

A composite image of the Crab nebula, showing a complex, filamentary structure of gas and dust. The image is a combination of X-ray (blue) and optical (red) data, with the blue X-ray emission forming a bright, central, and somewhat diffuse region, while the red optical emission forms a more structured, filamentary outer shell. The overall color palette is dominated by deep reds and purples, with bright blue and white highlights from the X-ray emission and stars.

e^-e^+ pair production in pulsar magnetospheres

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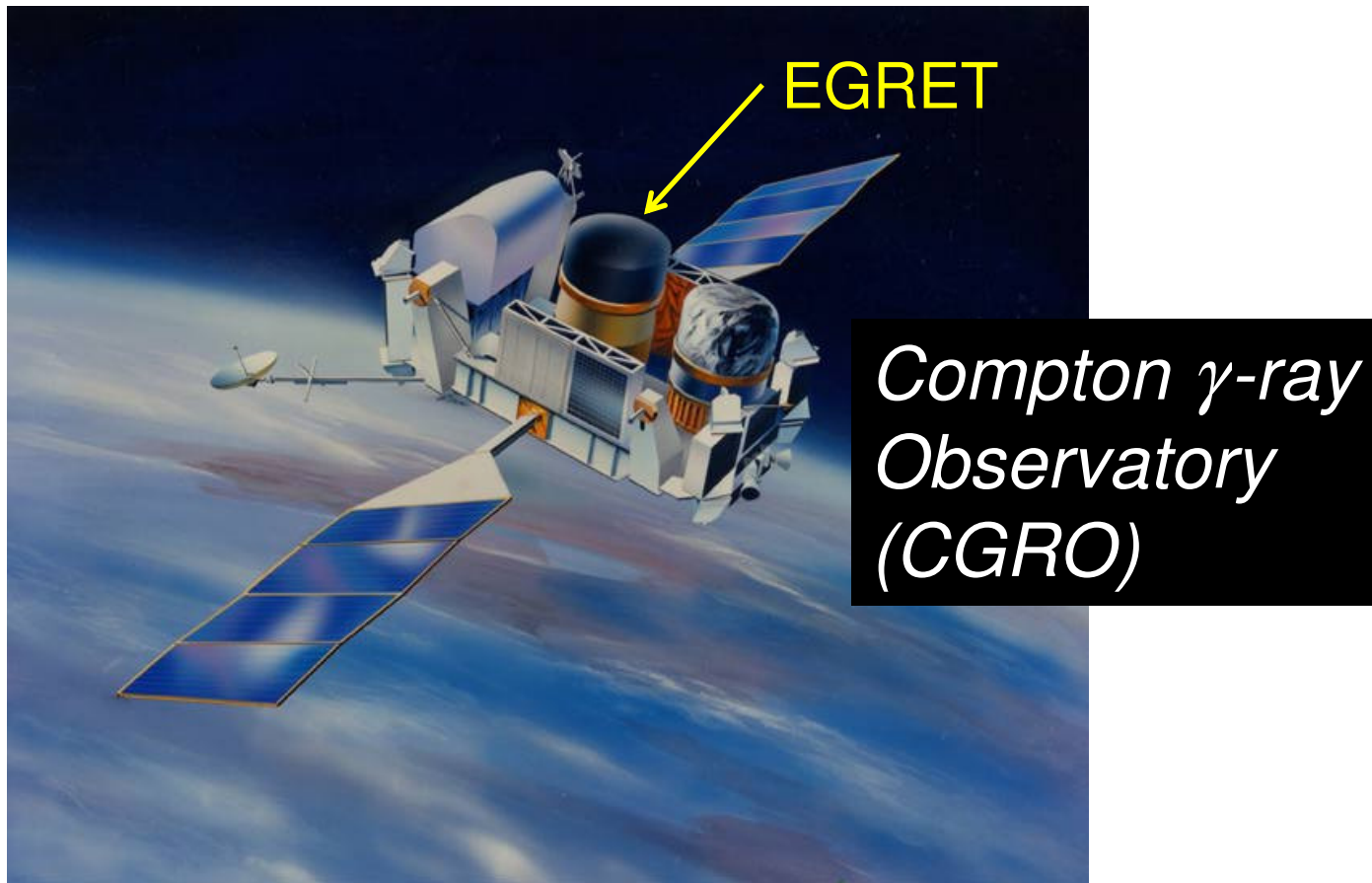
IPMU

December 8, 2009

Crab nebula: Composite image of X-ray [blue] and optical [red]

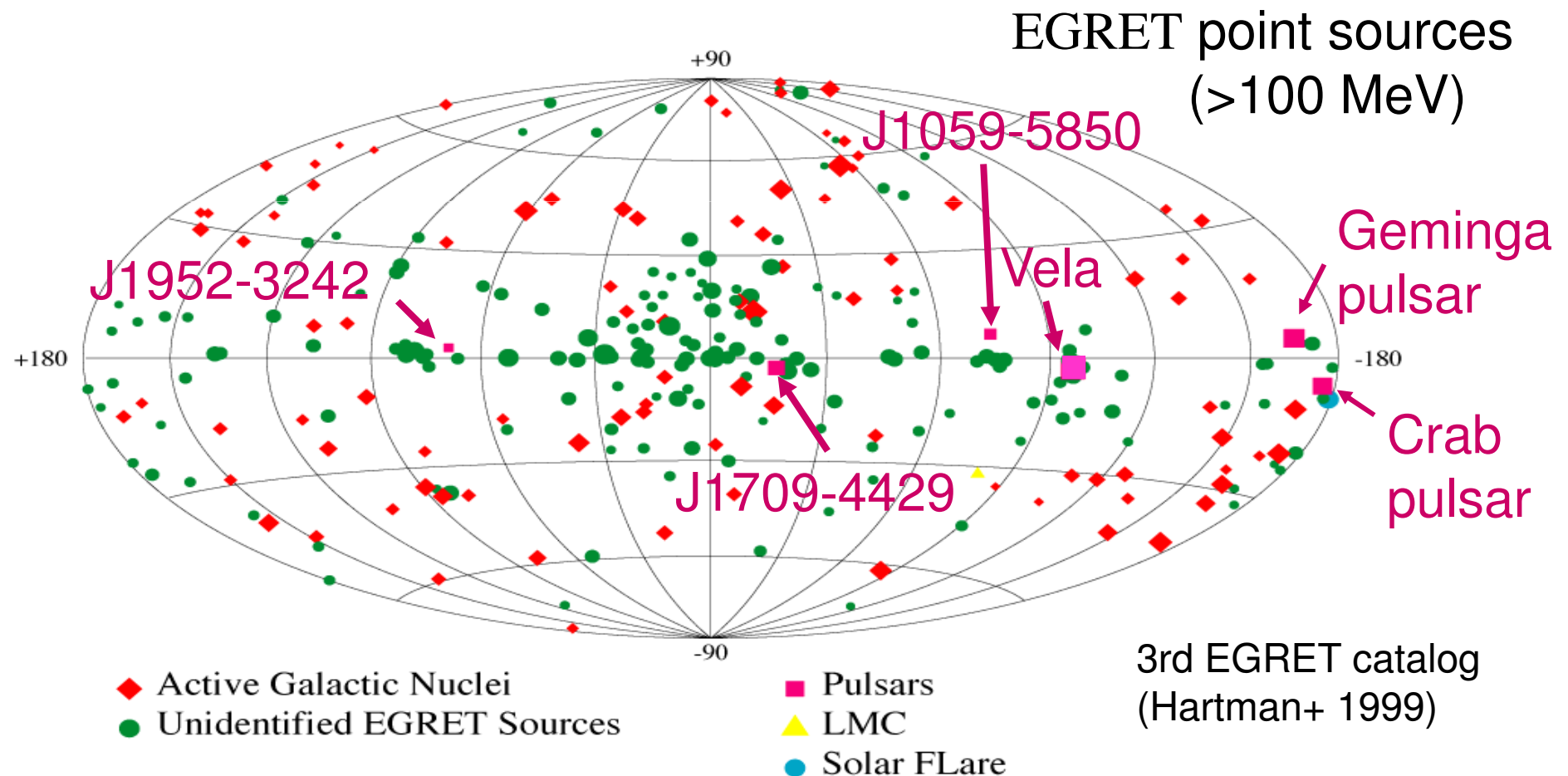
§1 Introduction: The γ -ray sky

Before 2008, EGRET aboard CGRO had detected six pulsars above 100 MeV.



§1 Introduction: The γ -ray sky

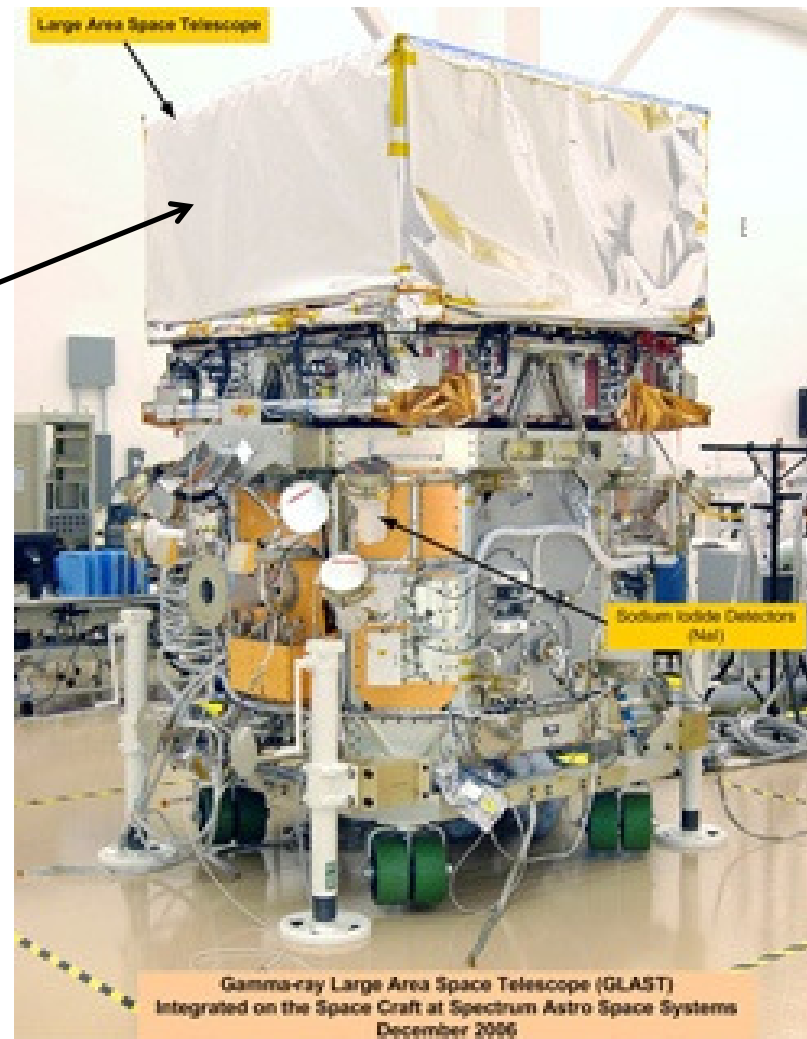
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§1 Introduction: The γ -ray sky

After 2008, LAT aboard Fermi has detected 46 pulsars above 100 MeV.

LAT
(Large
Area
Telescope)

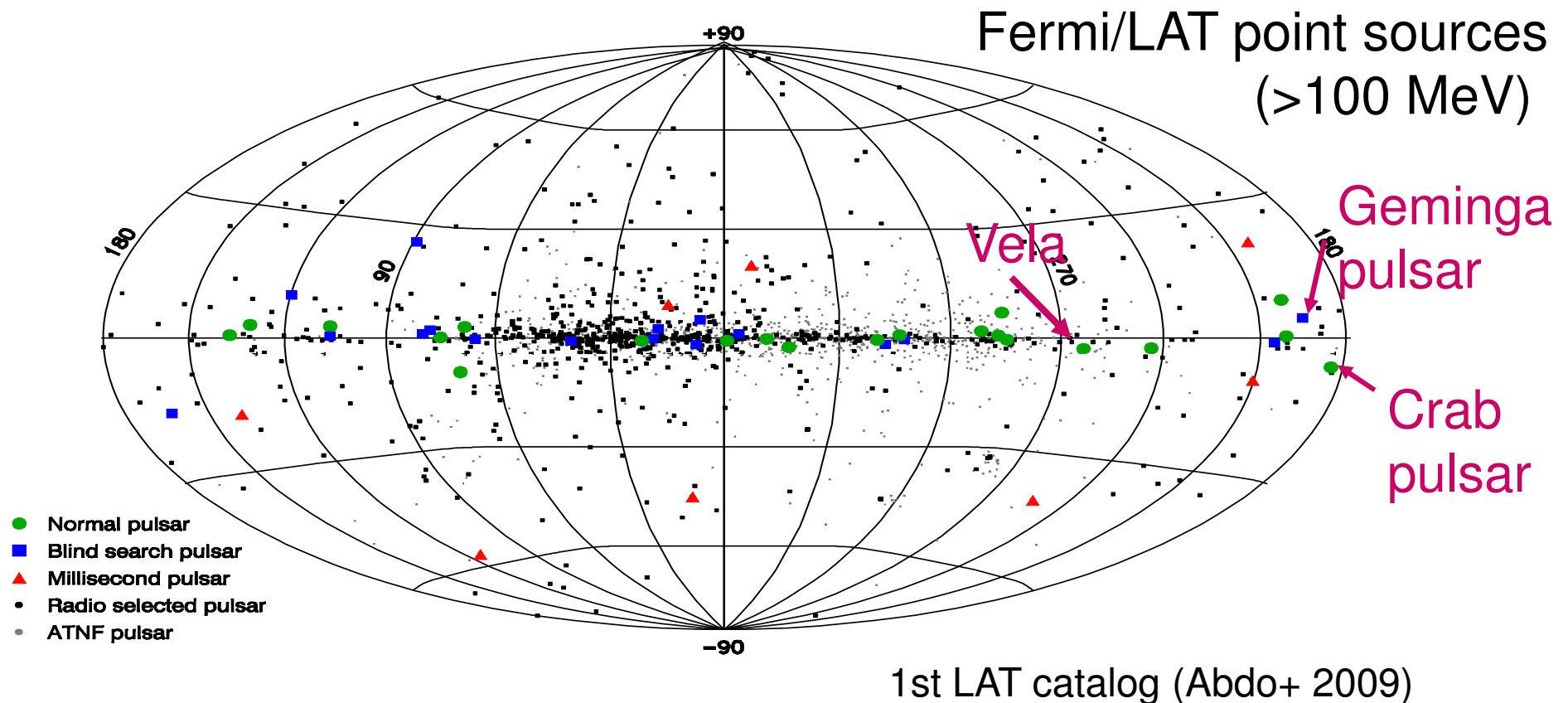


*Fermi γ -ray
space telescope*

Launched
June 2008.

