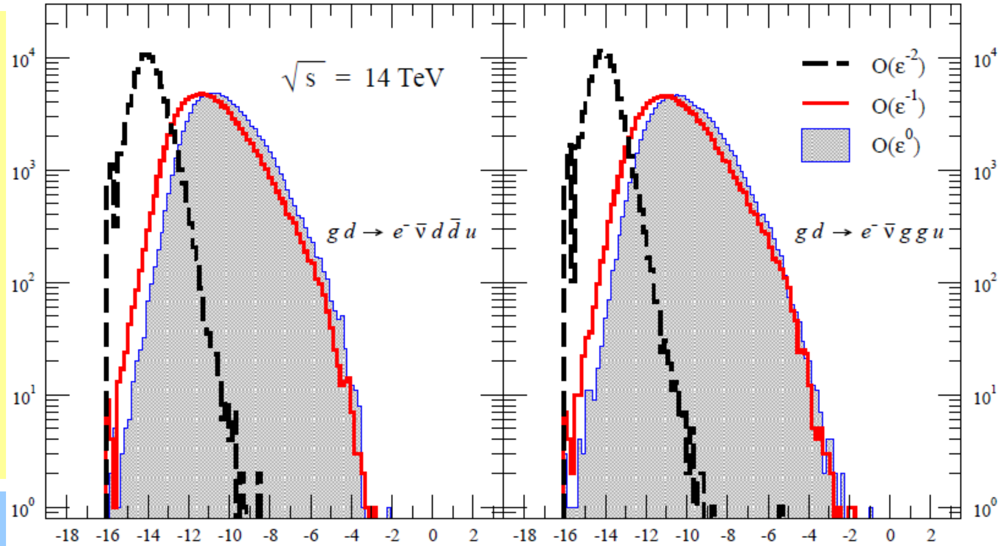
Numerical Stability of Virtual Terms

• Nontrivial because there are many kinematic regions where there are large cancellations between terms in this expansion, leading to roundoff error:

$$A^{1-\text{loop}} = \sum_i d_i \;\square\; + \sum_i c_i \;\triangle\; + \sum_i b_i \;\bigcirc\; + R + \mathcal{O}(\epsilon)$$

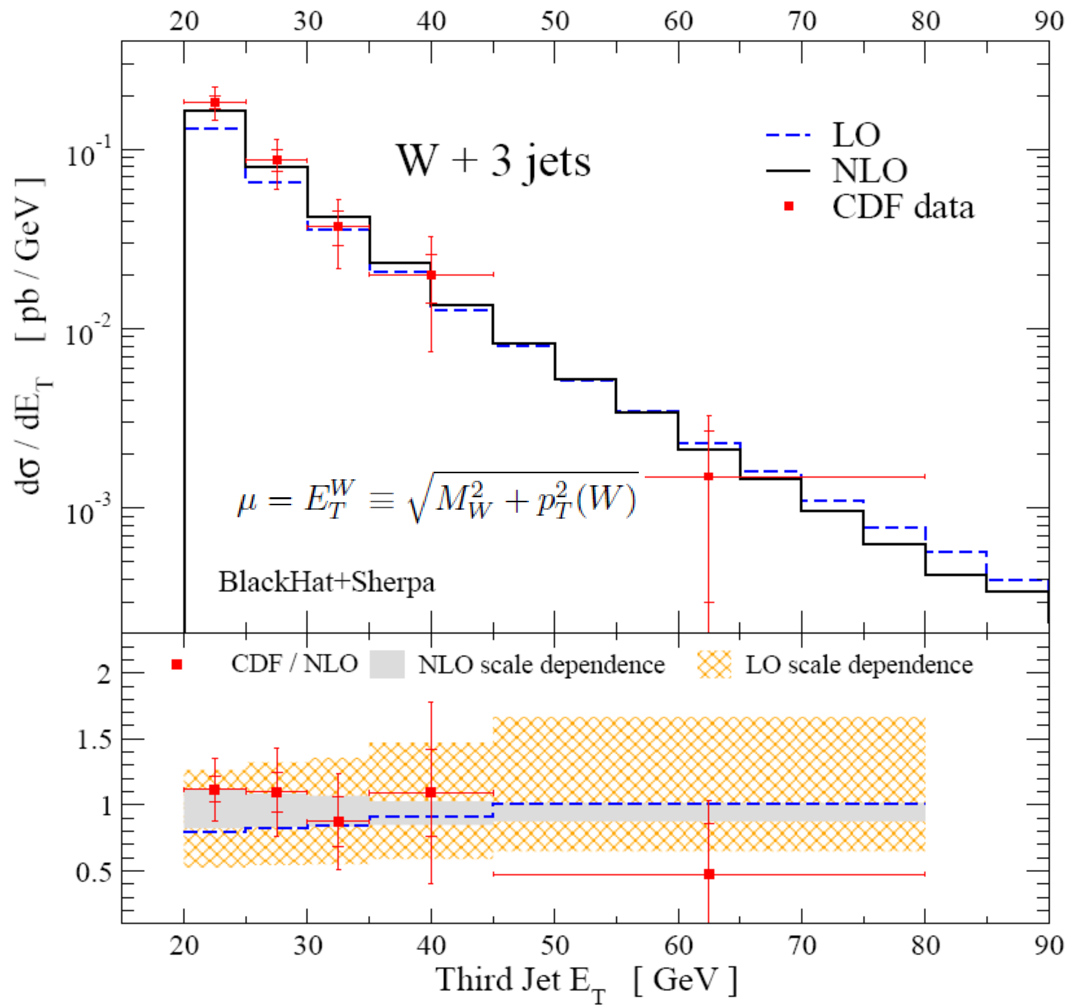
• BlackHat has a lot of tests for instability; if a piece of $A^{1\text{-loop}}$ is unstable, it recomputes that piece with higher precision (~32 digits).

• Resulting distributions of log(relative error) →



Real radiation handled using automated Catani-Seymour dipole subtraction Gleisberg, Krauss 0709.2881

W + 3 jets at Tevatron at NLO



same cuts as CDF

$E_T^{\text{jet}} > 20\,\text{GeV}\,, \qquad |\eta^{\text{jet}}| < 2$

$E_T^e > 20\,\text{GeV}\,, \qquad |\eta^e| < 1.1\,,$

$\not{E}_T > 30\,\text{GeV}\,, \qquad M_T^W > 20\,\text{GeV}$

$$M_T^W = \sqrt{2E_T^e E_T^\nu (1 - \cos(\Delta\phi_{e\nu}))}$$

Except: we use SISCone; CDF used IR unsafe JETCLU

- Much smaller uncertainties than at LO.
- Agrees well with data; more data coming soon!

