Flavor Mixing, Neutrino Masses, Neutrino Oycillations

Harald Fritzsch
LMU / MPI
Munich

Neutrinos:

1928, Wolfgang Pauli

neutrino mass ~ mass of electron

1968: V-A theory

massless neutrino

today:

neutrinos have a mass spectrum

~ very large mixing angles

neutrinos have a mass spectrum

Seminar

~ very large mixing angles

outline

fundamental constants
flavor mixing of quarks
2 generations
texture 0
3 generations
predictions for the angles

neutrino mixing
experiments
texture 0
mixing angles => mass ratios
neutrino masses
V(e3): Chooz, Daya Bay
double beta decay

since 1964:

electroweak gauge theory

SU(2,L) x U(1)

SU(2)xU(1)

weak interactions

electromagnetism

neutral current

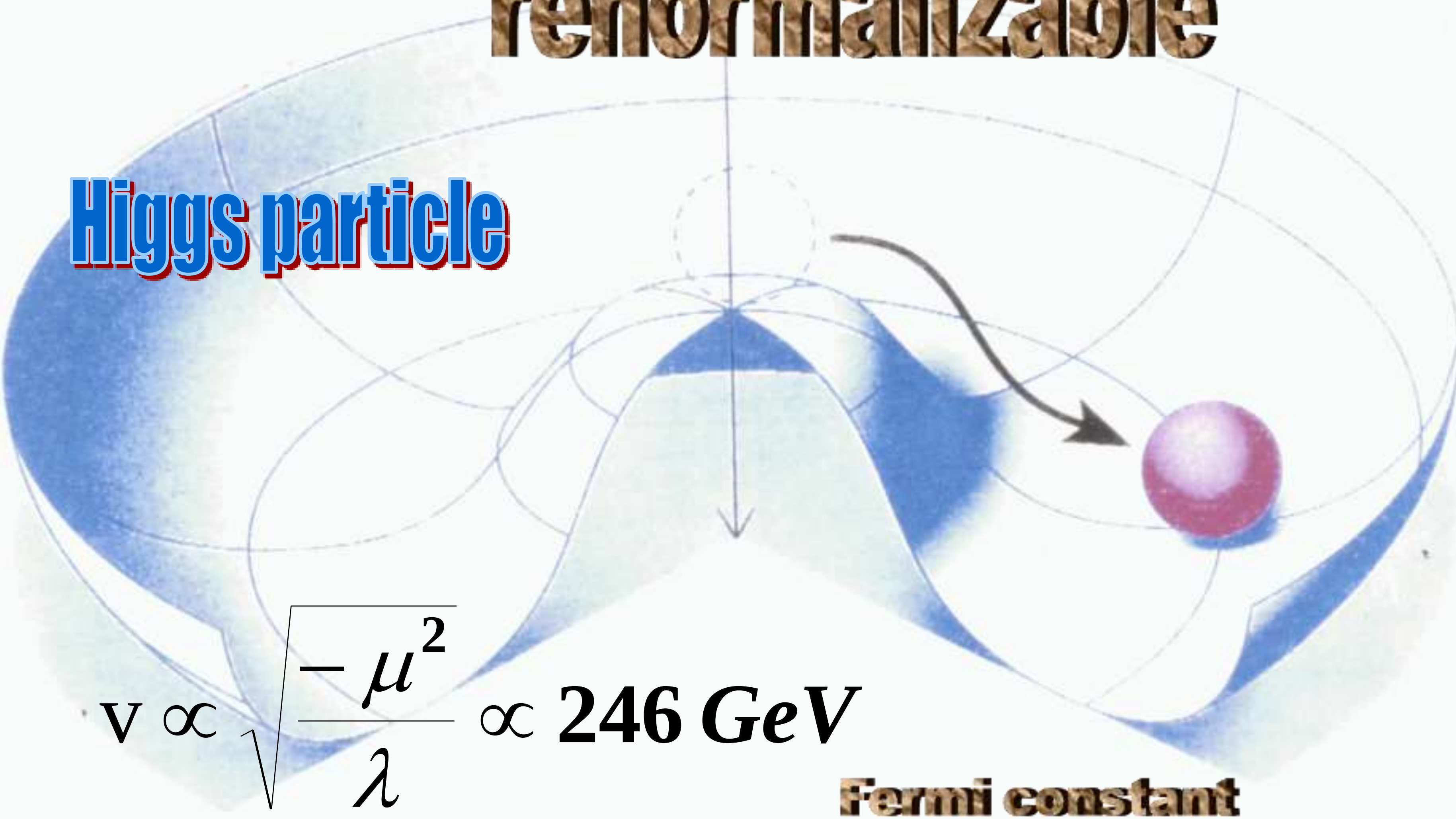
CERN 1972

Masses of W-Bosons are generated by symmetry breaking

(B-E-G-H-H-K) - Mechanism)