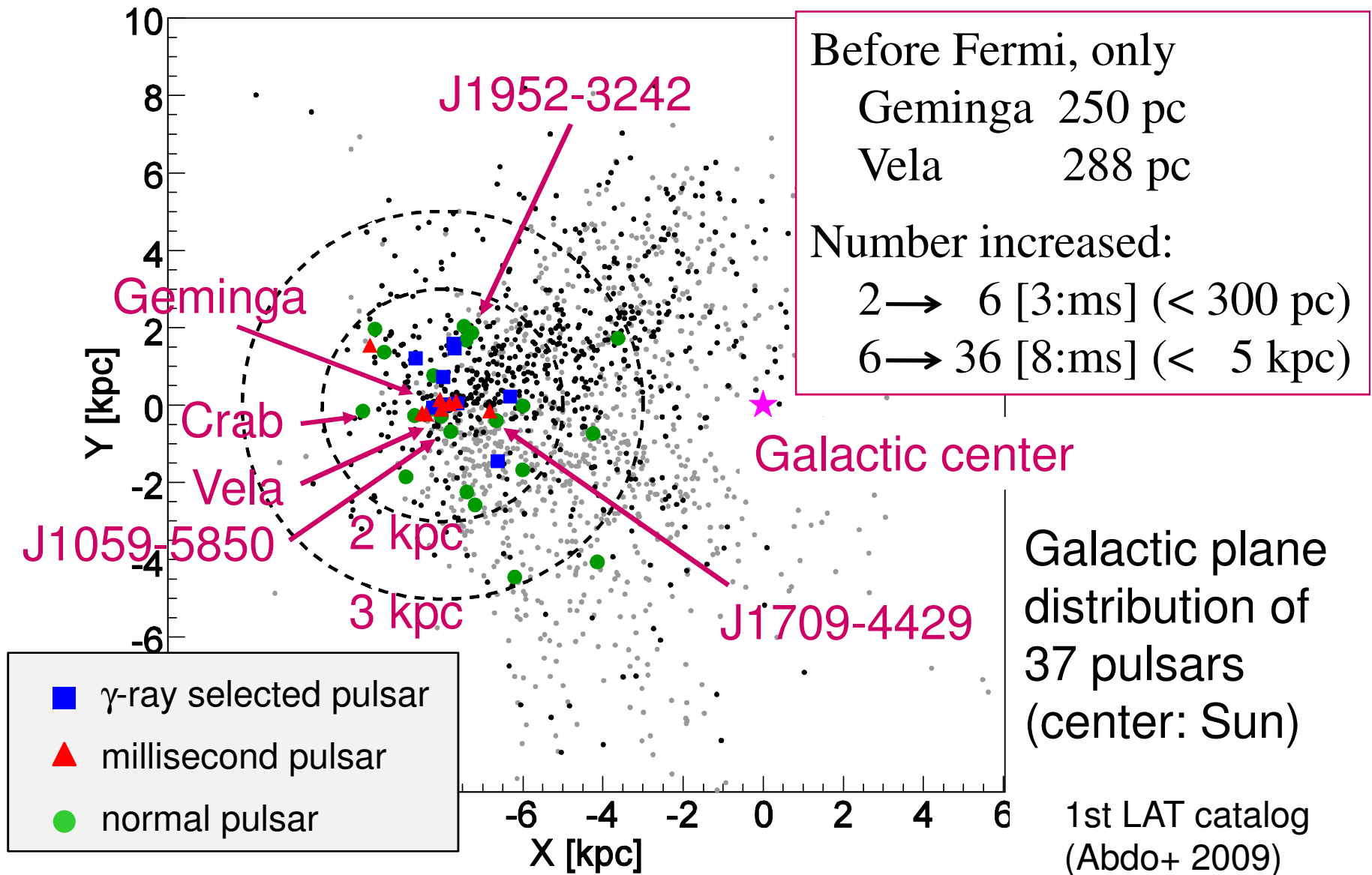
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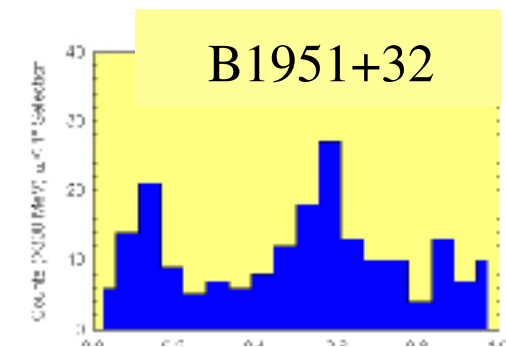
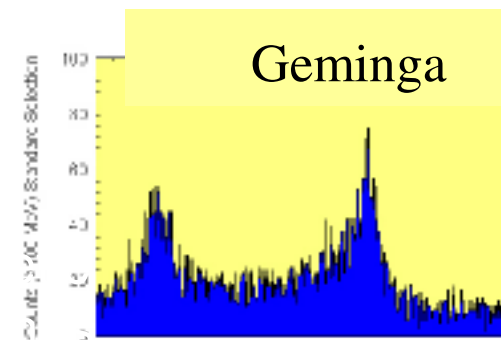
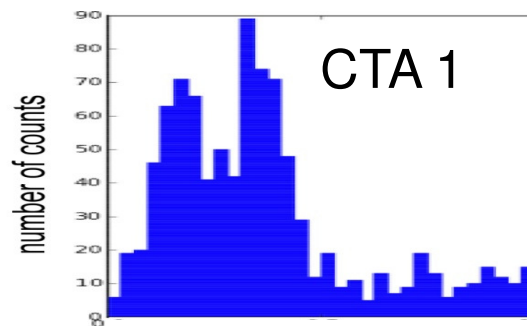
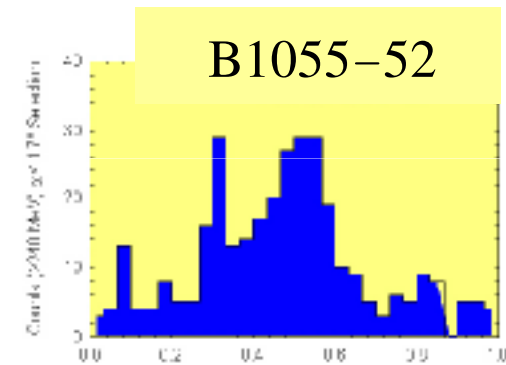
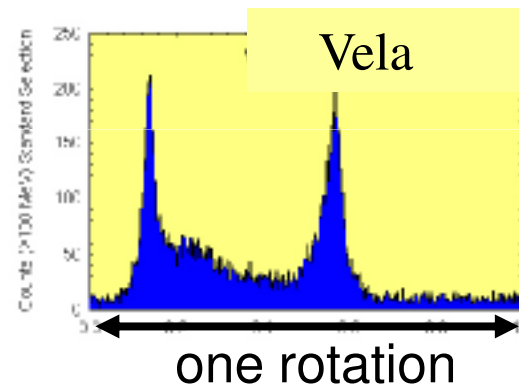
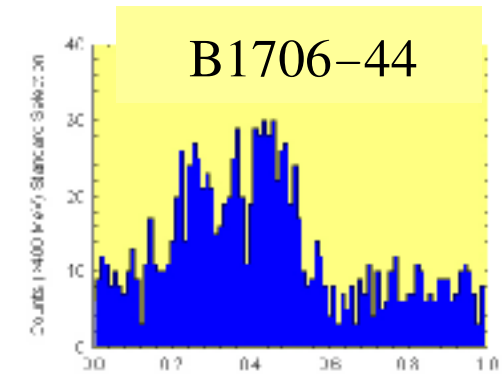
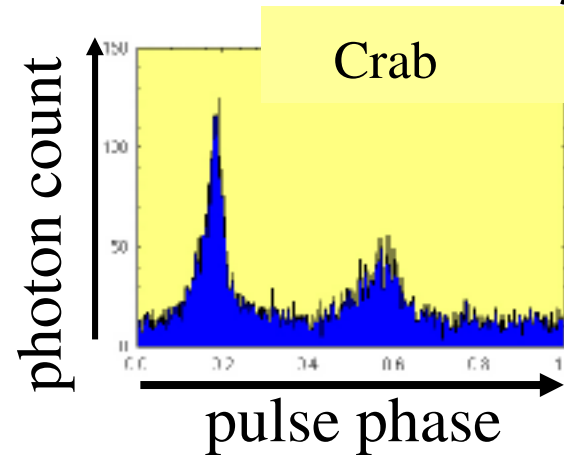
§1 Introduction: The γ -ray sky

Fermi/LAT is finding more and more **nearby γ -ray pulsars**.



§1 γ -ray Observations of Pulsars

Double-peak pulse profile is common.
(34 among 46 LAT pulsars)



LAT (Abdo+ 08)

EGRET (Thompson 01, Kanbach 02)

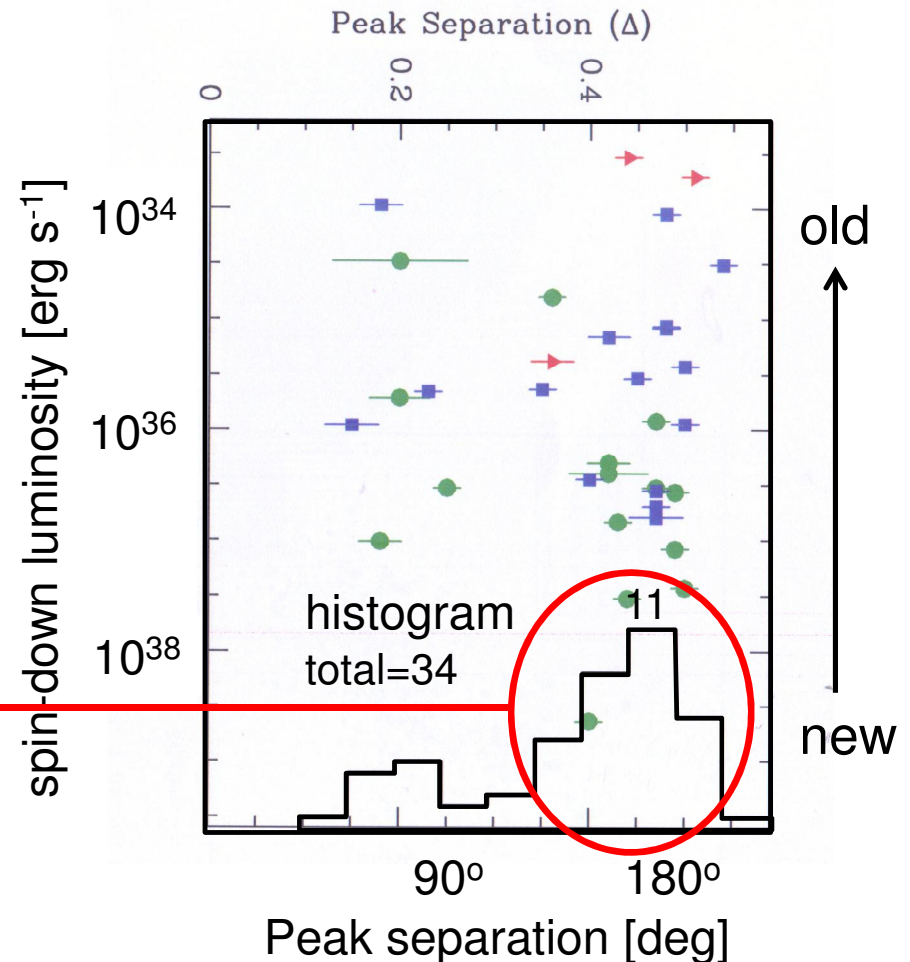
§1 γ -ray Observations of Pulsars

Double-peak pulse profile is common.

(34 among 46 LAT pulsars)

Peak separation is typically between 100° and 180° .

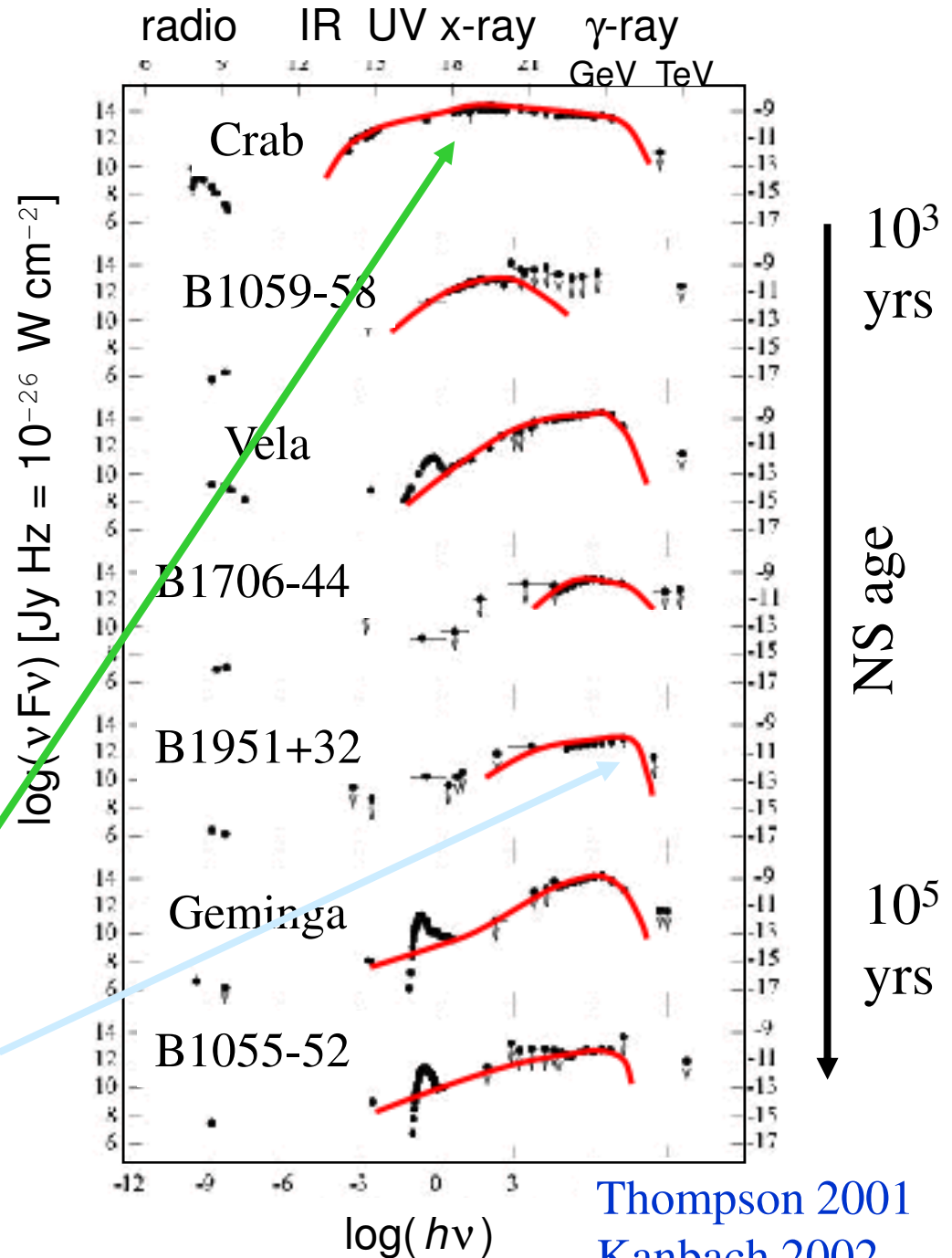
Peak separation has no strong dependence on pulsar age (i.e., E_{spin}).



1st LAT catalog (Abdo+ 2009)

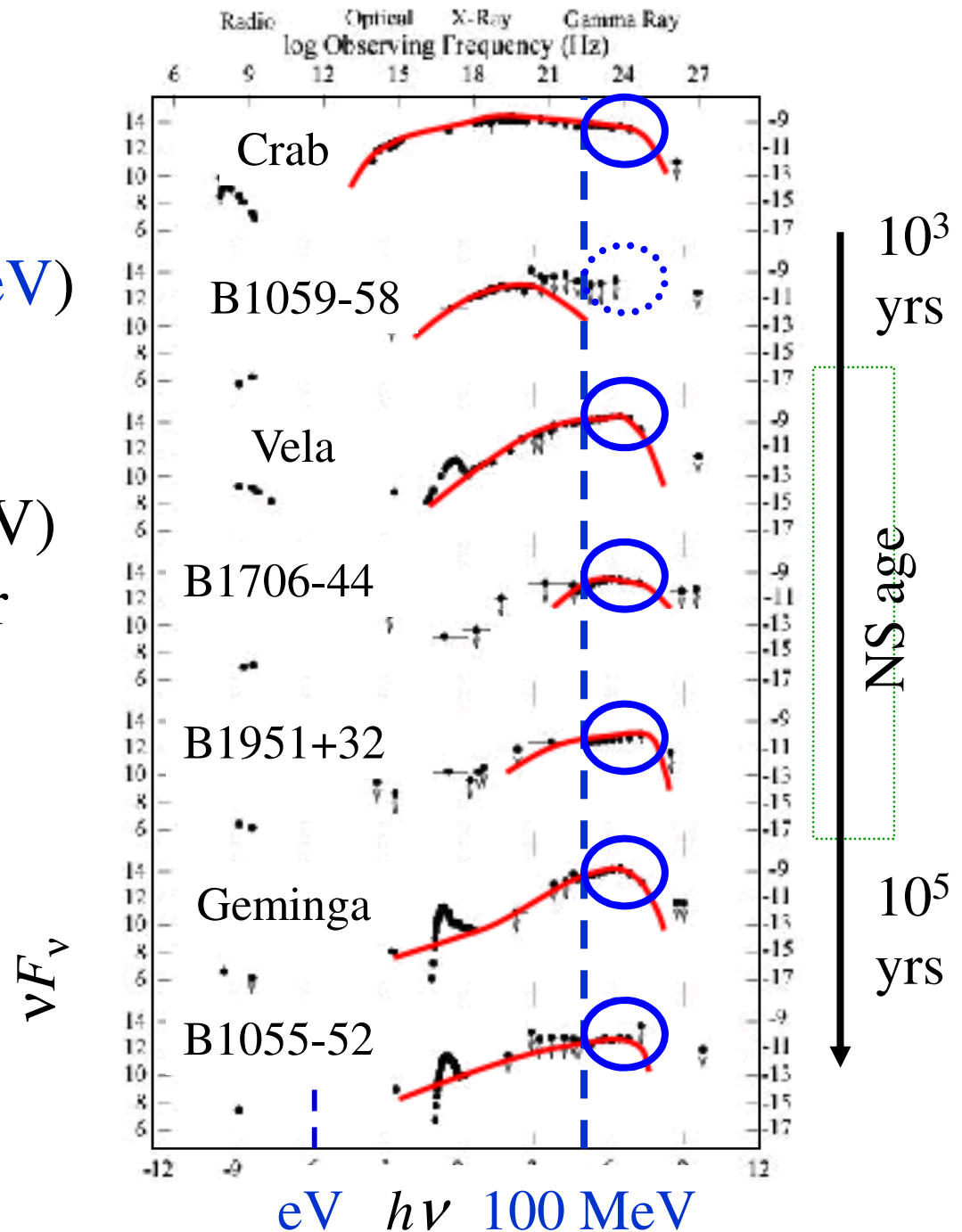
Broad-band spectra (pulsed)

- Power peaks in γ -rays
- No pulsed emission above 50 GeV
- High-energy turnover
- Spectrum gets harder as the NS ages. E.g., the **Crab** pulsar shows very soft γ -ray spectrum.
- B1951+32 shows the hardest spectrum.



Broad-band spectra (pulsed)

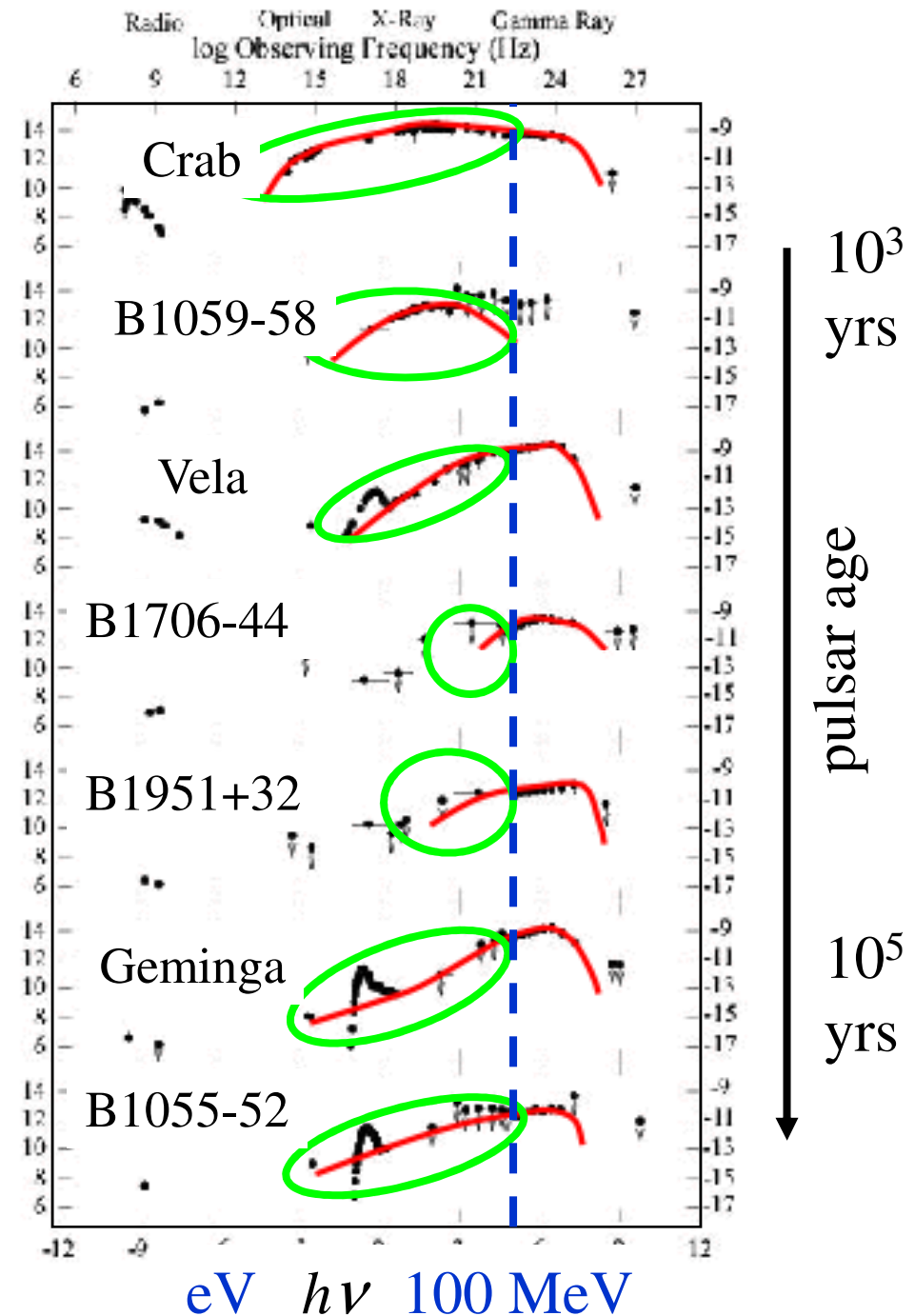
- High-energy ($> 100\text{MeV}$) photons are emitted via **curvature process** by **ultra-relativistic** ($\sim 10\text{TeV}$) e^\pm 's accelerated in pulsar magnetosphere.