W + n jets cuts at LHC

$$\sqrt{s} = 14 \text{ TeV}$$

$$|\eta^{\rm jet}| < 3\,, \qquad R = 0.4\,, \qquad |\eta^{e}| < 2.5\,, \qquad E_T^{e} > 20\,{\rm GeV}$$

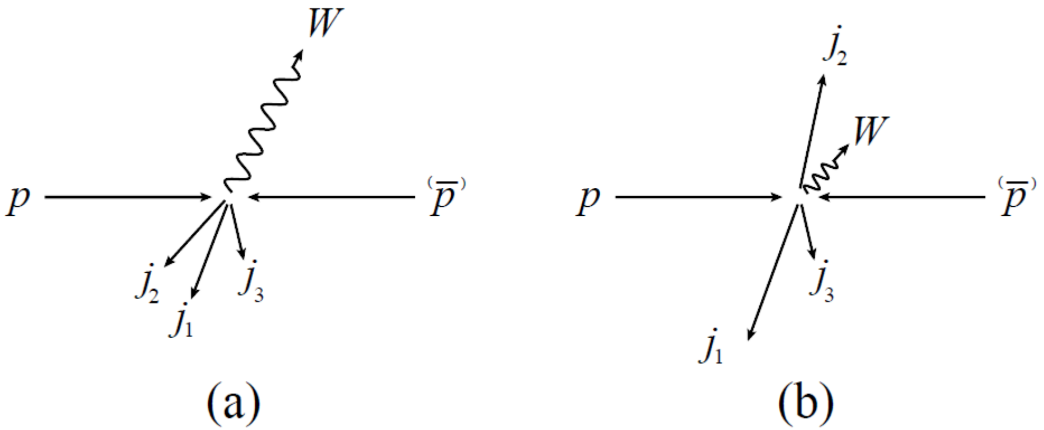
$$E_T^{\nu} > 30\,{\rm GeV}\,, \qquad M_T^{W} > 20\,{\rm GeV}\,.$$

$E_T^{\rm jet} > 30\,{\rm GeV}$ or $E_T^{\rm jet} > 40\,{\rm GeV}$

SISCone, $f = 0.5$ or kT

Better Scale Choices

What's going on?
Consider these
2 configurations:



- If (a) dominates, then $\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$ is OK

- But if (b) dominates, then the scale ***E_T^W* is way too low**.
- Looking at large E_T for the 2nd jet forces configuration (b).

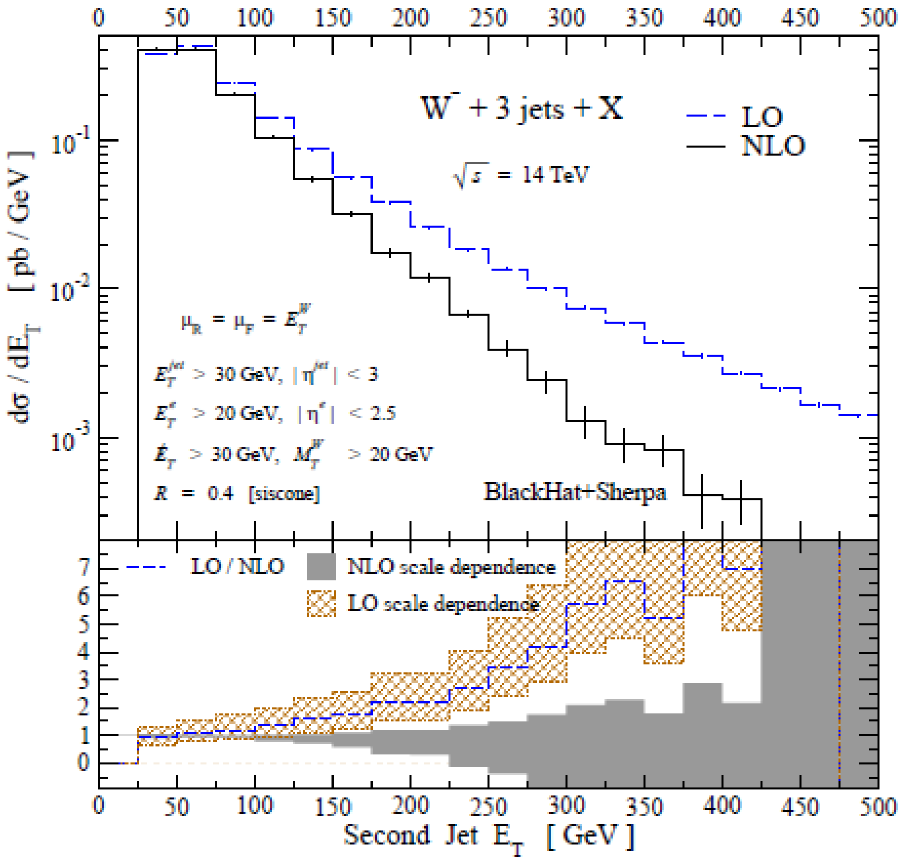
- The **total (partonic) transverse energy** is a **better variable**; gets large properly for both (a) and (b)

$$\hat{H}_T = \sum_p E_T^p + E_T^e + E_T^\nu$$

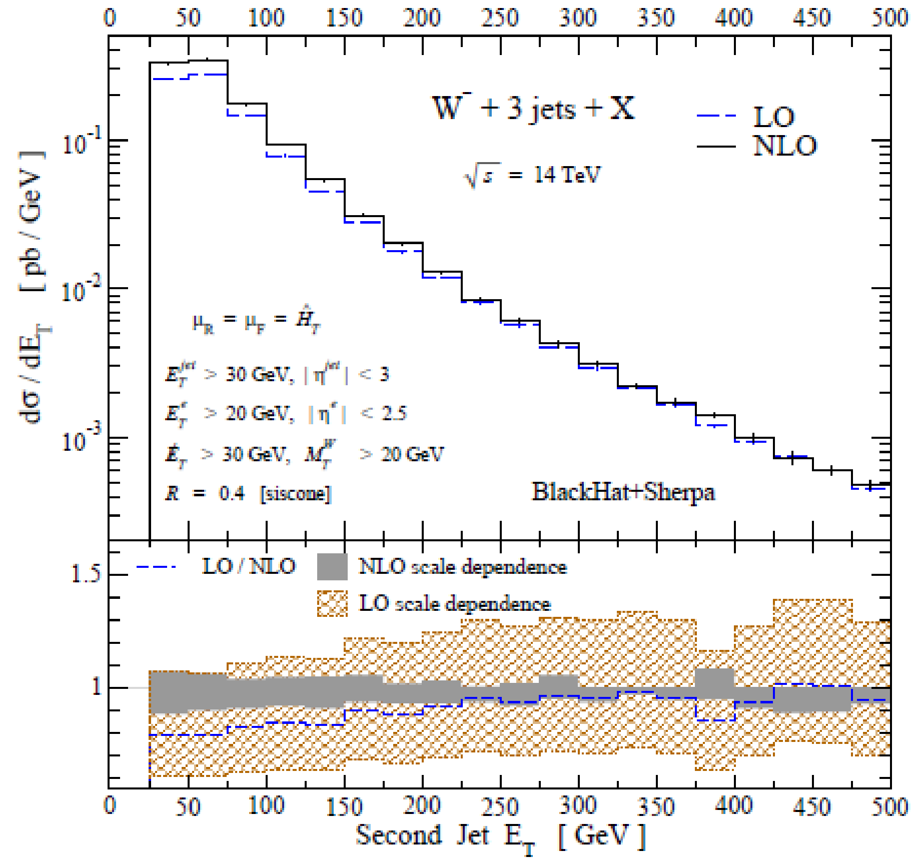
- Another reasonable scale is invariant mass of the *n* jets

Bauer, Lange
0905.4739

Compare the Two Scale Choices



$\mu = E_T^W$ very poor



$\mu = \hat{H}_T$ excellent!