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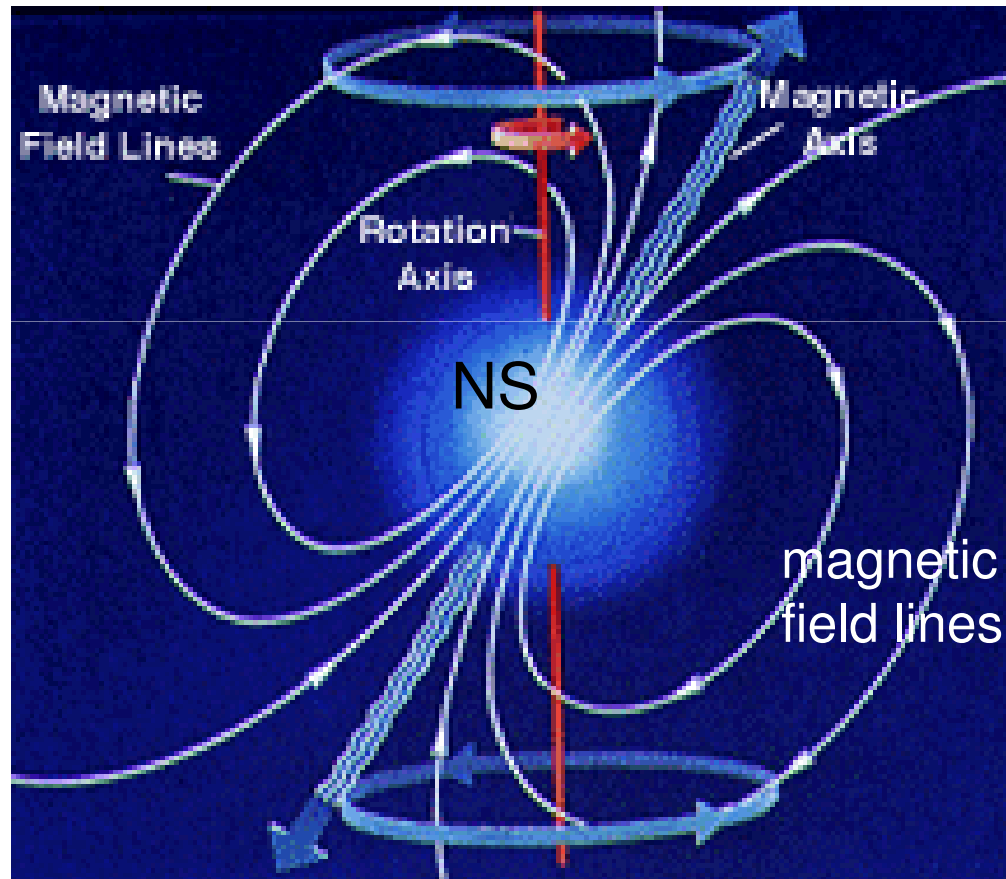
- Some of such **primary γ -rays** are absorbed in the NS magnetosphere and **reprocessed** in lower energies via **synchrotron** process.



§1 Introduction

Pulsars:

rapidly rotating, highly magnetized neutrons stars (NS)



Magnetic and rotation axes are misaligned.

Pulsars turn on and off as the beam sweeps our line of sight (e.g., lighthouse).



Double-peak light curve

§1 Introduction

Pulsar emissions result from electro-dynamical extraction of NS rotational energy.

(e.g., unipolar inductor)

The rotational energy loss rate:

$$\dot{E} = I\Omega\dot{\Omega} = -(2\pi)^2 I \dot{P} P^3,$$

where $\Omega \equiv 2\pi / P$: NS rotational angular freq.,

$I \sim 10^{45}$ g cm²: NS moment of inertia.

For typical high-energy pulsars,

$$P \sim 0.1 \text{ s}, \dot{P} \sim 10^{-13} \text{ s s}^{-1}, \text{ give } \left| \dot{E} \right| \sim 10^{33-38.7} \text{ ergs s}^{-1}$$

§1 Introduction

For such a small ($r_* \sim 10$ km) object to experience a large spin-down torque, it must have a strong coupling to their surroundings through ***B*** fields.

Spin-down luminosity: $L_{\text{spin}} = k \Omega^4 \mu^2 / c^3$.

For a dipole radiation, $k = 2 \sin^2 \alpha_i / 3$

(α_i : ***B*** inclination angle with respect to spin axis)

Equating L_{dip} and $-I\Omega(d\Omega/dt)$, we can infer ***B*** moment, μ .

For $P \sim 0.1$ s and $dP/dt \sim 10^{-13}$ s s⁻¹,

$\mu \sim 10^{30.5}$ G cm³ and $B_* \sim 10^{12.7}$ G.

§1 Introduction

$L_{\text{spin}} (< 10^{39} \text{ erg s}^{-1})$ is dissipated at ...

- Inner magnetosphere

($r < 3r_* \sim 30 \text{ km}$)

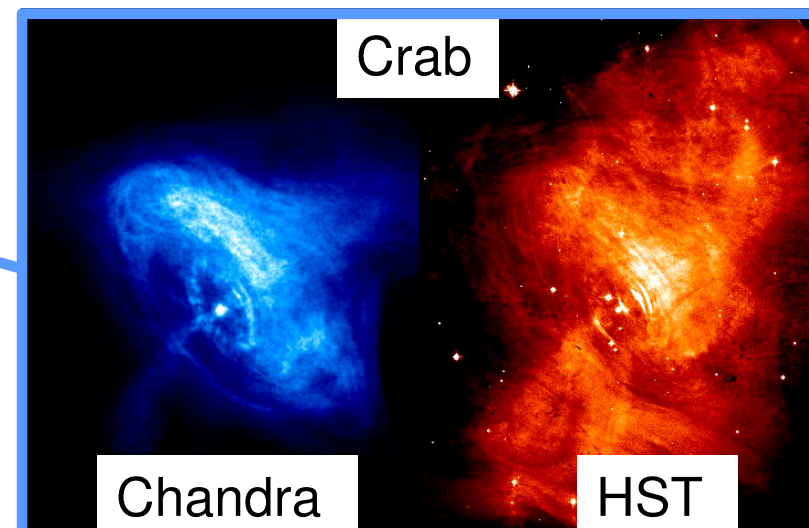
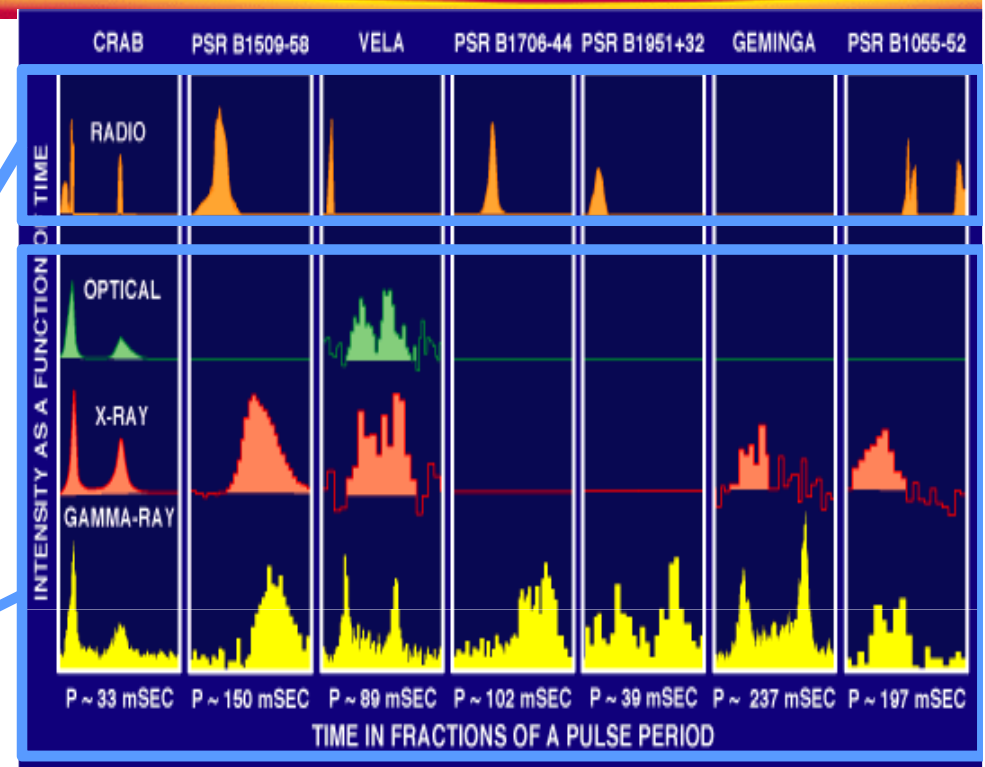
- Outer magnetosphere

($r \sim 1000 \text{ km}$)

- Wind region

($r \sim \text{pc}$)

80-99.9% of L_{spin} to be converted into e^\pm 's' E_{kin} .



§1 Introduction

$L_{\text{spin}} (< 10^{39} \text{ erg s}^{-1})$ is
dissipated at ...

pair energy @ rate
 $\gamma m_e c^2$ e^\pm 's s^{-1}

- Inner magnetosphere $\sim 0.001\%$ $< \text{GeV}$ $\sim 10^{40}$
($r < 3r_* \sim 30 \text{ km}$) \rightarrow pulsed radio \rightarrow wind e^\pm 's
- Outer magnetosphere $0.1\text{--}20\%$ $< 10 \text{ TeV}$ $\sim 10^{36}$
($r \sim 1000 \text{ km}$) \rightarrow pulsed X, γ -ray
- Wind region $99.9\text{--}80\%$ $> 10 \text{ GeV}$
($r \sim \text{pc}$) \rightarrow PWN emission
re-accelerated e^\pm 's into ISM

§1 Introduction

To interpret PAMELA and
ATIC results ...

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Nevertheless, investigating a **self-consistent treatment** of e^\pm creation/acceleration in pulsar magnetosphere, will help us understand an astrophysical origin of pair plasmas.
 \rightarrow Consider e^\pm creation/acceleration in outer magnetosphere.

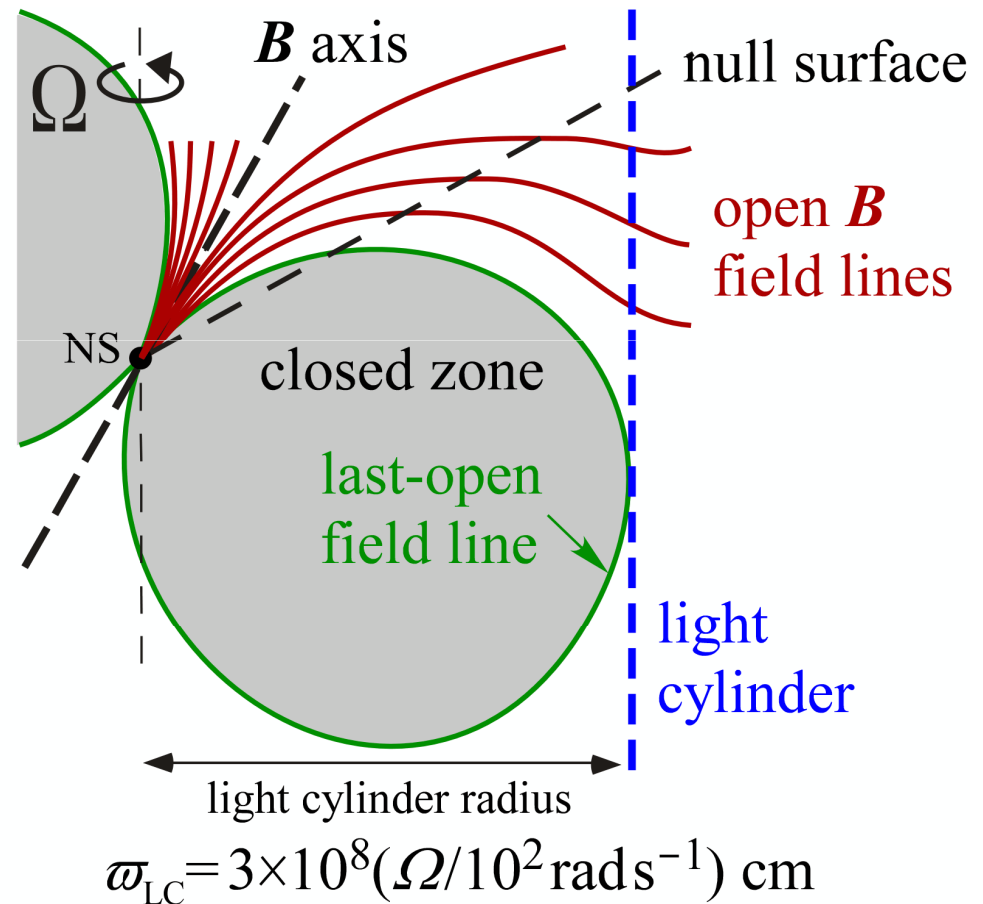
§2 Basic Emission Mechanism

A rotating NS magnetosphere can be divided into **open** and **closed zones**.

Last-open field lines form the boundary of them.

In the open zone, e^\pm 's escape through the **light cylinder** as a pulsar wind.

In the closed zone, on the other hand, an E_\parallel would be very quickly screened by the dense plasmas.



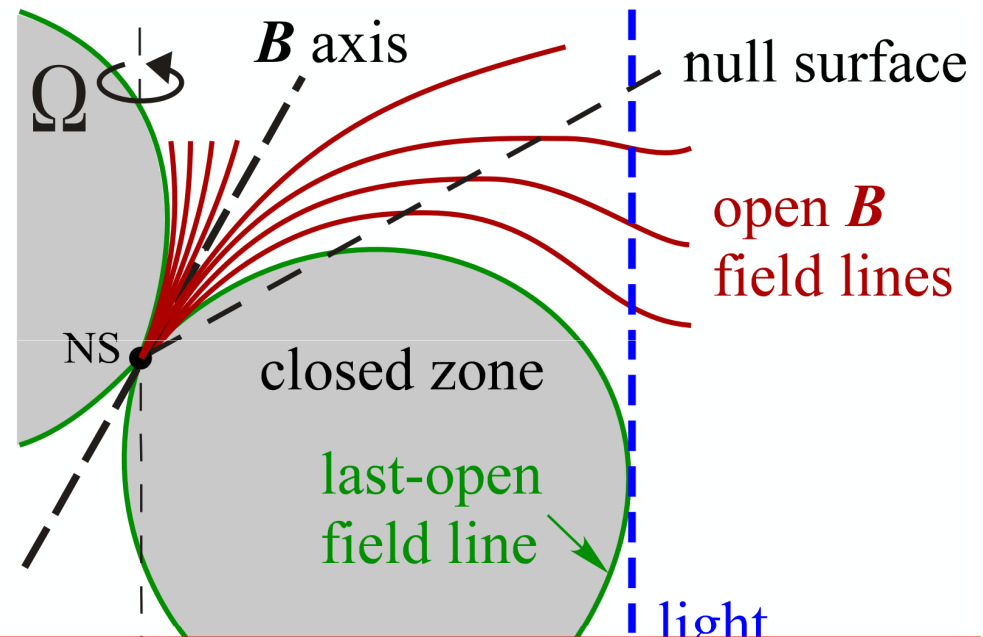
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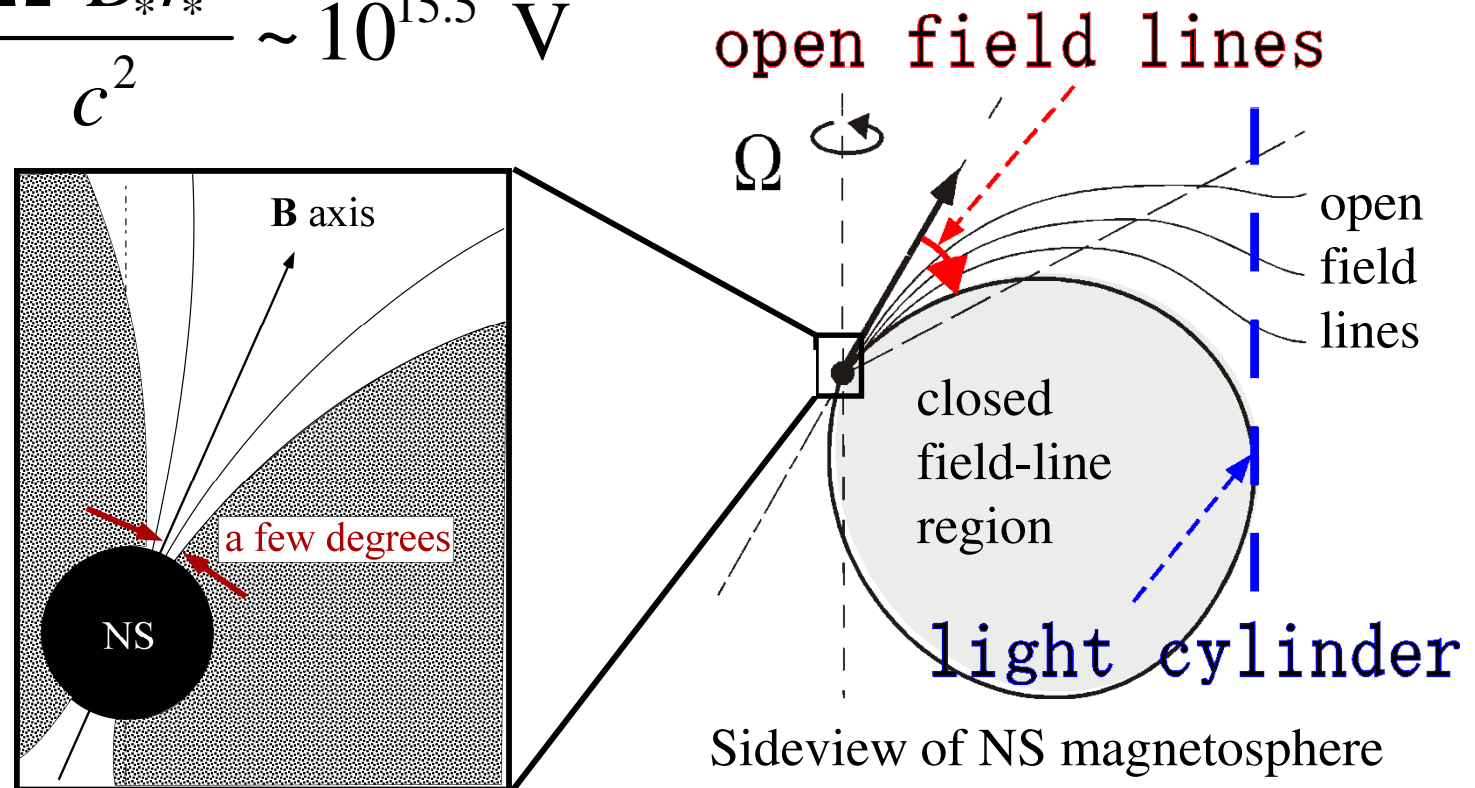
Thus, in all pulsar emission models, particle acceleration takes place only within the open zone.

§2 Basic Emission Mechanism

For typical high-energy pulsars, **open zone** occupies only a few degrees from ***B*** axis on the PC surface.

Available voltage in the open zone:

$$\text{EMF} \sim \frac{\Omega^2 B_* r_*^3}{c^2} \sim 10^{15.5} \text{ V}$$



§2 Basic Emission Mechanism

In a rotating NS magnetosphere, the **Goldreich-Julian charge density** is induced for a static observer. The inhomogeneous part of Maxwell eqs. give

$$\nabla \cdot \mathbf{E}_{\parallel} = 4\pi(\rho - \rho_{\text{GJ}}),$$

where $E_{\parallel} \equiv \mathbf{E} \cdot \mathbf{B}$, $\rho \equiv e(n_+ - n_-)$ and

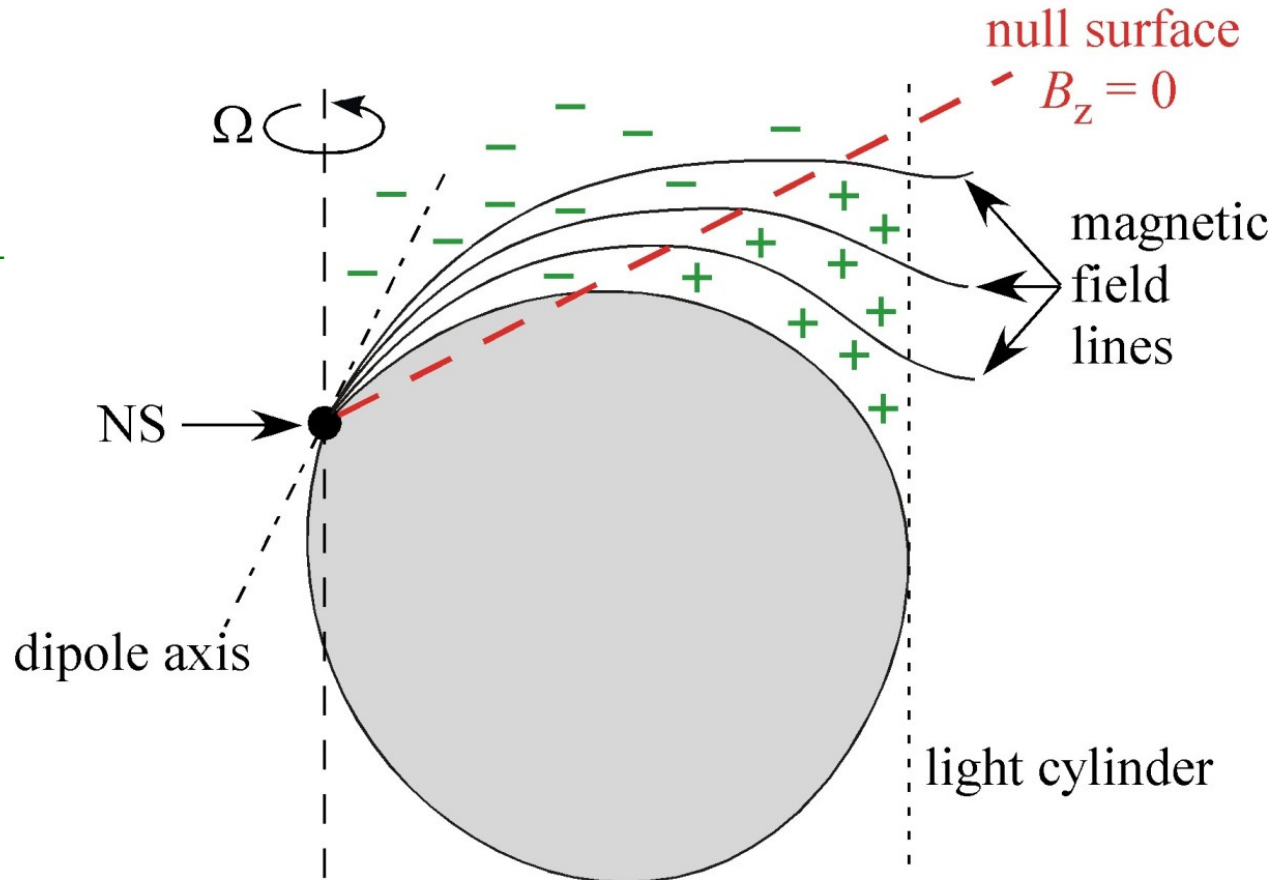
$$\rho_{\text{GJ}} \equiv \frac{1}{4\pi} \nabla \cdot \mathbf{E}_{\perp} = -\frac{\boldsymbol{\Omega} \cdot \mathbf{B}}{2\pi c}.$$

It follows that E_{\parallel} arises if $\rho \neq \rho_{\text{GJ}}$.

Note that ρ_{GJ} is uniquely determined by B-field geometry. For example, it changes at the so-called ‘null-charge surface’.

§2 Basic Emission Mechanism

$$\begin{aligned}\rho_{\text{GJ}} &\equiv \frac{1}{4\pi} \nabla \cdot \mathbf{E}_\perp \\ &= -\frac{\boldsymbol{\Omega} \cdot \mathbf{B}}{2\pi c}.\end{aligned}$$



Note that ρ_{GJ} is uniquely determined by B-field geometry. For example, it changes at the so-called ‘null-charge surface’.

§3 *Emission Models*

Next question:

Where is the particle accelerator, in which E_{\parallel} arises?

In this section, we geometrically consider three representative pulsar **high-energy emission models**:

(historical order)

1. Inner-gap (or polar-cap) model, 1982-1990's
2. Outer-gap model, 1986-present
3. Slot-gap model 2003-2009

Note: Inner-gap model still survives as the only theory that explains coherent **radio** pulsations.

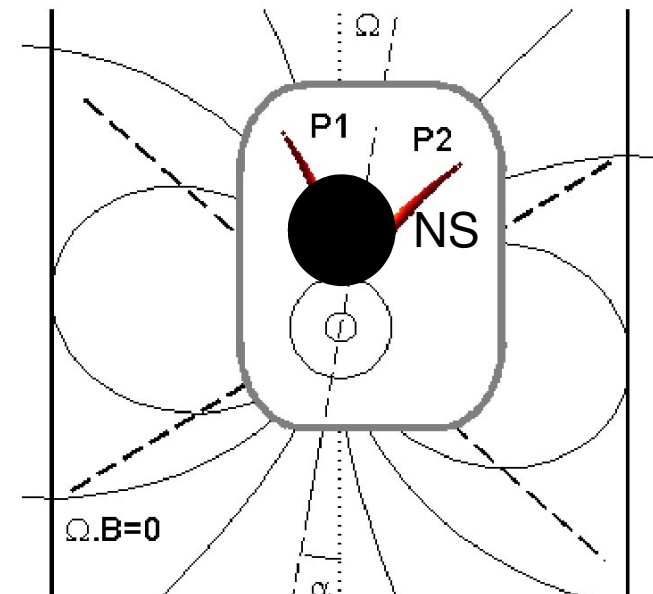
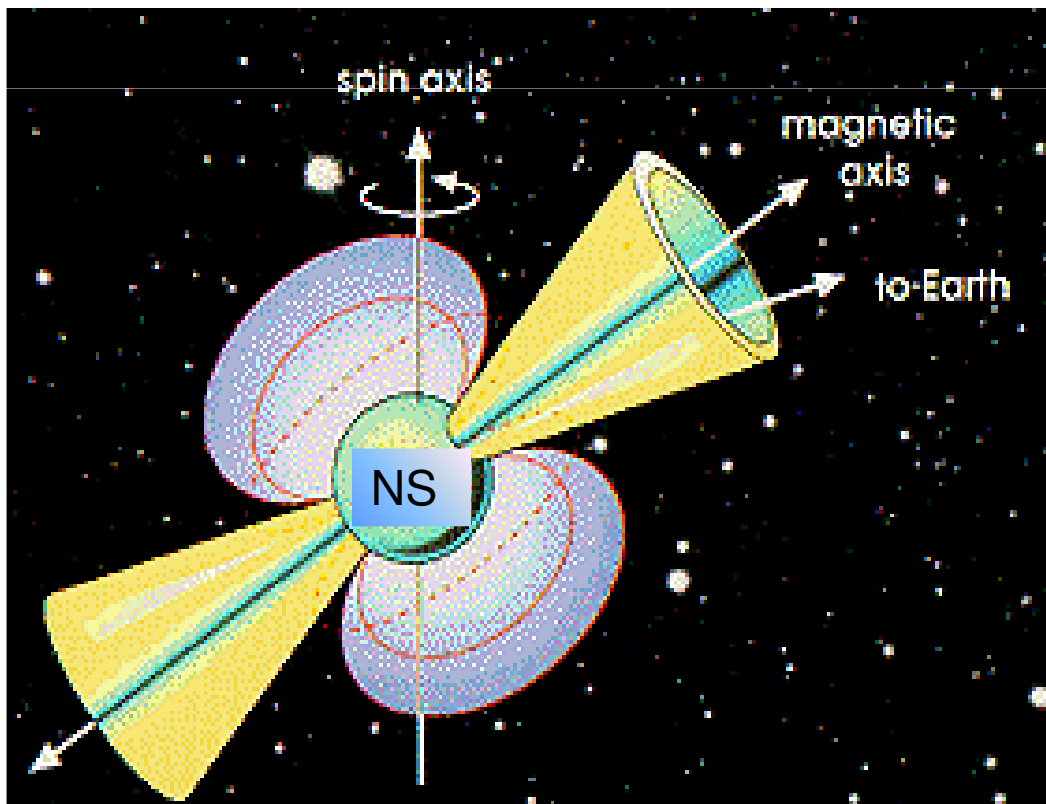
§3 Emission Models

Early 80's, **inner-gap model** was proposed.

(Daugherty & Harding ApJ 252, 337, 1982)

Emission altitude $< 3r_{\text{NS}}$ \longrightarrow pencil beam ($\Delta\Omega \ll 1$ ster)

Difficult to explain wide-separated double peaks
(in X-ray, γ -ray)



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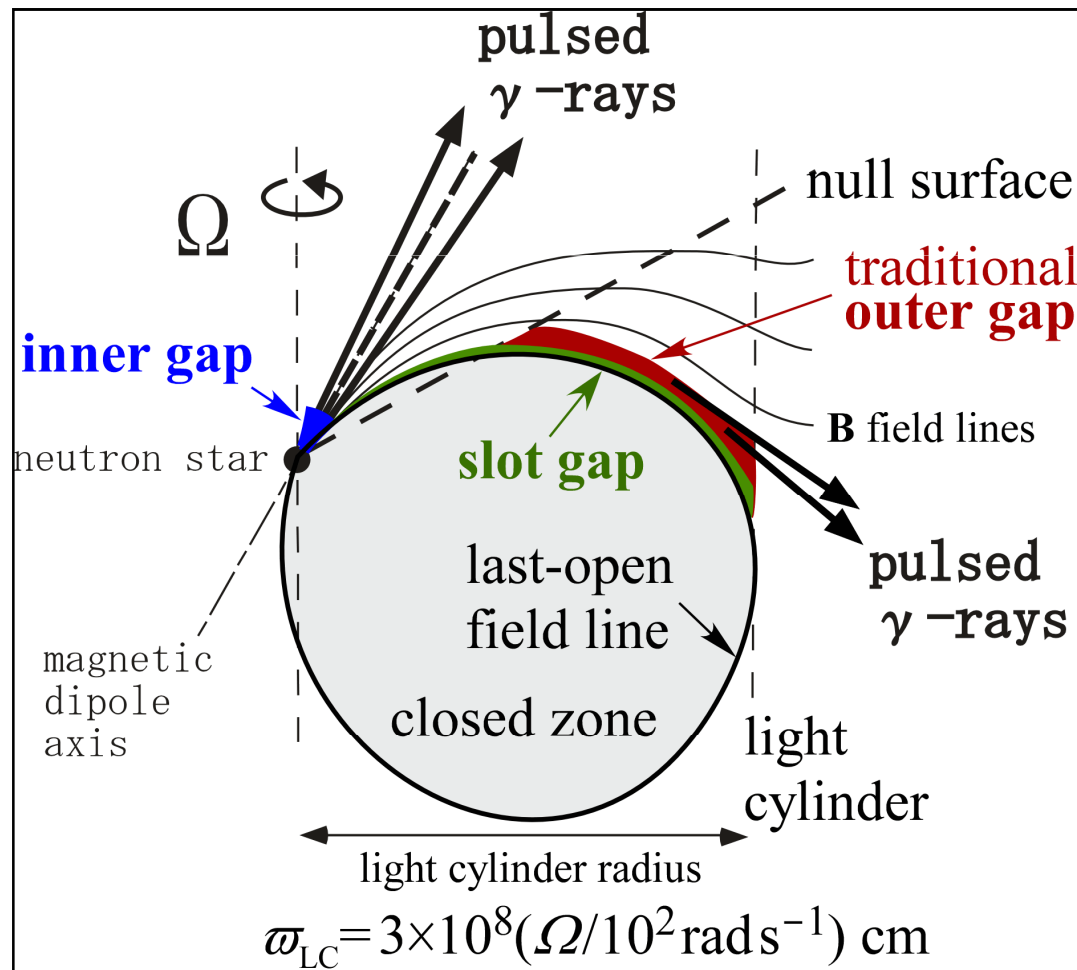
Thus, a **high-altitude emission** drew attention.

§3 Emission Models

Mid 80's, **outer-gap model** was proposed.

(Cheng, Ho, Ruderman ApJ 300, 500, 1986)

Emission altitude $> 100 r_{\text{NS}}$ \longrightarrow hollow cone emission
($\Delta\Omega \sim 1$ ster)



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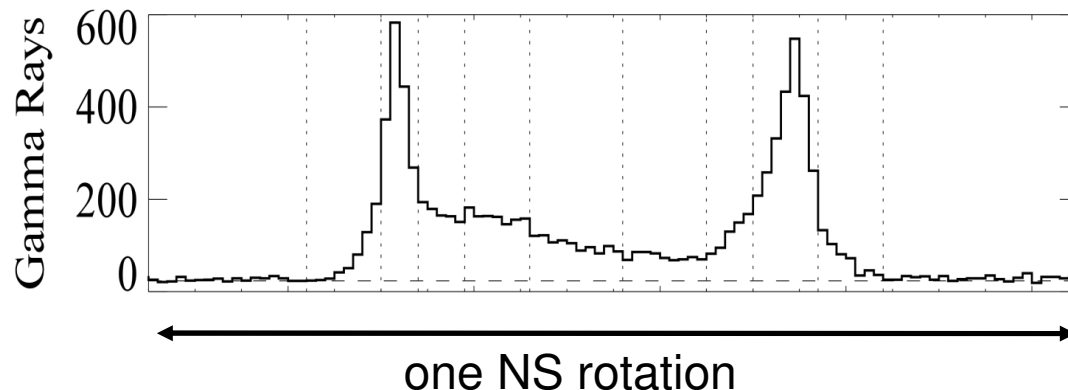
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Mid 90s', the outer-gap model was further developed by taking account of **special relativistic effects**.

(Romani ApJ 470, 469)

\longrightarrow Explains wide-separated double peaks.