– LO/NLO quite flat, and also for many other observables

"Berends Ratio"

Berends observed that $\frac{\sigma_{n+2\ \text{jets}}}{\sigma_{n+1\ \text{jets}}} \approx \frac{\sigma_{n+1\ \text{jets}}}{\sigma_{n\ \text{jets}}}$

We can compute $r_{B,1} \equiv \frac{\sigma_{3\ \text{jets}}\sigma_{1\ \text{jet}}}{\sigma^2_{2\ \text{jets}}}$ at LO, NLO

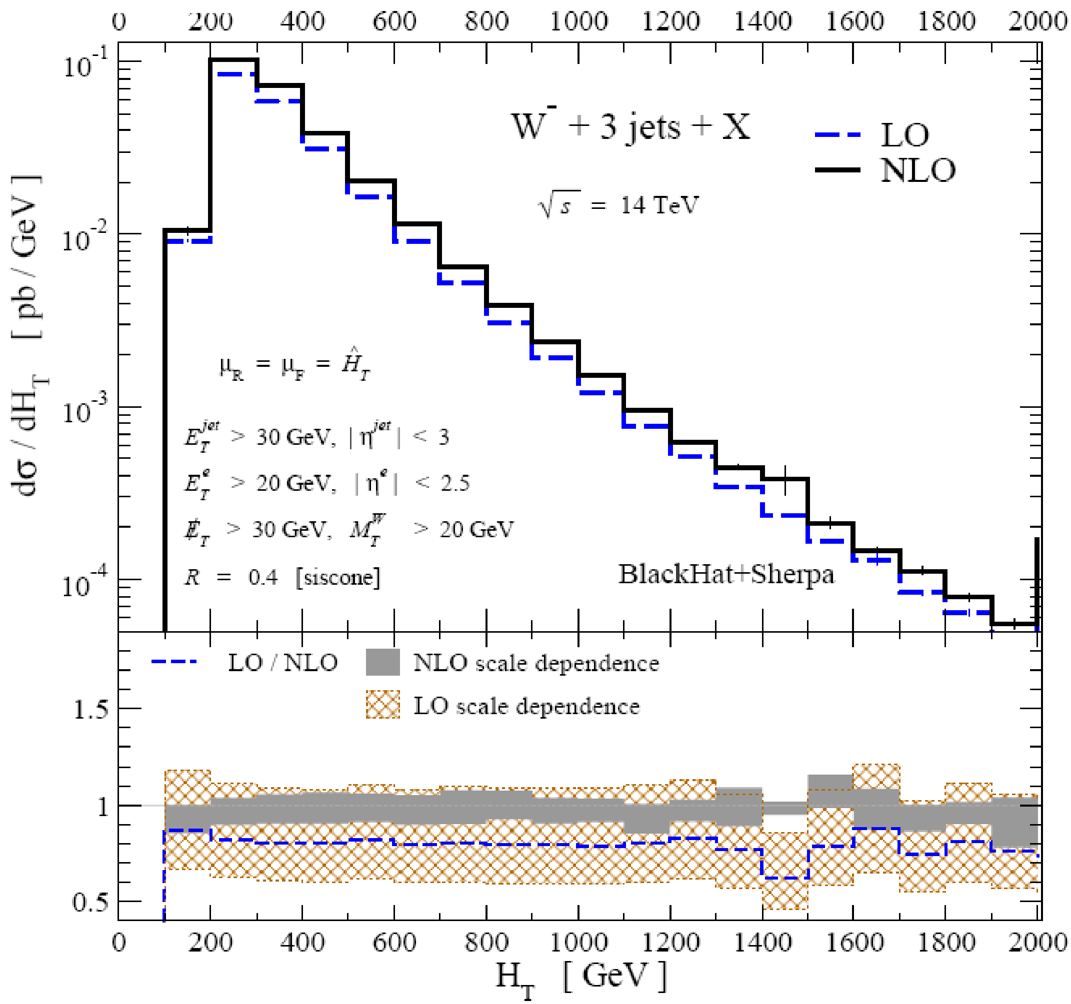
For *W* + n jets, and for SISCone and kT jet algorithms

$r_{B,1}$	LO	NLO
$E_T^{\text{jet}} > 30\,\text{GeV}$ SISCone	0.788	0.841
$E_T^{\text{jet}} > 40\,\text{GeV}$ SISCone	0.713	0.805
$E_T^{\text{jet}} > 30\,\text{GeV}$ kT	0.858	0.910
$E_T^{\text{jet}} > 40\,\text{GeV}$ kT	0.787	0.873