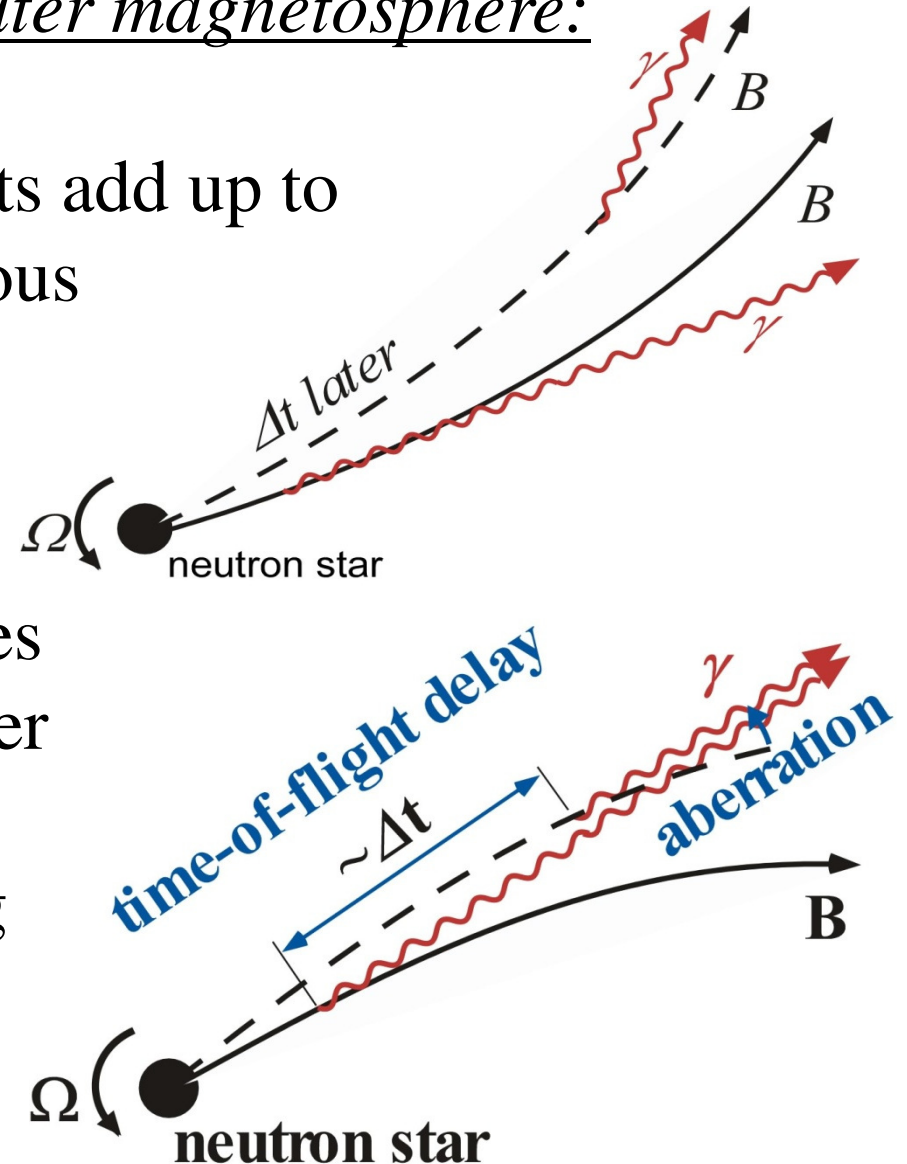


§3 Emission Models

Special relativistic effects in outer magnetosphere:

On the **leading** side, phase shifts add up to **spread** photons emitted at various altitudes over 140° in phase.

On the **trailing** side, photons emitted earlier at lower altitudes catch up with those emitted later at higher altitudes to **focus** in a small phase range 30° , forming caustics (strong intensity) in the phase plot.



§3 Emission Models

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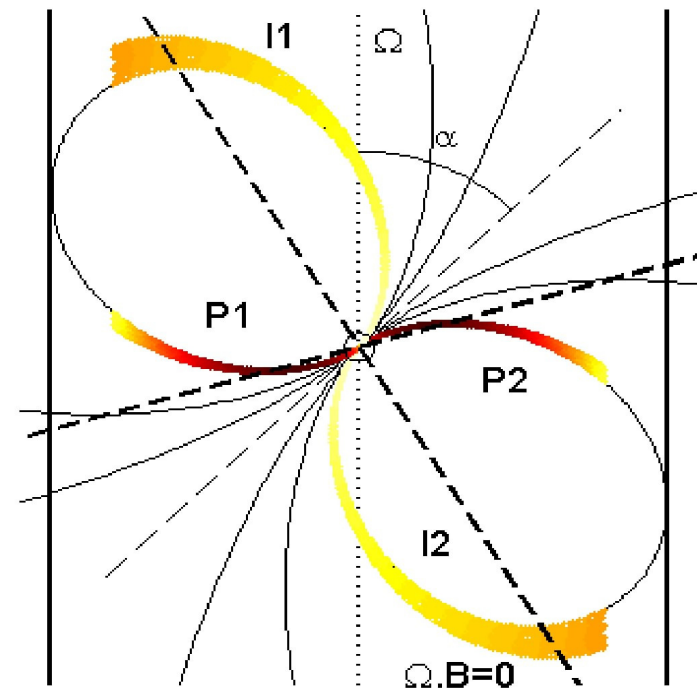
Outer-gap model became promising.

§3 Emission Models

Early 00's, an alternative mode, **slot-gap model**, was proposed. (Muslimov & Harding ApJ 588, 430, 2003)

They revisited the original idea of Arons (1983), extending his lower-altitude slot-gap model into the higher altitudes (by hand).

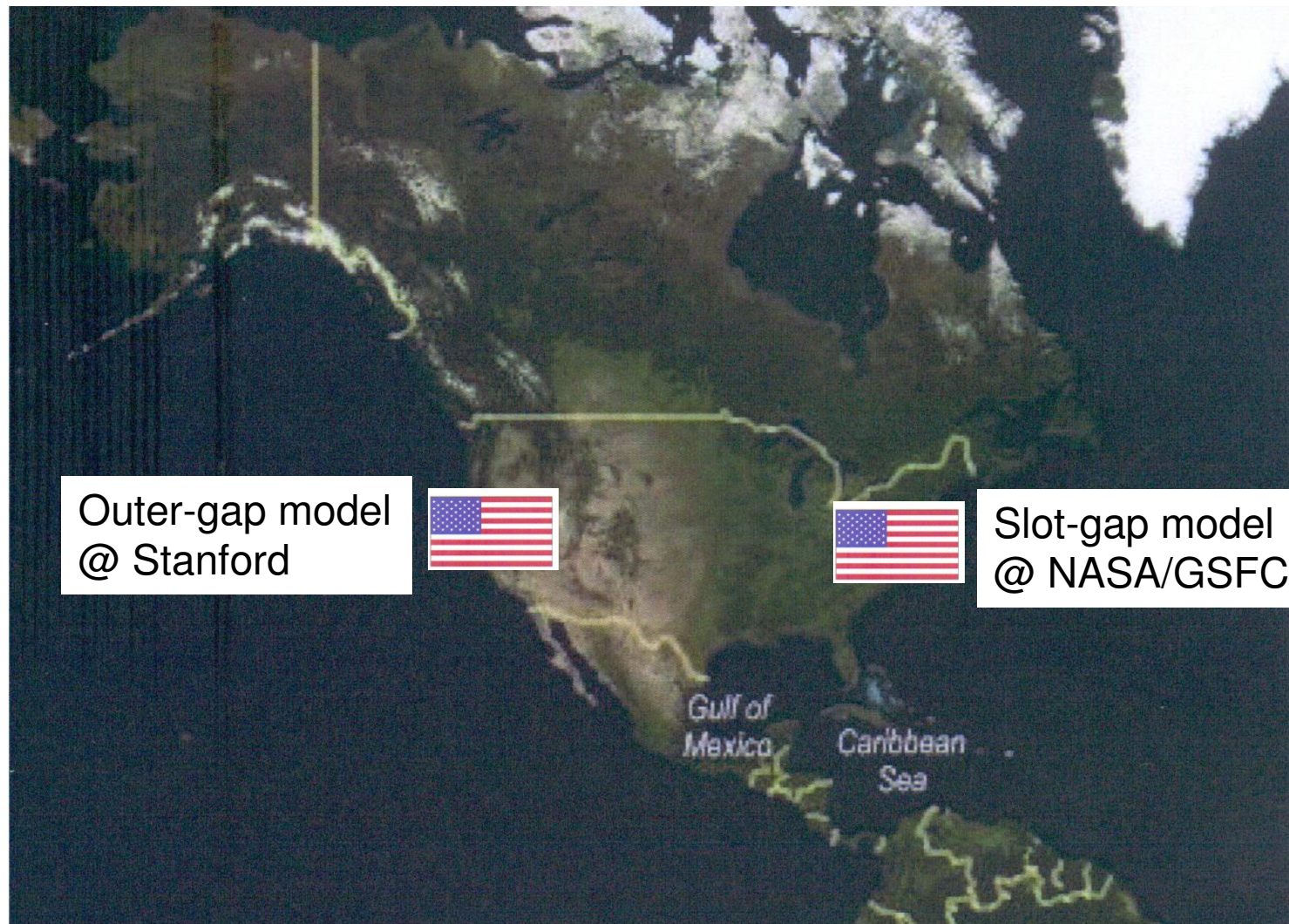
Due to special relativistic effects, wide-separated double peaks also appear, in the same way as in the outer-gap model (although the peak formation mechanism is slightly different).



§3 Emission Models

Early 00's, two models competed:

Outer-gap model vs. Slot-gap model



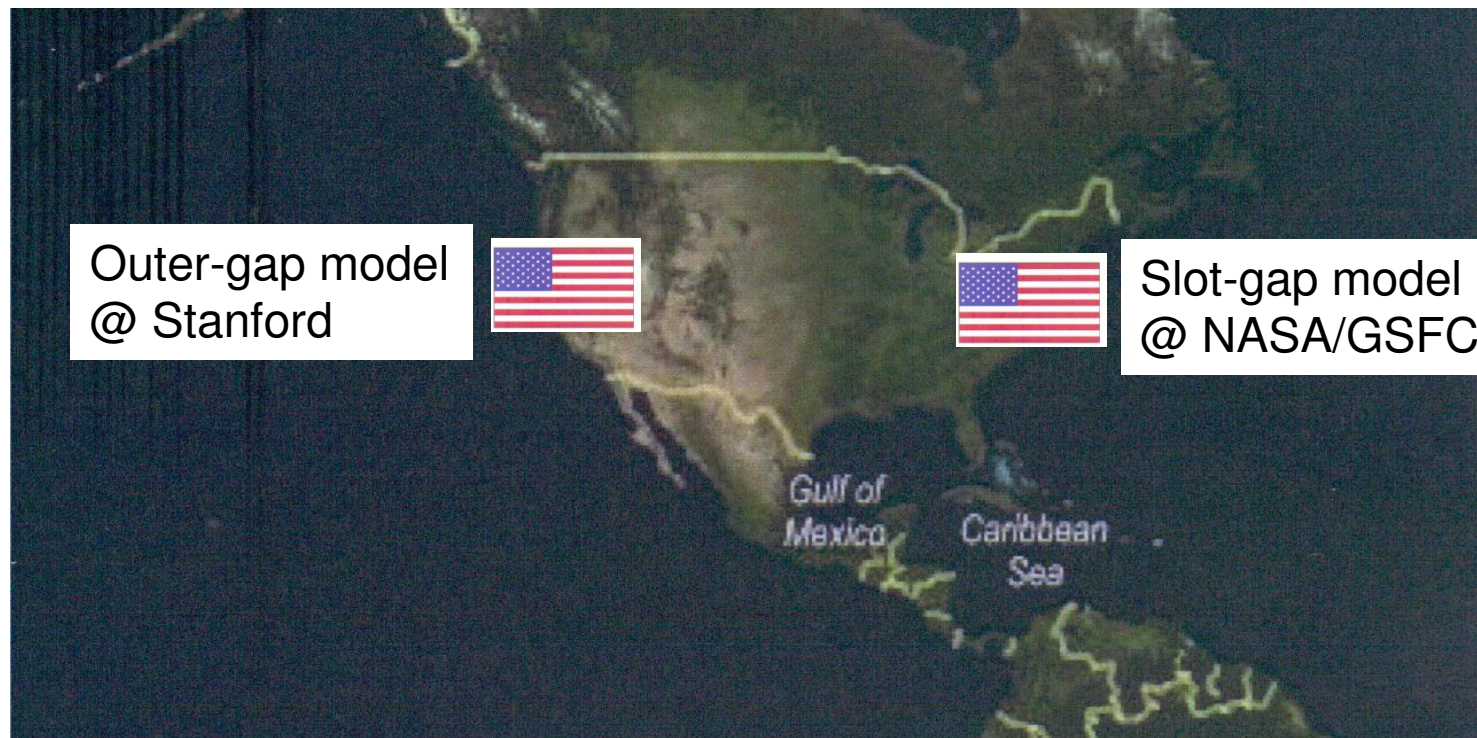
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A key science program of **Fermi** ():

To **discriminate the two models** by detailed observations.



§3 Emission Models

Early 00's, two models competed:

Outer-gap model vs. Slot-gap model

A key science program of **Fermi**:

To **discriminate the two models** by detailed observations.

However, we can **rule out the slot-gap model by theoretical consideration**, before comparing with observations.

(Hirotani ApJ 688, L25, 2008)

(Hirotani Open Astron. in press, 2009)

§4 Problems in Slot-gap model

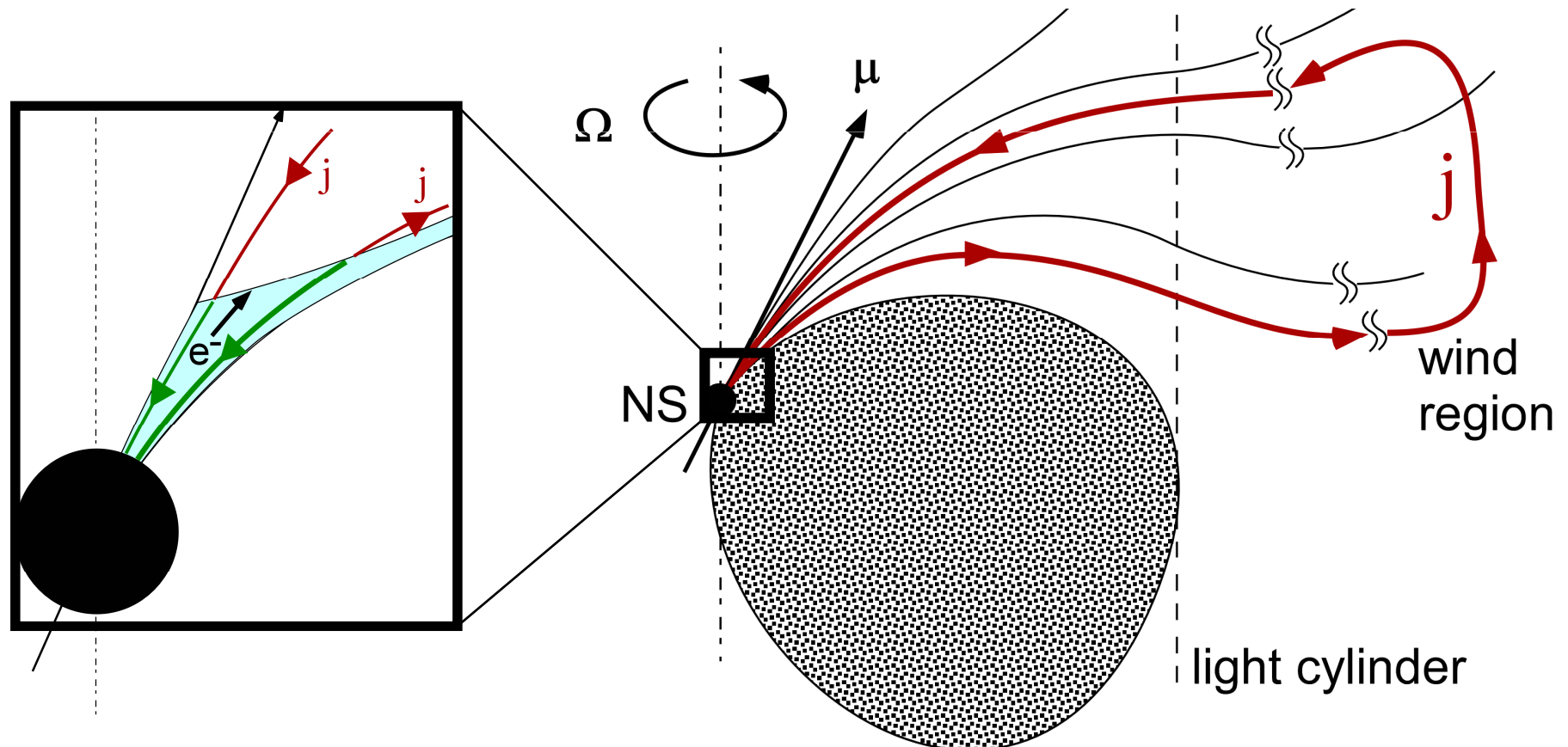
Serious electro-dynamical problems in the SG model:

- (1) Electric current closure,
- (2) Unphysical assumption of the Goldreich-Julian charge density,
- (3) Over-estimated electron Lorentz factors,
- (4) Insufficient γ -ray luminosity

§4. Problems in Slot-gap model

(1) Current closure:

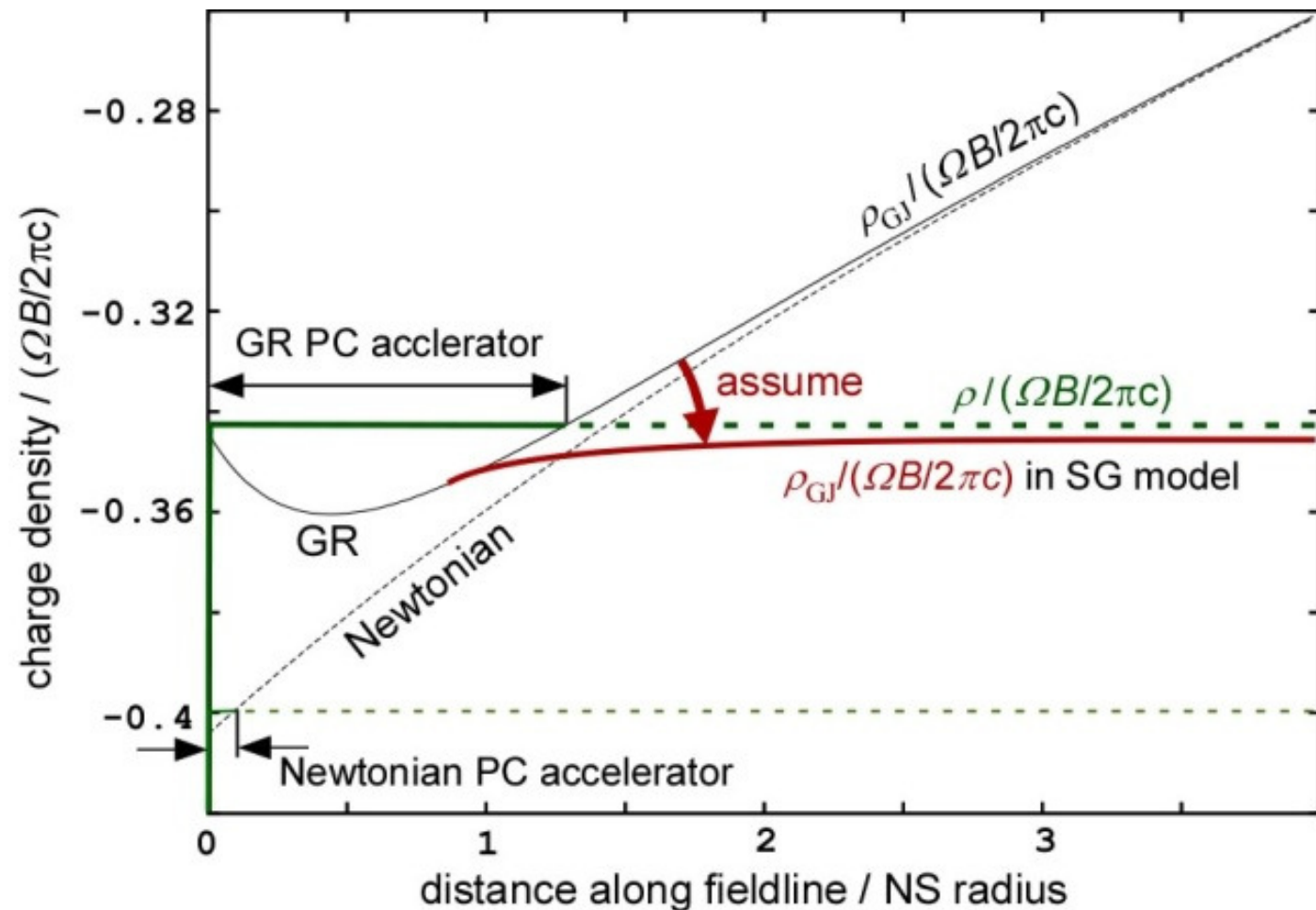
SG model (outward extension of IG model) predicts a negative E_{\parallel} when $\mathbf{\Omega} \cdot \mathbf{\mu} > 0$. $E_{\parallel} < 0$ induces an **opposite** gap **current** from the global current flow patterns.



§4. Problems in Slot-gap model

(2) Artificial Goldreich-Julian charge density:

Unphysical GJ charge density is assumed in the higher altitudes.

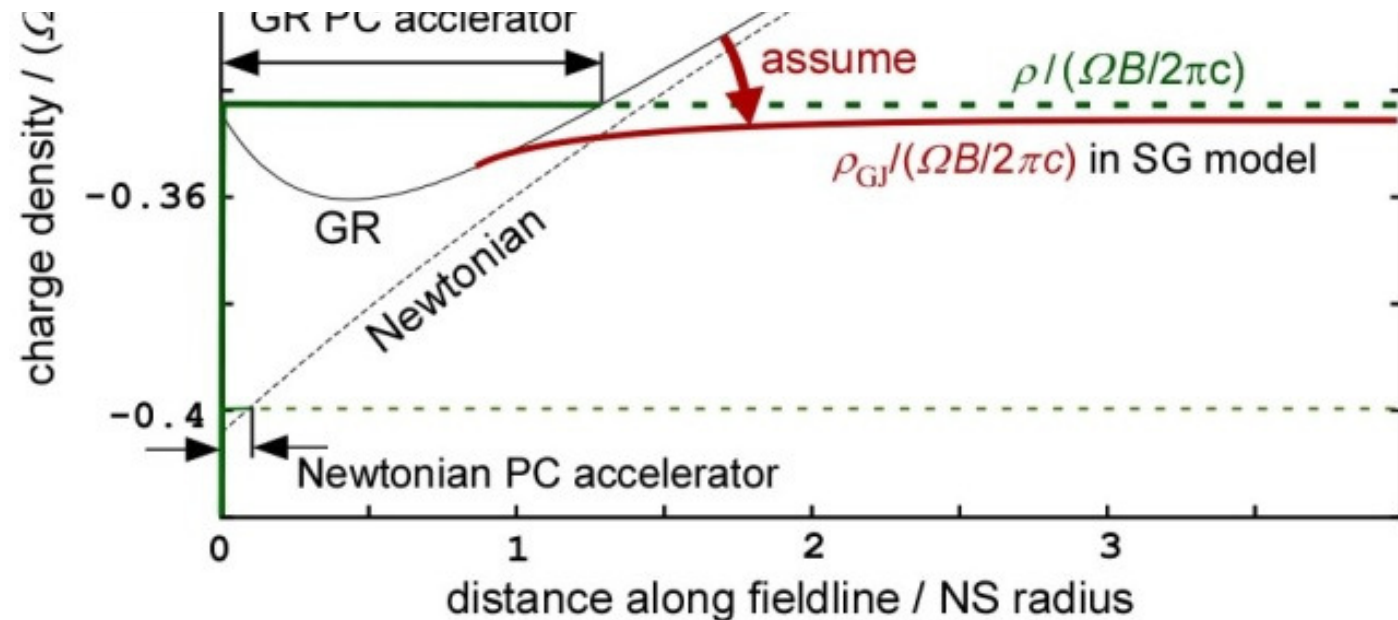


§4. Problems in Slot-gap model

(2) Artificial Goldreich-Julian charge density:

Unphysical GJ charge density is assumed in the higher altitudes. Note that ρ_{GJ} is geometrically determined.

$$\rho_{\text{GJ}} = \frac{c^2}{4\pi\sqrt{-g}} \partial_\mu \left(\frac{\sqrt{-g}}{\rho_w^2} g^{\mu\nu} g_{\phi\phi} (\Omega - \omega) F_{\phi\nu} \right).$$



§4. Problems in Slot-gap model

(3) Systematically over-estimated electrons' Lorentz factors:

Muslimov & Harding (2003, ApJ 588, 430)

$$\Delta\Phi \approx 1.5 \times 10^{12} \left[1 - \frac{3}{8}(1 + 7\delta) \right] B_{12}^{\delta} \left(\frac{I_{45}}{\lambda^8} \right)^{1/7} \mathcal{F} \text{ V}, \quad (22)$$

By subs $1.5 \times 10^{12} / 0.5 \times 10^6$ (21) into equation (22), we can get the characteristic Lorentz factor of primary electrons, $\gamma = e\Delta\Phi/m_e c^2$,

$$\gamma \approx 4 \times 10^7 \left(\frac{I_{45}}{\lambda_{0.1}^8} \right)^{1/7} \left[1 - \frac{3}{8}(1 + 7\delta) \right] B_{12}^{\delta} \left(1 - \frac{1}{\eta_{\text{acc}}^3} \right), \quad (24)$$

$< 3 \times 10^6$

Lorentz factor: more than 13 times over-estimated.

→ L_{γ} is over-estimated more than 30,000 times.

§4. Problems in Slot-gap model

(3) Systematically over-estimated electrons' Lorentz factors:

Muslimov & Harding (2004, ApJ 606, 1143)

$$E_{\parallel, \text{high}} \approx -\frac{3}{8} \left(\frac{\Omega R}{c} \right)^3 \frac{B_0}{f(1)} \nu_{\text{SG}} \left\{ \left[1 + \frac{1}{3} \kappa \left(5 - \frac{8}{\eta_c^3} \right) + 2 \frac{\eta}{\eta_{\text{lc}}} \right] \right. \\ \left. \right\} (1 - \xi_*^2). \quad (53)$$

$$\gamma = \left(\frac{3}{2} \frac{\rho_c^2}{e} E_{\parallel} \right)^{1/4}$$

By using equation (53) in the above equation, we arrive at

$$\gamma \sim 3 \times 10^7 \left[(2.5 + 0.6 \kappa_{0.15}) B_{12} \frac{R_6^3}{P_{0.1}} \frac{\eta}{\eta_{\text{lc}}} \right]^{1/4}$$

$$\sim 1.35 \times 10^7$$

L_{γ} is over-estimated ~ 25 times.

(56)

§4. Problems in Slot-gap model

(3) Systematically over-estimated electrons' Lorentz factors:

Harding et al. (2008, ApJ 680, 1378)

HIGH-ALTITUDE EMISSION FROM PULSAR SLOT GAPS: THE CRAB PULSAR

Received 2007 November 30; accepted 2008 March 1

We present results of a 3D model of optically thin emission from the slot gap accelerator of a rotation-powered pulsar. Primary electrons accelerating to high altitudes in the slot gap reach radiation reaction limited Lorentz factors of $\sim 2 \times 10^7$, which flow along field lines interior to the slot gap. The curvature of primary electrons and pairs produce a broad spectrum of emission and pairs undergo cyclotron resonant absorption of radio pulses. Synchrotron radiation from pairs with a power-law spectrum up to ~ 10 MeV. Synchrotron emission from the slot gap is $\Phi_{\text{SG}} = \Phi_{\text{low}}^{\text{SG}}(\eta_c) + \Phi_{\text{high}}^{\text{SG}}$. We examine the energy-dependence of the slot gap emission as a function of magnetic field and acceleration parameter.

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Terminal Lorentz factor

$$\gamma = \left(\frac{3}{2} \frac{\rho_c^2}{e} E_{\parallel} \right)^{1/4} \quad (6)$$

L_{γ} is over-estimated ~ 40 times.

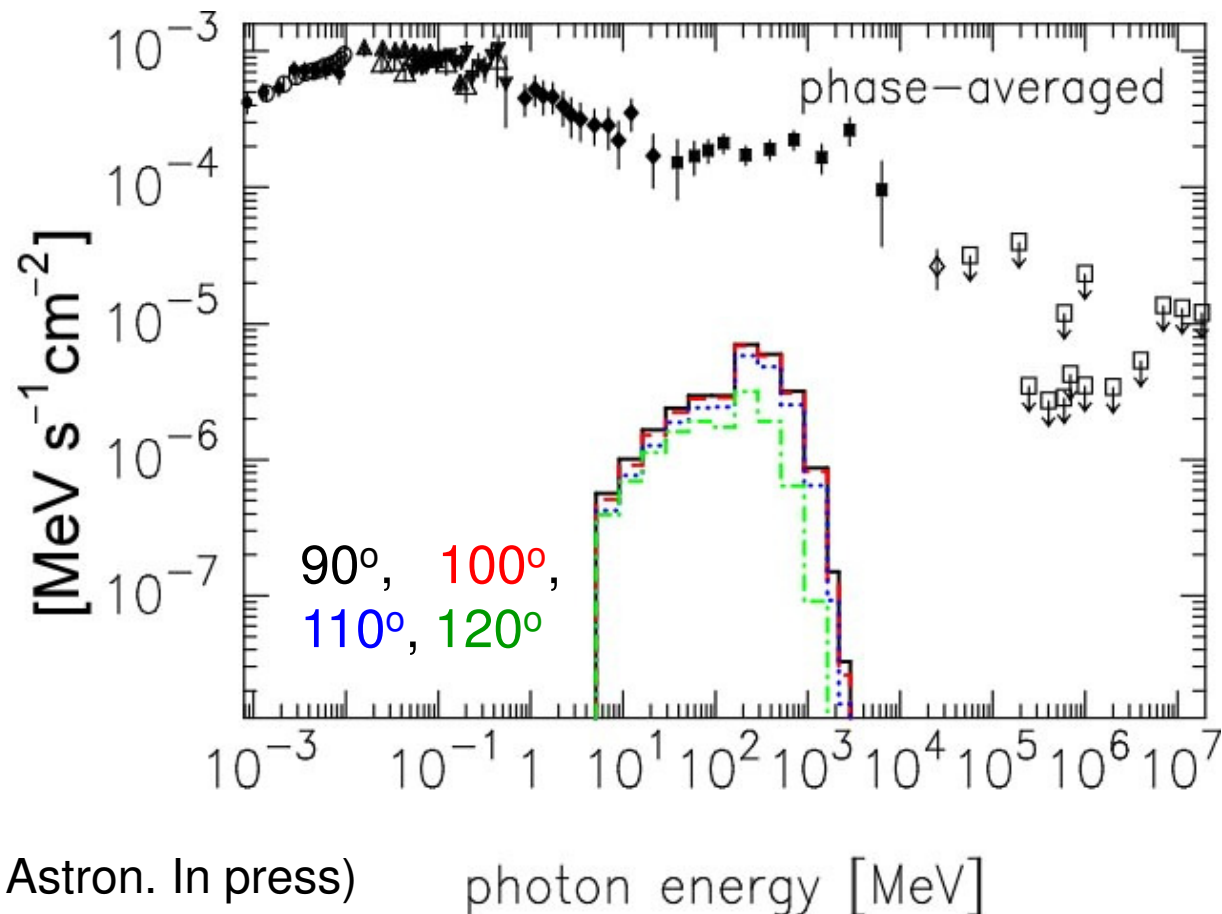
profiles, as well as the possibility of a model with $\alpha = 45^\circ$ and $\zeta \sim 100^\circ$ or 80° . This model predicts that the slot gap emission below 200 MeV will exhibit correlations in time and phase with the radio emission.

§4. Problems in Slot-gap model

(4) Insufficient γ -ray luminosity:

If we adopt the same parameter as Harding+('08), the predicted γ -ray flux is turned to be much less than the observed one.

Fig. Phase-averaged SG spectrum for four discrete viewing angles, 90° , 100° , 110° , and 120° .



Hirotsu (2009, Open Astron. In press)

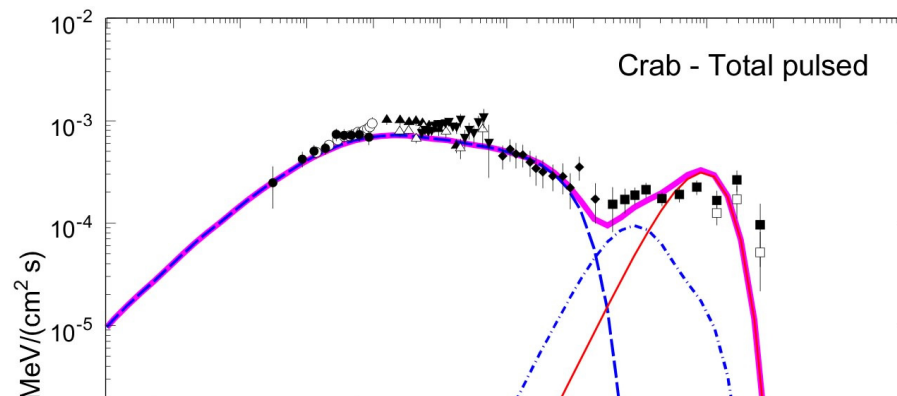
photon energy [MeV]

§4. Problems in Slot-gap model

(4) Insufficient γ -ray luminosity:

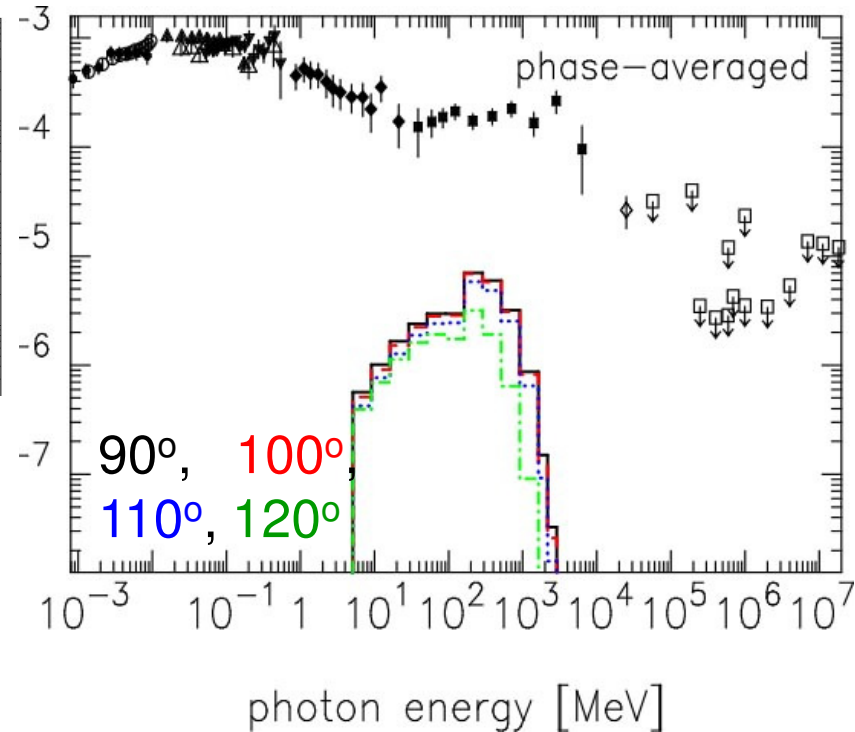
SG prediction with the same parameters:

Harding + (08)



L_γ was **overestimated** by the overestimated Lorentz factors and, very unfortunately, by a programming bug (priv. comm)

Hirotsu (09)



§4. Problems in Slot-gap model

(4) Insufficient γ -ray luminosity:

The same conclusion can be obtained analytically.

(Hirotani 2008, ApJ 688, L25)

Predicted γ -ray flux of the Crab pulsar:

$$(\nu F_\nu)_{\text{peak}} \approx 0.0450 f^3 \kappa \underbrace{\frac{\mu^2 \Omega^4}{c^3} \frac{1}{d^2}}_{\propto \dot{E}/d^2 : \text{spin-down flux}}, \quad \kappa \sim 1.$$

f : fractional gap width ($f \ll 1$ denotes a thin gap)

§4 Problems in Slot-gap model

$$(\nu F_\nu)_{\text{peak}} \approx 0.0450 f^3 \kappa \frac{\mu^2 \Omega^4}{c^3} \frac{1}{d^2}$$

Apply this general result to the Crab pulsar ($\Omega=190 \text{ rad s}^{-1}$).