Total Transverse Energy H_T at LHC

$$H_T = \sum_j E_{T,j}^{\rm jet} + E_T^e + E_T^\nu$$

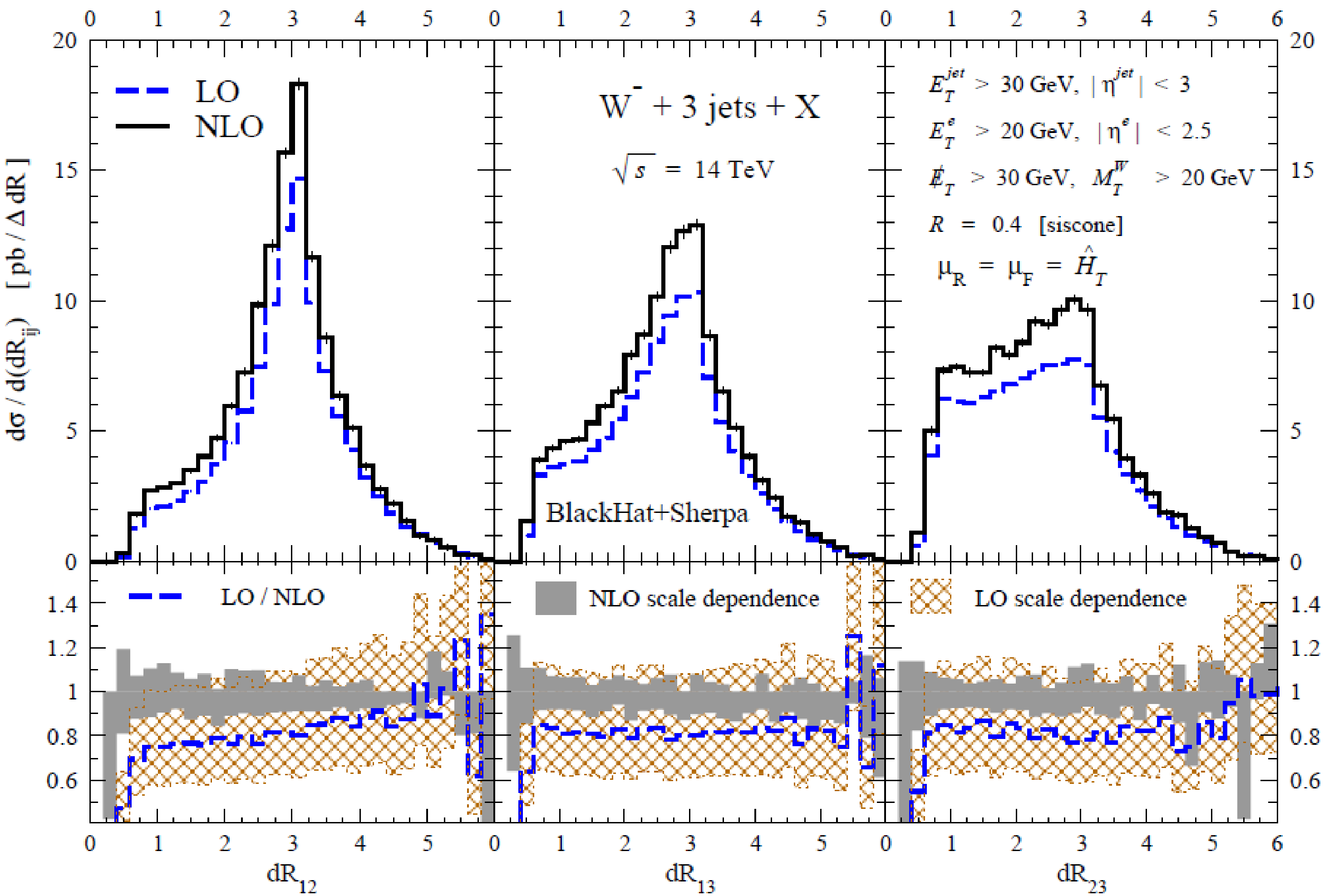
often used in supersymmetry searches



0907.1984

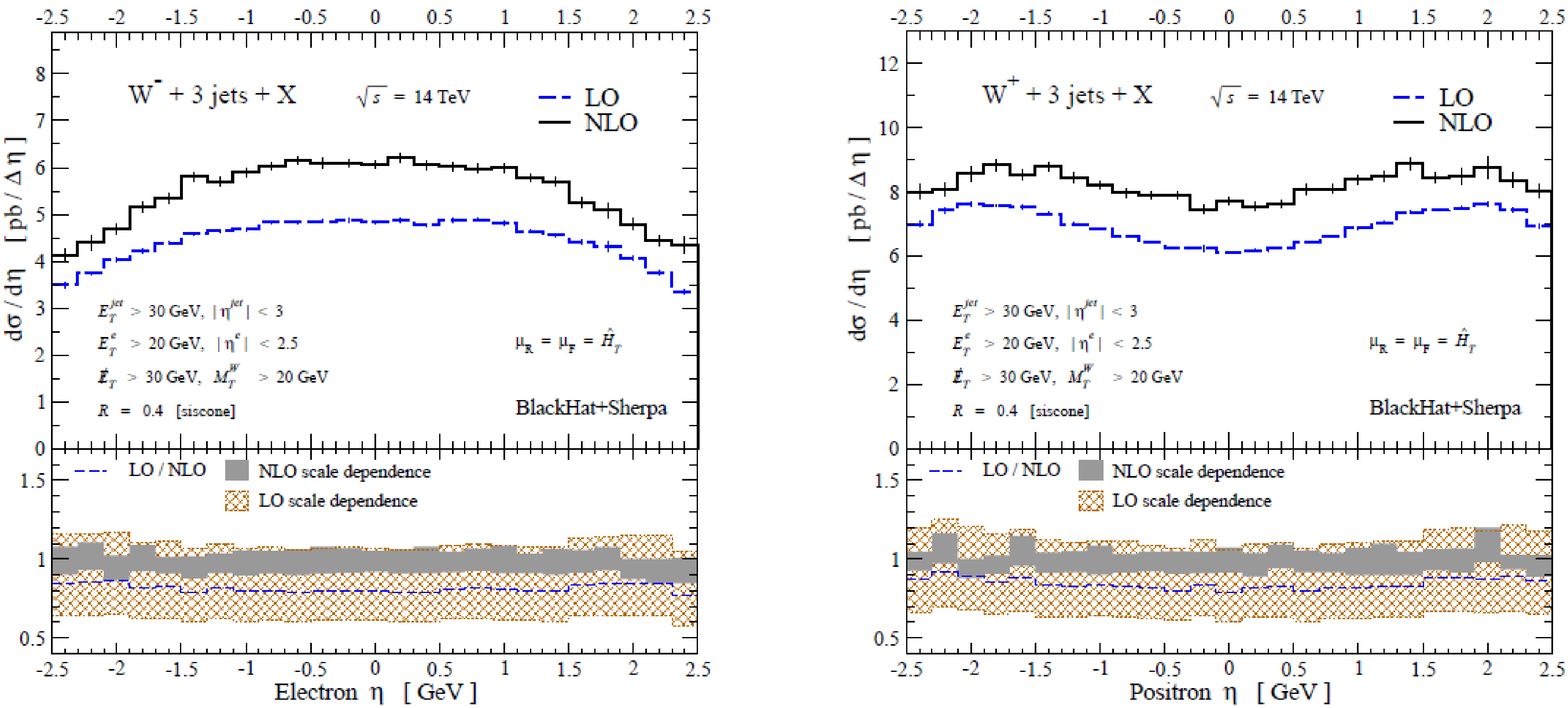
flat LO/NLO ratio due to good choice of scale $\mu = H_T$