Hirotani (2008) *ApJ* 688, L25

(I) For **OG** model ($f \sim 0.14$, $\kappa \sim 0.3$, $\mu = 4 \times 10^{30} \text{ G cm}^3$),

$$(\nu F_\nu)_{\text{peak}} \sim 4 \times 10^{-4} \text{ MeV s}^{-1} \text{ cm}^{-2} \sim \text{EGRET flux.}$$

(II) For **SG** model ($f \sim 0.04$, $\kappa \sim 0.2$), even with a large μ ,

$$(\nu F_\nu)_{\text{peak}} \sim 3 \times 10^{-5} (\mu / 8 \times 10^{30})^2 \text{ MeV s}^{-1} \text{ cm}^{-2} \\ < 0.1 \text{ EGRET flux.}$$

OG model remains as the only possible γ -ray pulsar model.

§4 Problems in Slot-gap model

We will confirm these analytical conclusions by numerical computations in the next section.

Apply this general result to the Crab pulsar ($\Omega=190 \text{ rad s}^{-1}$).

Hirotani (2008) *ApJ* 688, L25

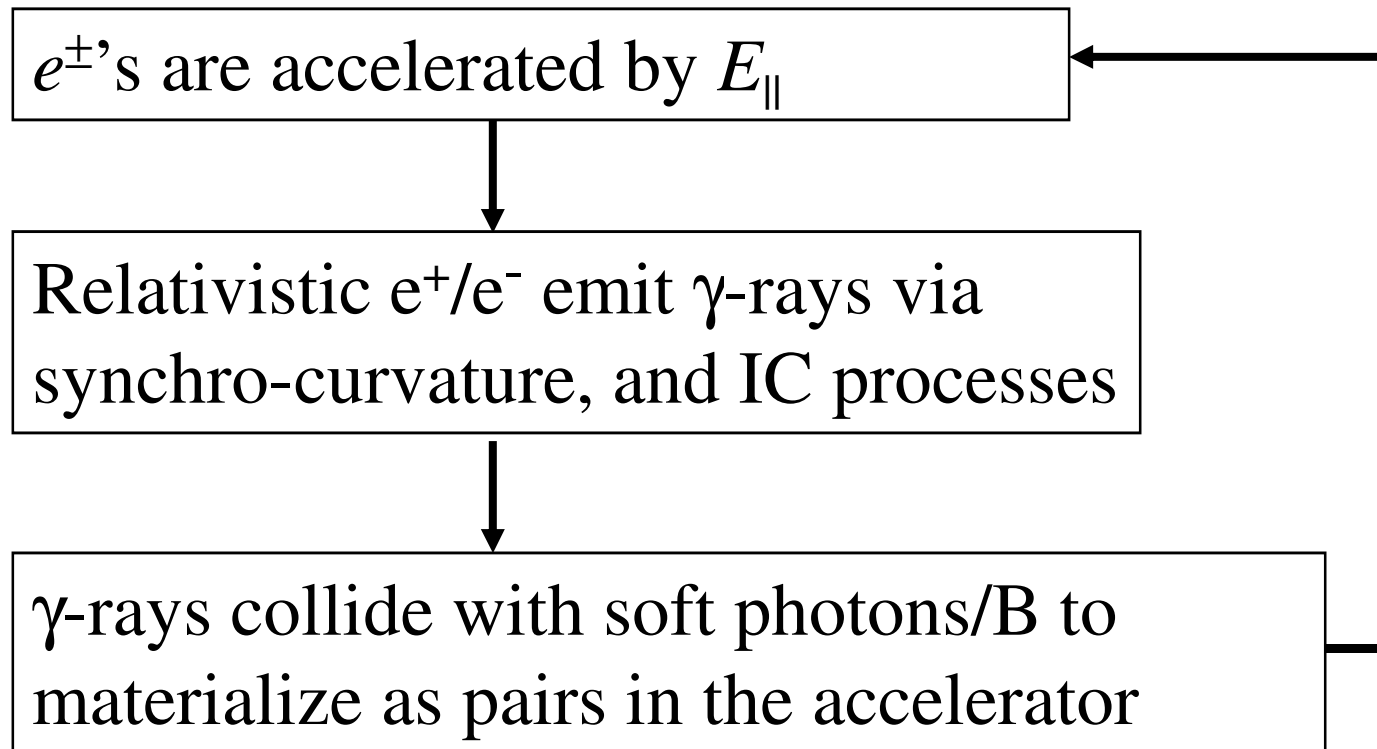
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 $< 0.1 \text{ EGRET flux}.$

OG model remains as the only possible γ -ray pulsar model.

§5 *New gap theory: Self-consistent approach*

Stationary, self-sustained pair-production cascade in a rotating NS magnetosphere:



§5 New gap theory: Self-consistent approach

The **Poisson equation** for the electrostatic potential ψ is given by

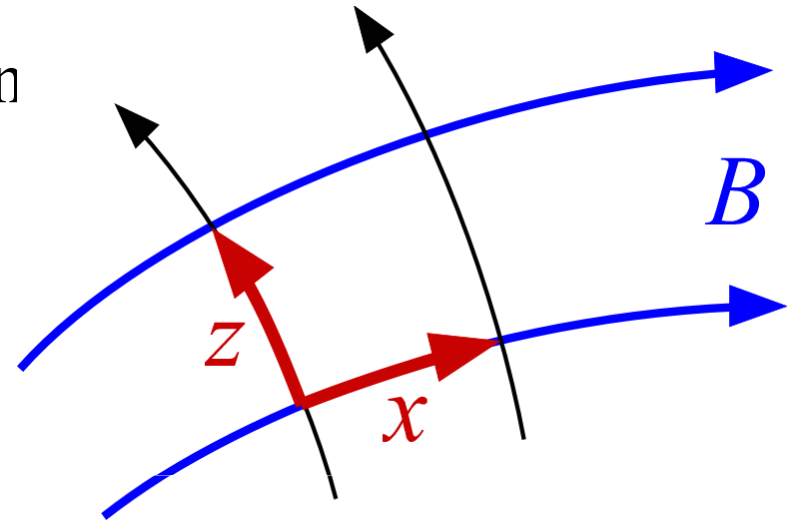
$$-\nabla^2\psi = 4\pi(\rho - \rho_{\text{GJ}}) ,$$

$$\text{where } E_{\parallel} \equiv -\frac{\partial\psi}{\partial x} ,$$

$$\rho \equiv e \int_0^{\infty} d\Gamma [N_+(x, z, \Gamma) - N_-(x, z, \Gamma)] + \rho_{\text{ion}} ,$$

$$\rho_{\text{GJ}} \equiv -\frac{\mathbf{\Omega} \cdot \mathbf{B}}{2\pi c} .$$

N_+/N_- : distrib. func. of e^+/e^-
 Γ : Lorentz factor of e^+/e^-



§5 New gap theory: Self-consistent approach

Assuming $\partial_t + \Omega \partial_\phi = 0$, we solve the e^\pm 's Boltzmann eqs.

$$\frac{\partial N_\pm}{\partial t} + \vec{v} \cdot \nabla N_\pm + \left(e \vec{E}_\parallel + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \frac{\partial N_\pm}{\partial \vec{p}} = S_{IC} + \int \alpha_\nu d\nu \int \frac{I_\nu}{h\nu} d\omega$$

together with the radiative transfer equation,

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

N_\pm : positronic/electronic spatial # density,

E_\parallel : magnetic-field-aligned electric field,

S_{IC} : ICS re-distribution function, $d\omega$: solid angle element,

I_ν : specific intensity, l : path length along the ray

α_ν : absorption coefficient, j_ν : emission coefficient

§5 New gap theory: Self-consistent approach

Specify the three parameters: (period, P , is known.)

- magnetic inclination (e.g., $\alpha_{\text{inc}}=45^\circ, 75^\circ$),
- magnetic dipole moment of NS (e.g., $\mu=4\times 10^{30}\text{Gcm}^3$)
- neutron-star surface temperature (e.g., $kT_{\text{NS}}=50\text{ eV}$)

Solve Poisson eq. + Boltzmann eqs. in **6-D** phase space (i.e., 3-D config. + 3-D mom. space) + RTE.

I first solved ([Hirotani '08, Open Astron., in press](#))

- gap geometry,
- acceleration electric field distribution,
- particle density and energy spectrum,
- γ -ray flux and energy spectrum,

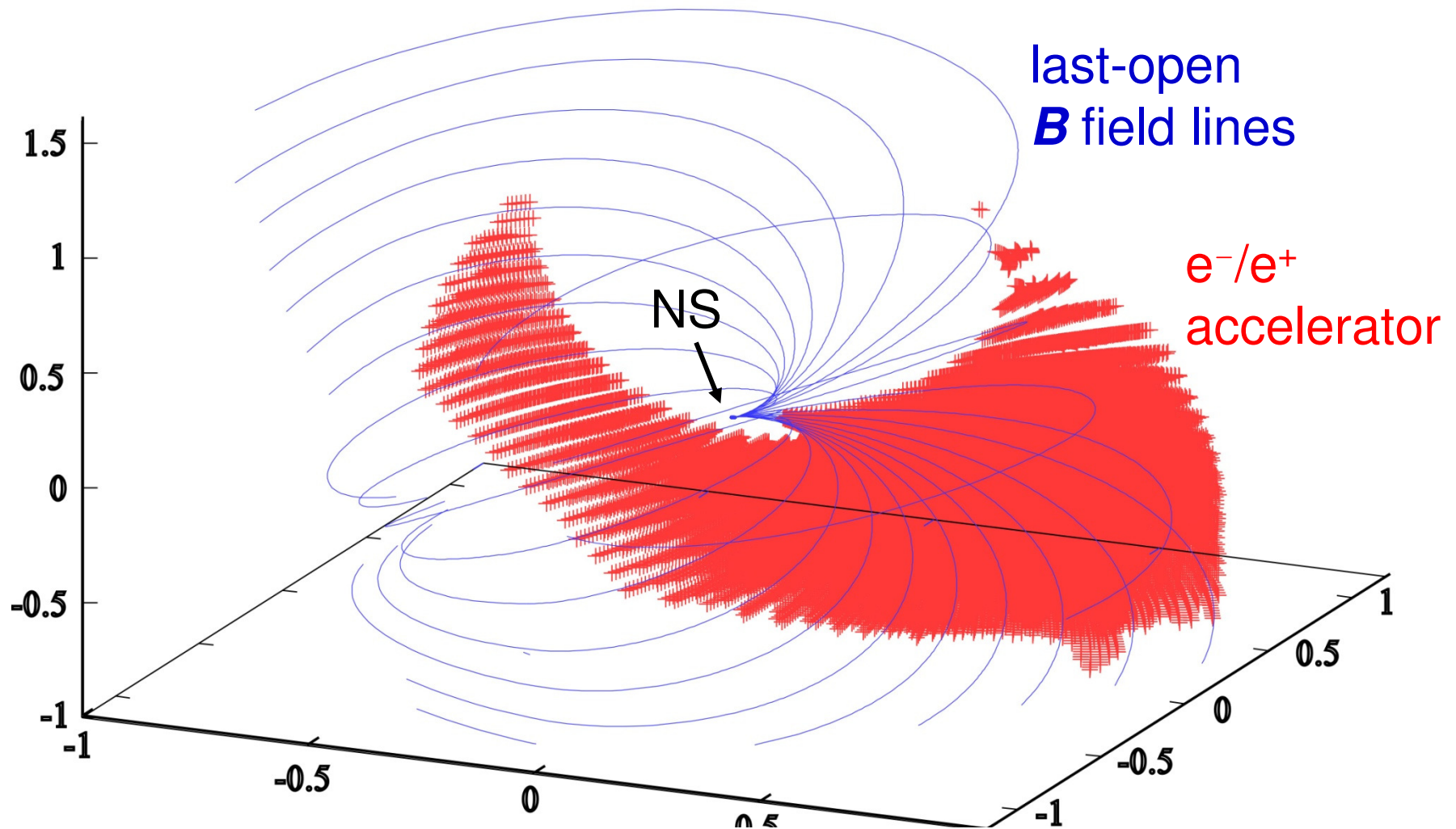
by specifying these three parameters.

§5 New gap theory: Self-consistent approach

I applied the theory to the Crab pulsar.

§5 *New gap theory: Self-consistent approach*

3-D distribution of the particle accelerator (i.e., high-energy emission zone) is solved from the Poisson eq.



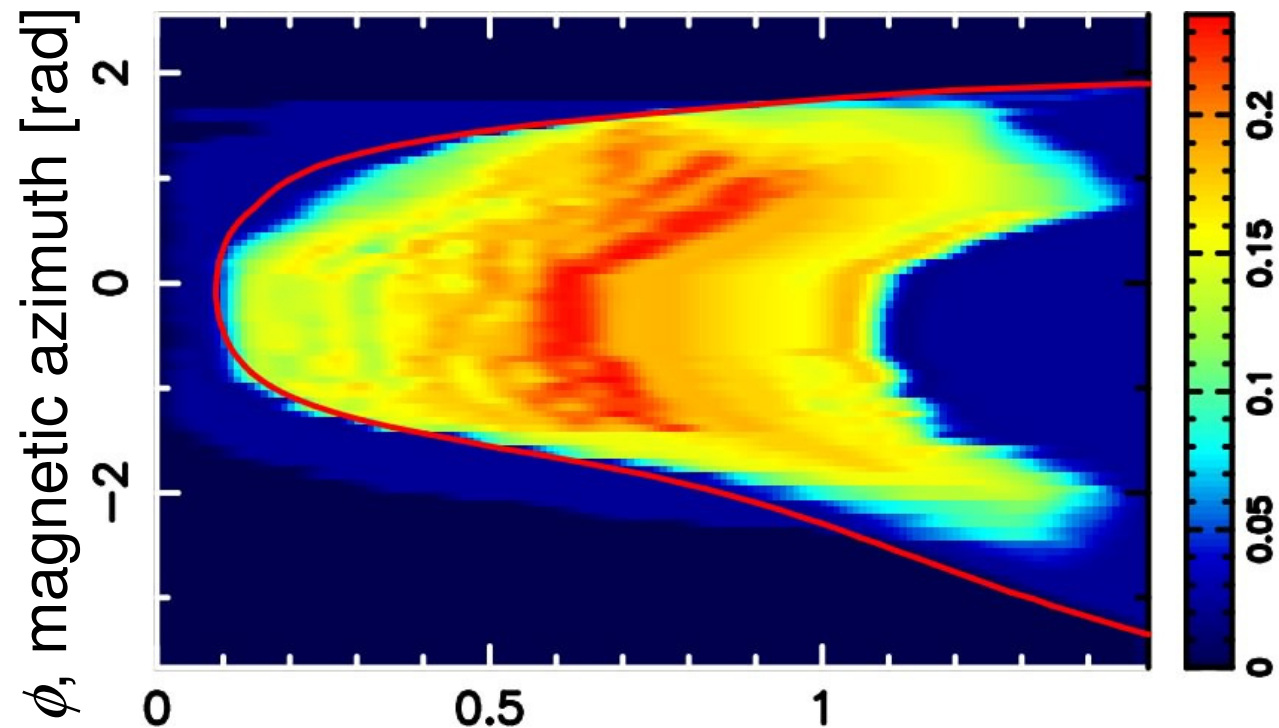
§5 New gap theory: Self-consistent approach

The gap activity is controlled by f^3 .

meridional thickness, $f = f(s, \phi)$. [ϕ : magnetic azimuth]

- Previous models: **assume** or estimate f by dim. analysis.
- This work: **solve** f from the basic eqs. in 3-D mag. sphere.

Fig.) gap trans-
field thickness
on 2-D last-
open-field-line
surface



s , distance along field line / light cylinder rad.

§5 New gap theory: Self-consistent approach

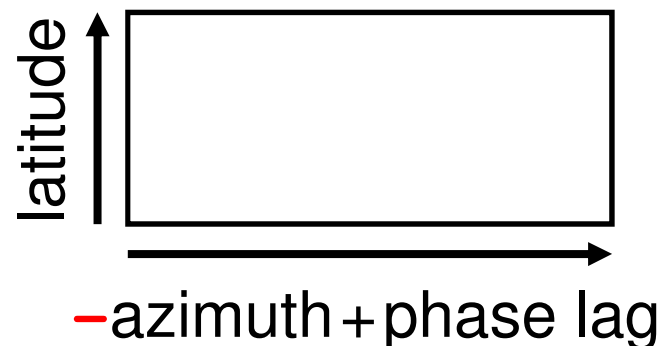
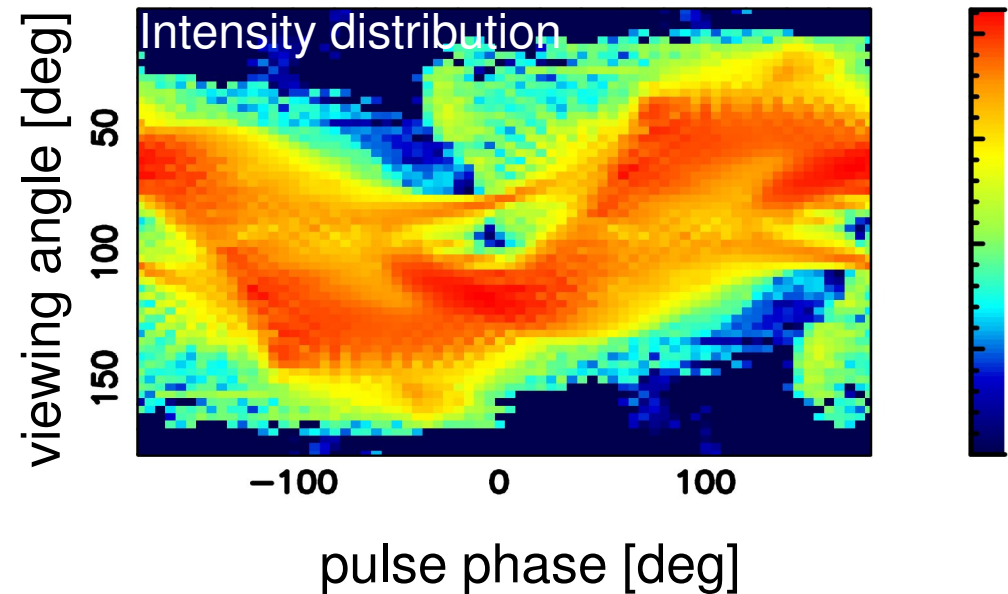
Intrinsic quantities (e.g., gap 3-D geometry, f , E_{\parallel} , e^{\pm} distribution functions, specific intensity at each point) of an OG is self-consistently solved if we give B inclination, NS magnetic moment, NS surface temperature, without introducing any artificial assumptions.