Jet Separations $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$



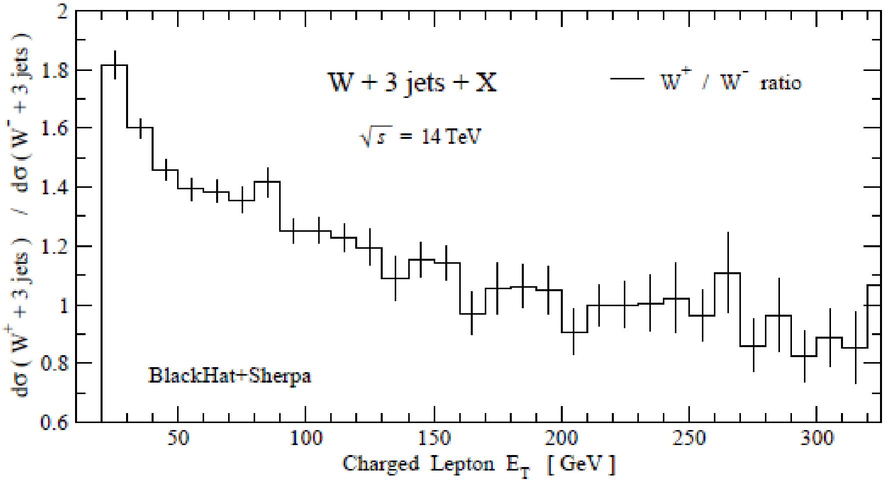
Pretty well modeled by LO, except for first 2 jets

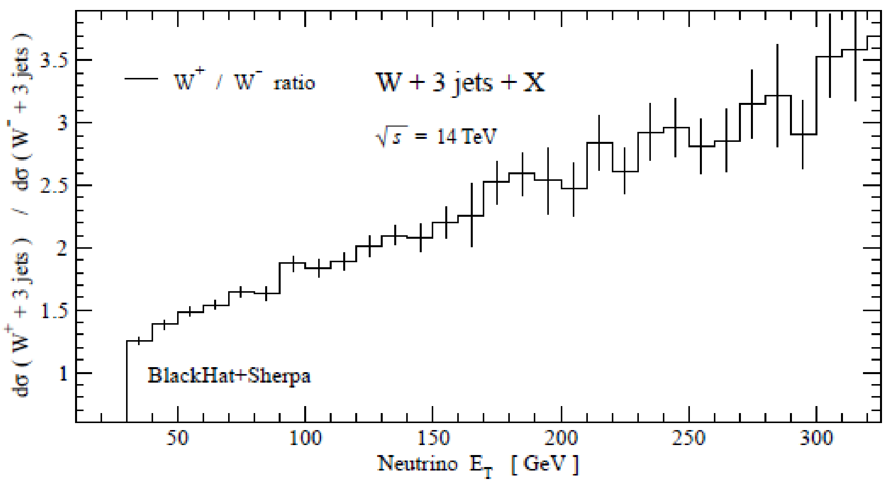
Lepton Rapidity in *W* + 3 jets at LHC



Rapidity distributions “remember” that *u(x)/d(x)* gets very large as $x \rightarrow 1$

Leptonic E_T in *W* + 3 jets at LHC





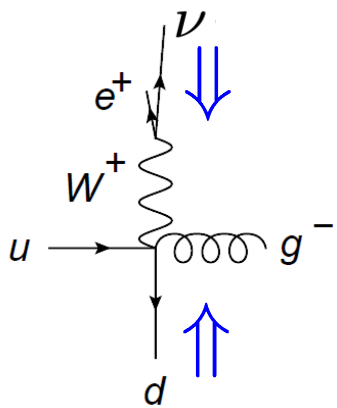
W^+/W^- transverse lepton ratios trace a remarkably large and stable left-handed *W* polarization at large $p_T(W)$
– independent of number of jets
– will be useful to separate *W* + n jets from top, maybe also from new physics

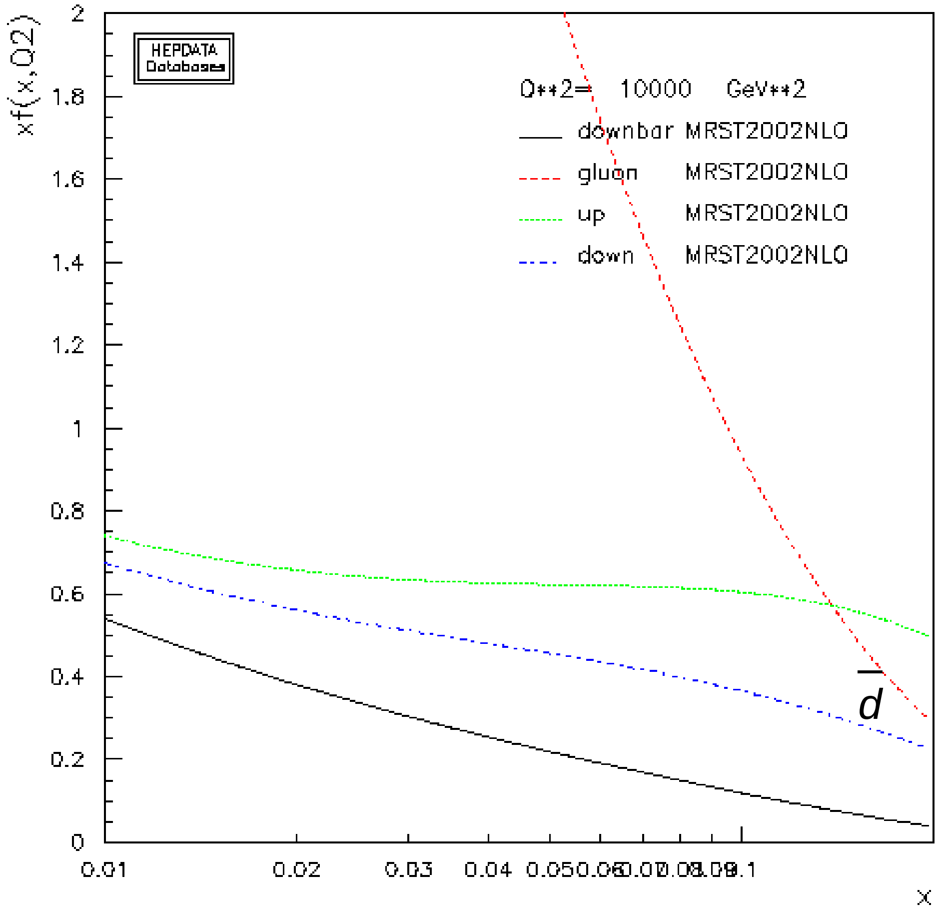
Transverse spin can be confusing

(stolen from recent talk by W. Vogelsang)

Origin of W polarization in LO W + 1 jet

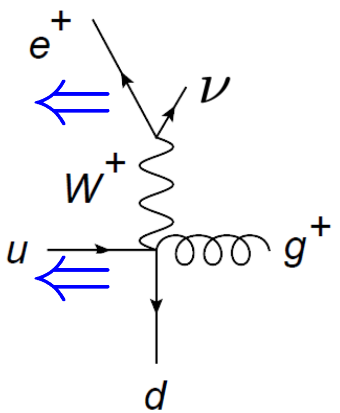
SU(2)$_L$ + valence quark dominance





$$A^{\text{tree}} \propto \frac{\langle d\,\nu\rangle^2}{\langle u\,g\rangle\langle g\,d\rangle}$$

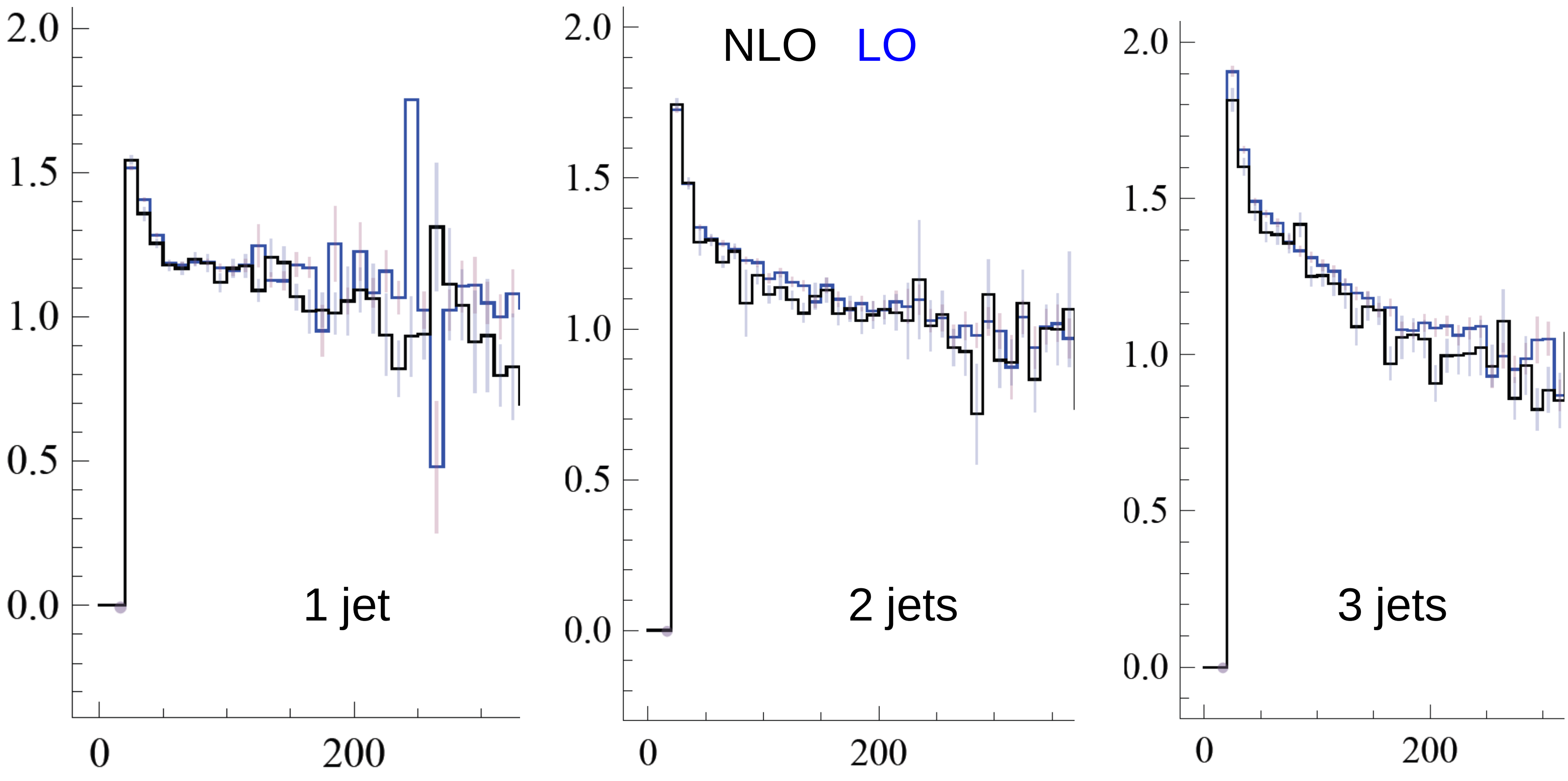
$$d\sigma \;\; \propto (k_d \cdot k_\nu)^2$$