If we additionally give the distance and observer's viewing angle, we can predict the luminosity, pulse profiles, and the photon spectrum in each pulse phase.

§5 New gap theory: Self-consistent approach

Photons are emitted along the local \mathbf{B} field lines (in the co-rotating frame) by relativistic beaming and propagate in a hollow cone.

The hollow cone emission is projected on the 2-D propagation directional plane.



one NS rotation

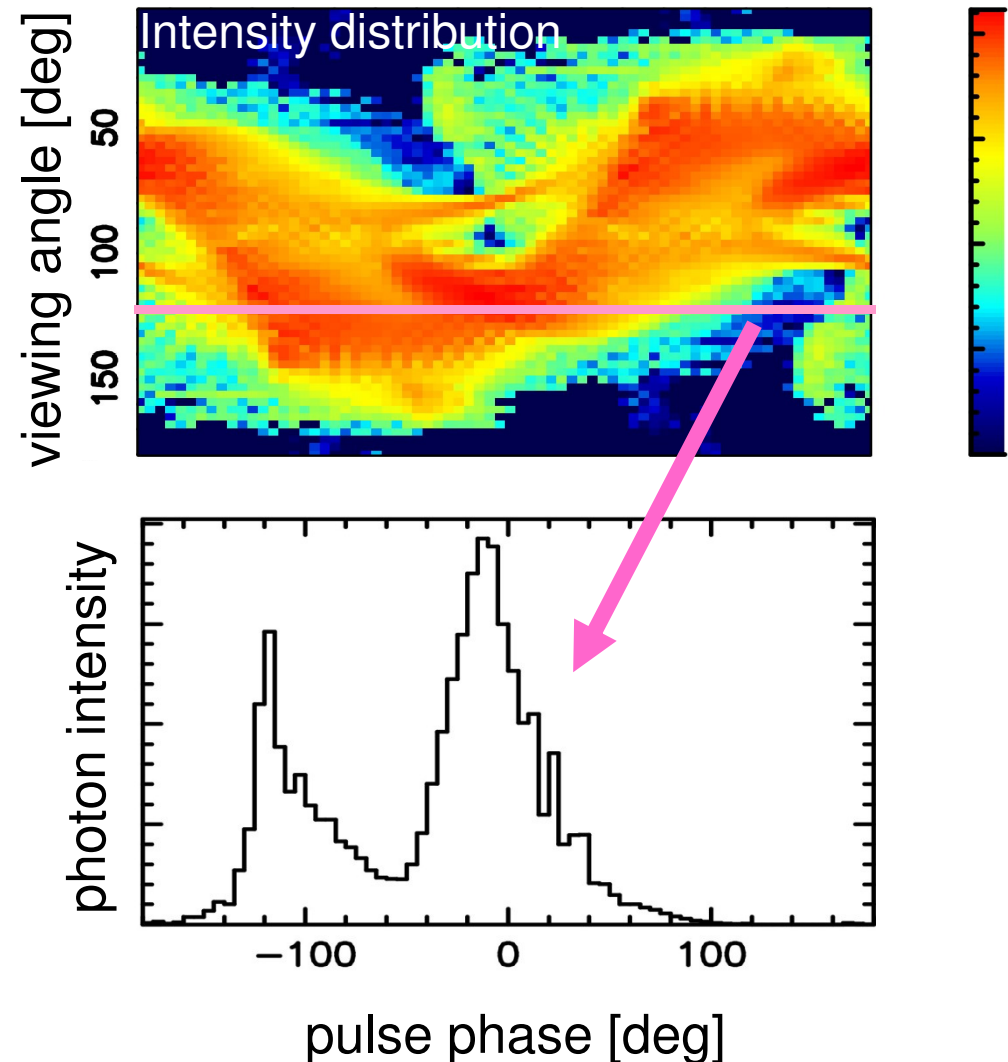
Photons emitted at **smaller azimuth arrives earlier.**

§5 *New gap theory: Self-consistent approach*

Photons are emitted along the local \mathbf{B} field lines (in the co-rotating frame) by relativistic beaming and propagate in a hollow cone.

The hollow cone emission is projected on the 2-D propagation directional plane.

If we **specify** the observer's **viewing angle**, we obtain the **pulse profile**.



§5 New gap theory: Self-consistent approach

Predicted spectra reproduce observations, if we assume appropriate viewing angle (e.g., $\sim 100^\circ$).

Fig.) OG
prediction of
Crab νF_ν spectra

solid:

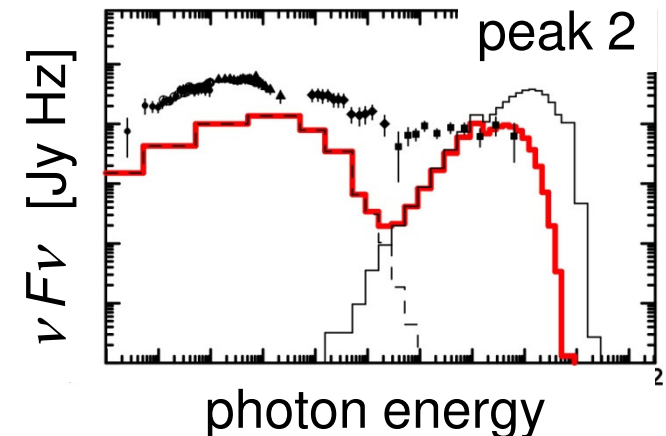
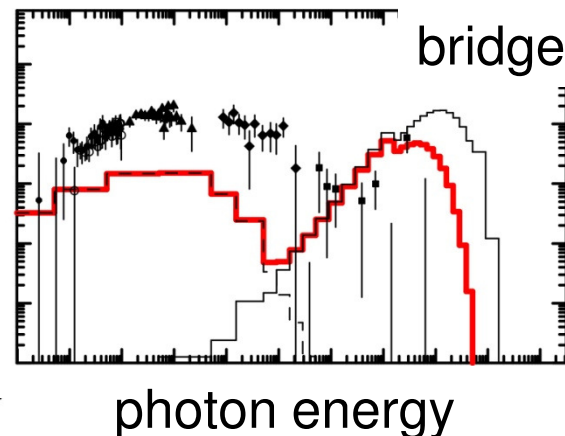
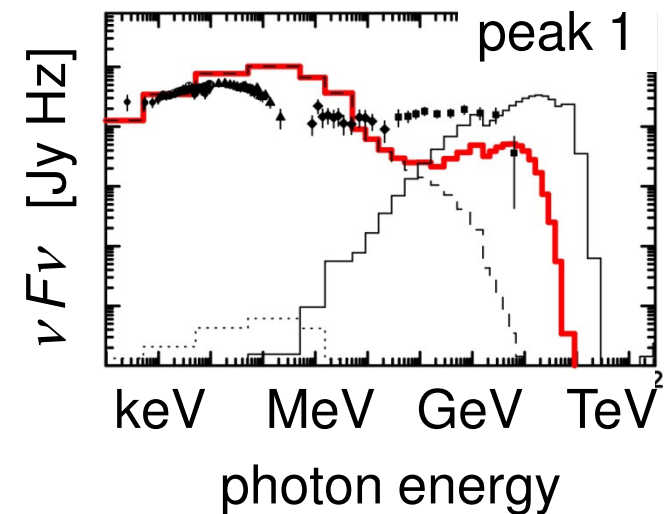
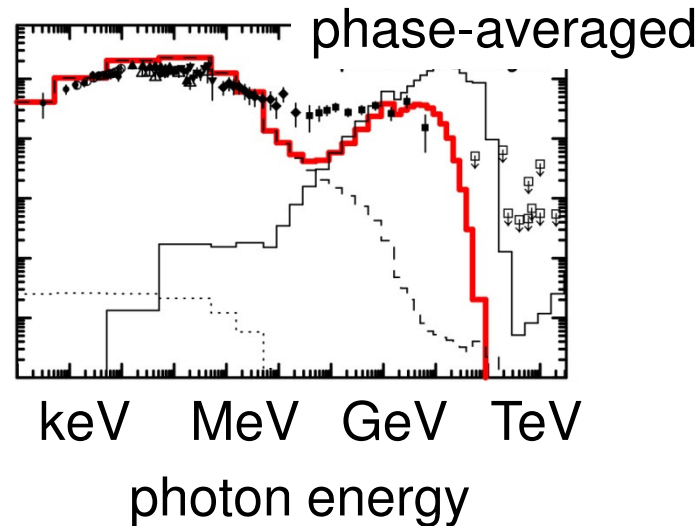
un-abs. primary

dashed:

un-abs. 2ndary

red:

to be observed,
prim.+2nd+3rd

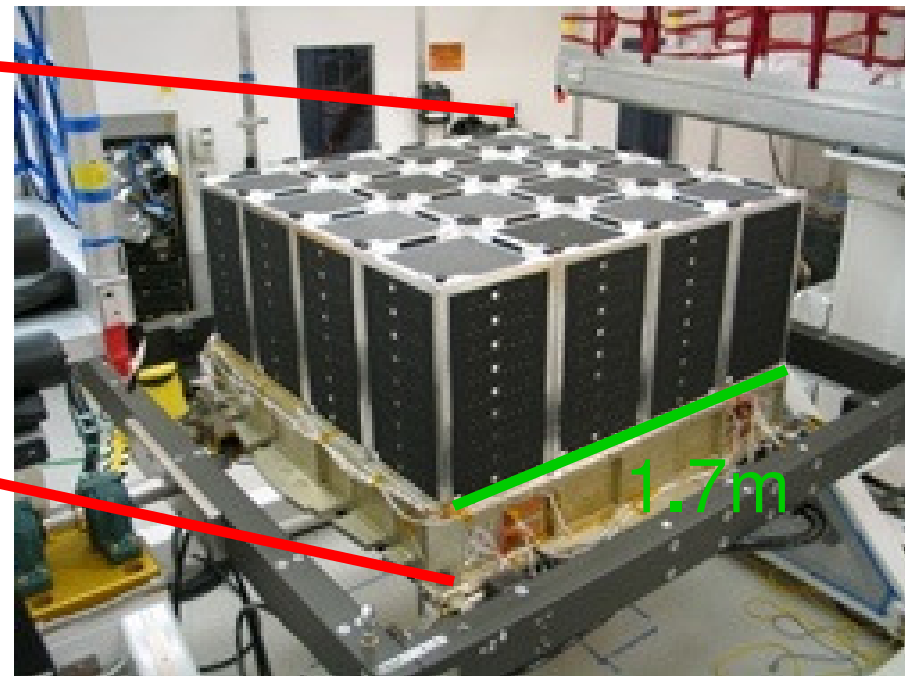


Summary

- High-energy emissions from pulsar magnetospheres are first solved from the set of Maxwell ($\text{div}\mathbf{E}=4\pi\rho$ only) and Boltzmann eqs., if we specify P , dP/dt , α_{incl} , kT_{NS} . We no longer have to assume the gap geometry, E_{\parallel} , e^{\pm} distribution functions. (\mathbf{B} field \leftarrow vacuum rotating dipole solution)
- The obtained solution for the Crab pulsar corresponds to a quantitative extension of the previous, phenomenological OG models, and qualitatively reproduces the observations in IR-VHE.
- SG model can account for less than 10% of the observed Crab γ -ray flux.
- The same scheme can be applied for arbitrary rotation-powered pulsars.

§1 Introduction: The γ -ray sky

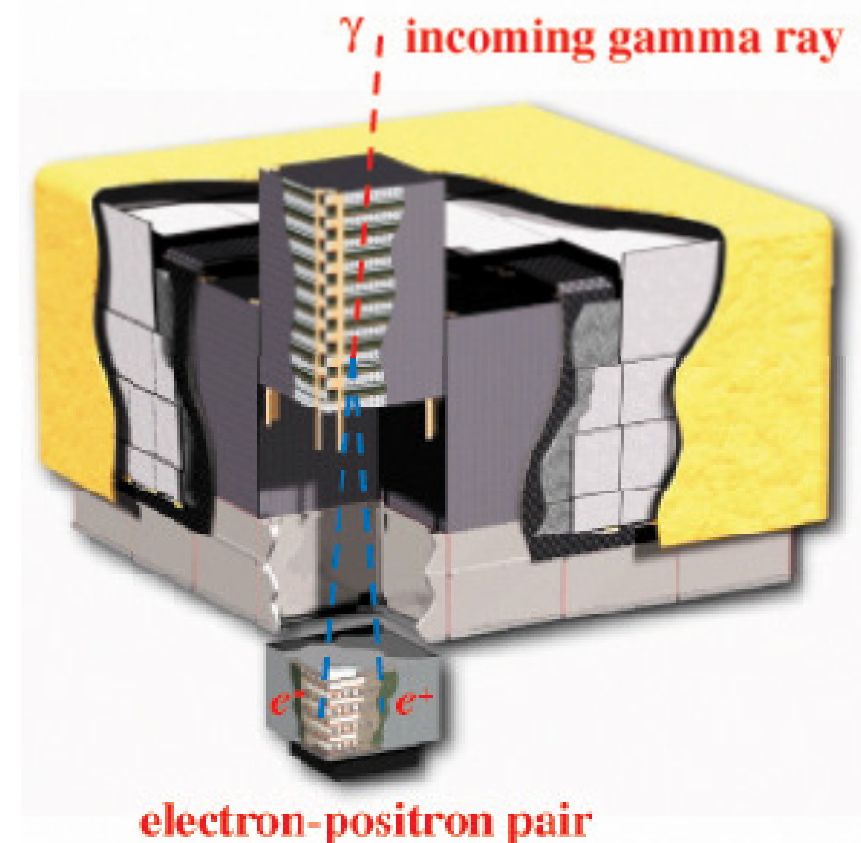
The *Large Area Telescope* (20 MeV – 300 GeV) aboard the *Fermi Gamma-Ray Space Telescope*.



LAT PSF $\sim 0.1^\circ$ @ 1 GeV
FOV ~ 2.5 ster
sensitivity $\sim 30 \times$ EGRET

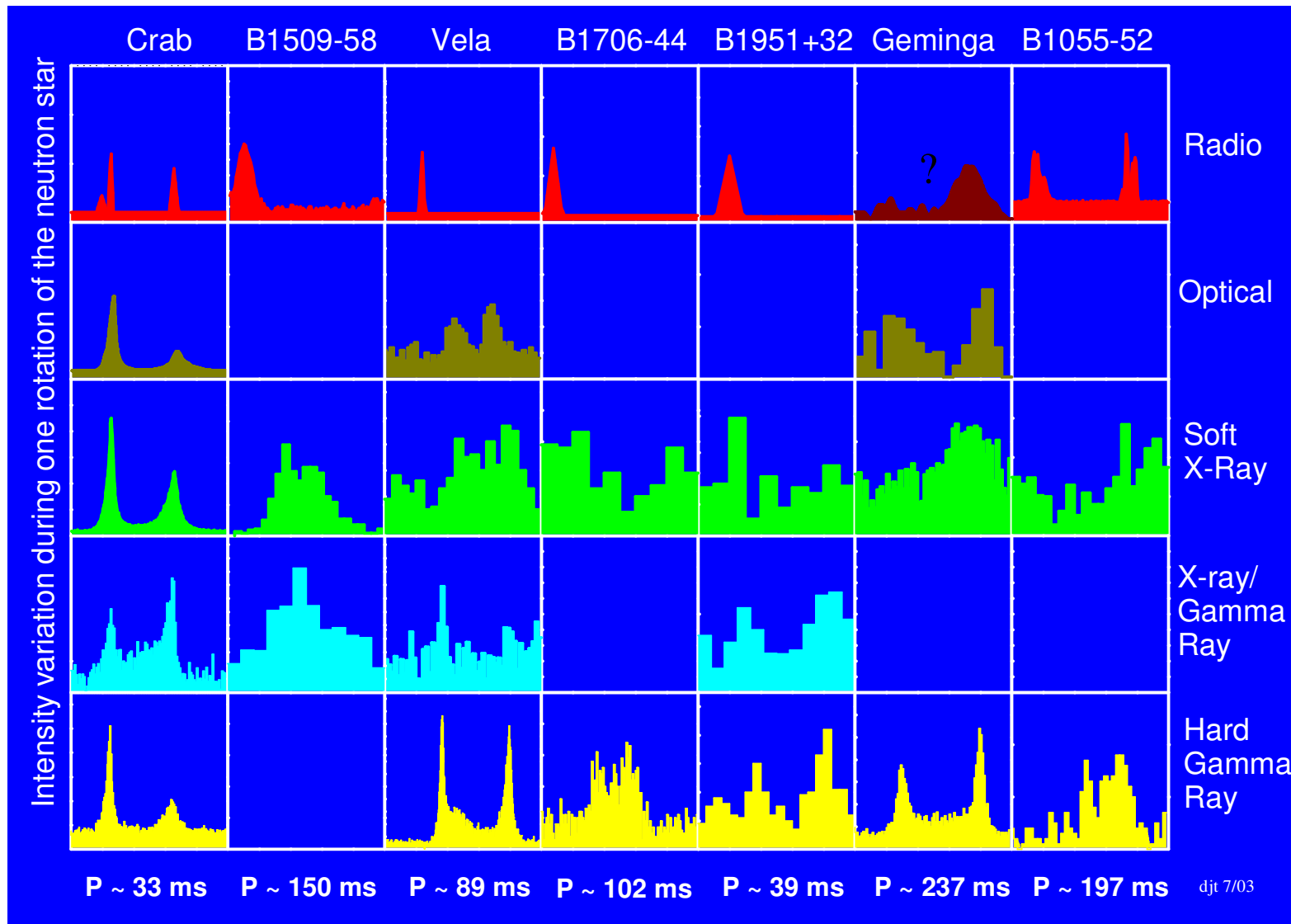
§1 Introduction: The γ -ray sky

The *Large Area Telescope* (20 MeV – 300 GeV) aboard the *Fermi Gamma-Ray Space Telescope*.



§1 Introduction: CGRO observations

γ -ray pulsars emit radiation in a wide frequency range:



Thompson 2003, astro-ph/0312272