$$A^{\text{tree}} \propto \frac{[u\,e]^2}{[u\,g][g\,d]}$$

$$d\sigma \;\; \propto (k_u \cdot k_e)^2$$

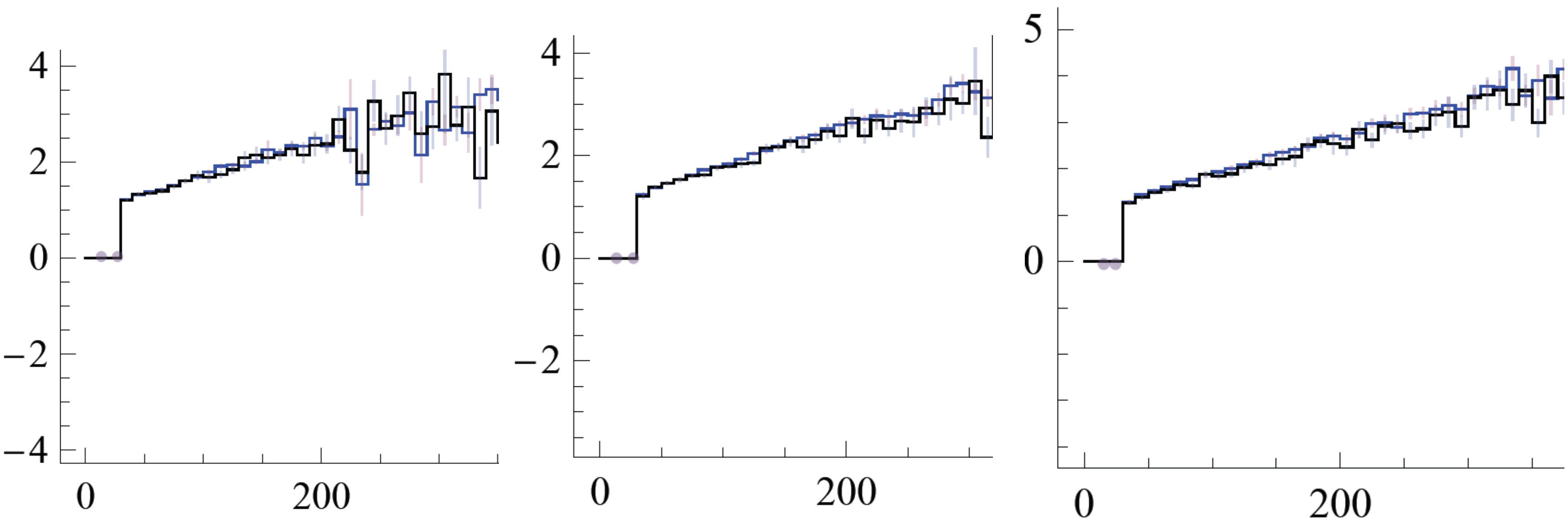
$W^{+/-}$ + n jets: e^+/e^- E_T ratio



- LO → NLO hardly affects ratios

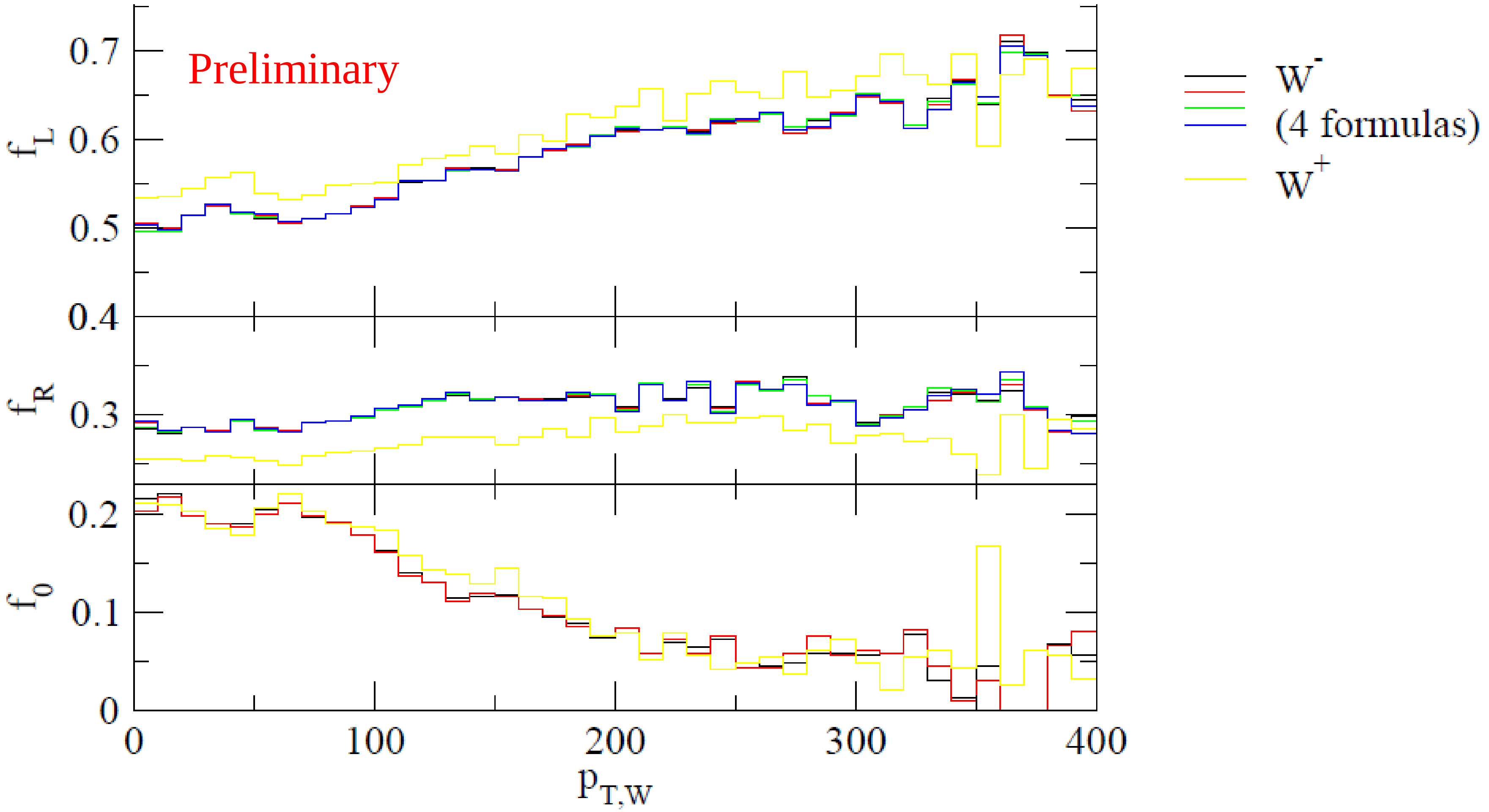
$W^{+/-}$ + n jets: Neutrino E_T

NLO LO



1 jet 2 jets 3 jets

Actual *W* polarization – LO W + 2 jets



Top quark pairs very different

Main production channels are C invariant:

$$g\bar{g} \to t\bar{t} \qquad q\bar{q} \to t\bar{t}$$

Semi-leptonic decay involves (partially) left-handed W^+

$$t\bar{t} \to bW^+\bar{b}W^- \to b\,e^+\nu\,\bar{b}jj$$

But charge conjugate decay involves (same degree) right-handed W^-

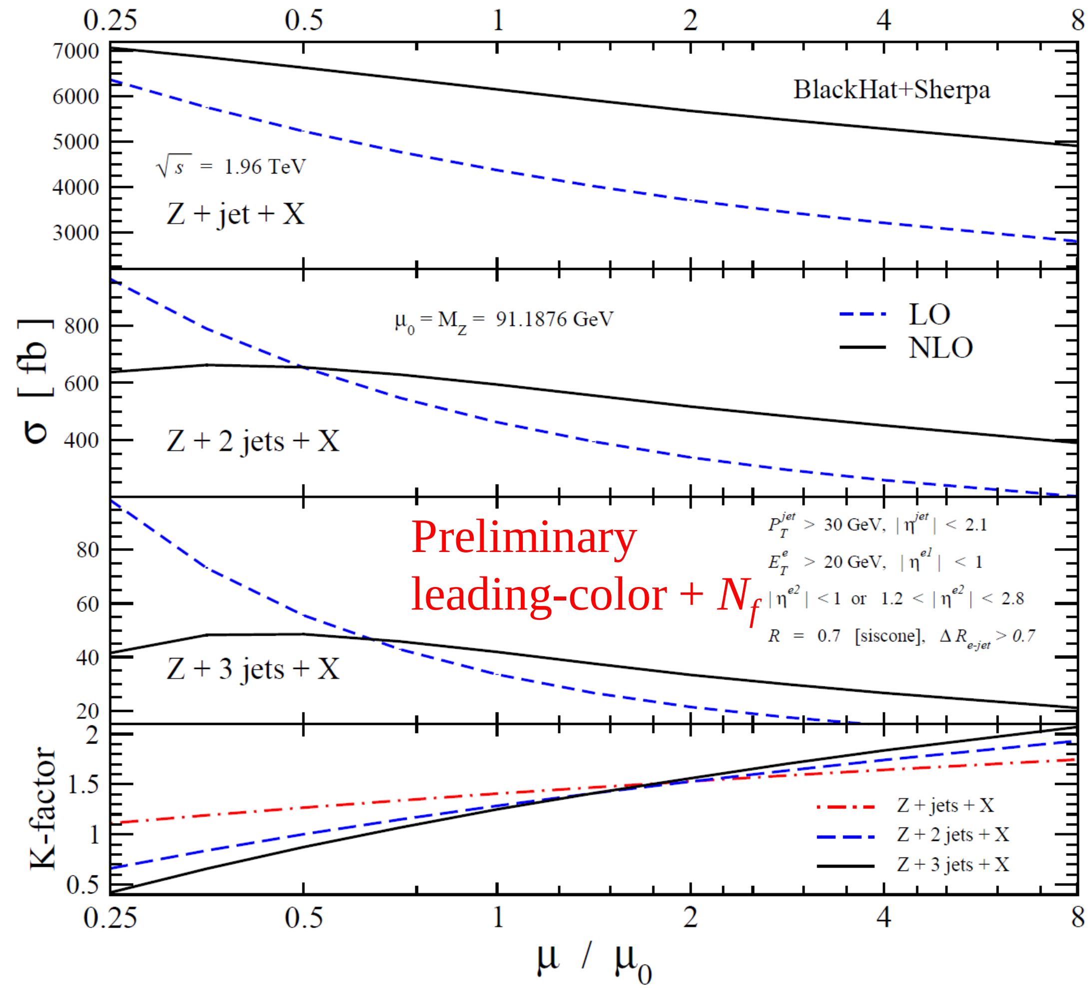
$$t\bar{t} \to bW^+\bar{b}W^- \to bjj\,\bar{b}\,e^-\bar{\nu}$$

→ electron and positron have almost identical p_T distributions

→ A nice handle on separating *W* + jets from top

Supersymmetry may be like top – or not – depends on $qg \to \tilde{q}\tilde{g}$

First NLO Z + 3 jets Results



SISCone
$f = 0.75$
$R = 0.7$

K-factor at $\mu = M_V$ is 20% larger than in W + 3 jets, but this was for $E_T > 20$ GeV, and SISCone with $f = 0.5$, $R = 0.4$

Algorithm Dependence of Z + n jets

SISCone $f = 0.75$, $R = 0.7$

# of jets	LO parton	NLO parton
1	$4205.67(1.91)^{+800.69}_{-615.94}$	$6062.55(7.85)^{+495.80}_{-462.51}$
2	$422.22(0.33)^{+168.22}_{-108.99}$	$575.17(1.30)^{+71.22}_{-75.88}$
3	$28.66(0.03)^{+17.87}_{-9.97}$	$39.46(0.19)^{+8.29}_{-8.32}$

Berends ratio: 0.676 0.728

Anti-kT, $R = 0.7$

# of jets	LO parton	NLO parton
1	$4205.67(1.91)^{+800.69}_{-615.94}$	$5819.67(7.89)^{+422.60}_{-411.10}$
2	$469.37(0.36)^{+185.46}_{-120.30}$	$583.56(1.34)^{+50.23}_{-66.39}$
3	$43.28(0.05)^{+26.59}_{-14.92}$	$47.72(0.22)^{+3.46}_{-7.78}$

Berends ratio: 0.826 0.816

kT, $R = 0.7$

# of jets	LO parton	NLO parton
1	$4205.67(1.91)^{+800.69}_{-615.94}$	$5819.67(7.89)^{+422.60}_{-411.10}$
2	$469.37(0.36)^{+185.46}_{-120.30}$	$587.58(1.29)^{+52.14}_{-67.55}$
3	$43.28(0.05)^{+26.59}_{-14.92}$	$48.72(0.22)^{+4.23}_{-8.17}$

Berends ratio: 0.826 0.821

Preliminary
leading-color + N_f

Conclusions

- New and efficient computational approaches to one-loop QCD amplitudes needed for important Tevatron and LHC backgrounds:
  - exploit **analyticity**: build loop amplitudes up out of trees
  - implemented numerically in C++ program **BlackHat**, as well as **CutTools** and **Rocket**
- NLO *W* + 3 jets agrees well with Tevatron data
- LHC kinematics and pp initial state ➔ different effects
- Valuable lessons already learned about scales and *W* polarization
- Preliminary [leading-color + N_f] NLO *Z* + 3 jets results too
- *W/Z* + 4 jets also now feasible
- Other groups have produced NLO results for several other processes using similar methods (VVV, $t\bar{t}\,b\bar{b}$, …)
- Will aid in optimal exploitation of LHC data!

Extra slides

Infrared safety

Cones tricky to get right. Seeds can cause problems.

- JETCLU (CDF) + D0 cone algorithms were IR unsafe for NLO W + 2 jets
- Midpoint OK for W + 2 jets, but (probably) fails for W + 3 jets

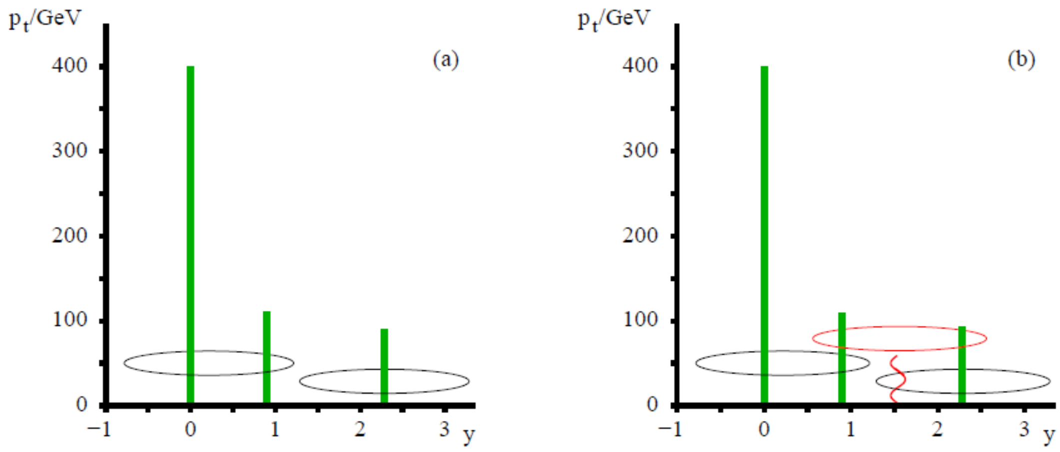


Figure 1: Configuration illustrating one of the IR unsafety problems of the midpoint jet algorithm ($R = 1$); (a) the stable cones (ellipses) found in the midpoint algorithm; (b) with the addition of an arbitrarily soft seed particle (red wavy line) an extra stable cone is found.

SIScone is a practical (fast enough) seedless cone algorithm that avoids these problems
Salam, Soyez