(Brout - Englert - Guralnik - Hagen - Higgs - Kibble)

mass and symmetry breaking



$$v \propto \sqrt{\frac{-\mu^2}{\lambda}} \propto 246\,GeV$$

Strong interactions

1971

q ==>

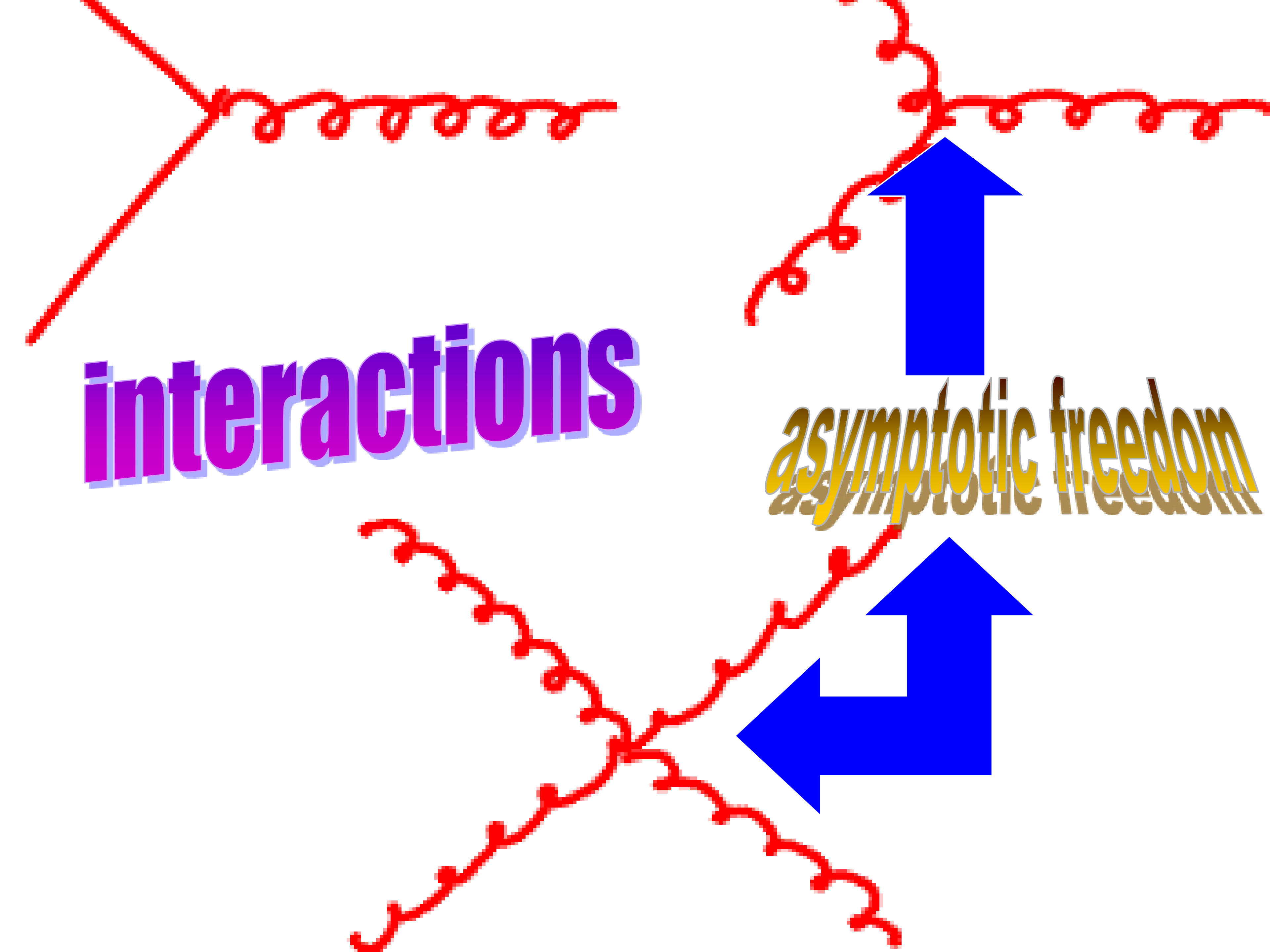
q_r q_g q_b

$SU(3)_c$

1972

QCD

Fritzsch Gell-Mann


interactions
asymptotic freedom

1973:

Standard Model

SU(3,c) x SU(2,L) x U(1)

The Standard Model

"PERIODIC SYSTEM" OF ELEMENTARY PARTICLES

	QUARKS			LEPTONS			
ELECTRIC CHARGE	STRONG NUCLEAR FORCE			NO STRONG NUCLEAR FORCE			ELECTRIC CHARGE
$+2/3$	u	c	t	ν_e	ν_μ	ν_τ	0
	UP	CHARM	TRUTH	ELECTRON-NEUTRINO	MUON-NEUTRINO	TAU-NEUTRINO	
$-1/3$	d	s	b	e^-	μ^-	τ^-	-1
	DOWN	STRANGE	BEAUTY	ELECTRON	MUON	TAU	
	I	II	III	I	II	III	

? 3 families < ==> 3 colors ?

====> problem

32 fundamental constants

Fundamental constants

number of space dimensions	1
number of time dimensions	1
number of colors	1
number of families	1
Newtons constant G	1
fine structure constant	1
coupling constant of strong interaction	1
coupling constant of weak interaction	1
mass of W boson	1
mass of Higgs boson	1
masses of 6 quarks and 6 leptons	12
flavor mixing of quarks	4
flavor mixing of leptons	6
	32

Masses -
Charged leptons and quarks (MeV)

electron: 0.51 muon: 105.7 tau: 1 777

u: 5.3 c: 1 200 t: 173 000

d: 7.8 s: 130 b: 4 300

(u, d, c, s - quark masses at 1 GeV)

relations between lepton masses?

Bjorken:

$$m_e / m_\mu = \frac{3\alpha}{\pi} \ln 2 + O(\alpha^2) + \ldots$$

$$= 0.00483 + O(\alpha^2) + \ldots$$

observed: 0.00484

relations between quark masses ?

Quark Masses:

- Observed:

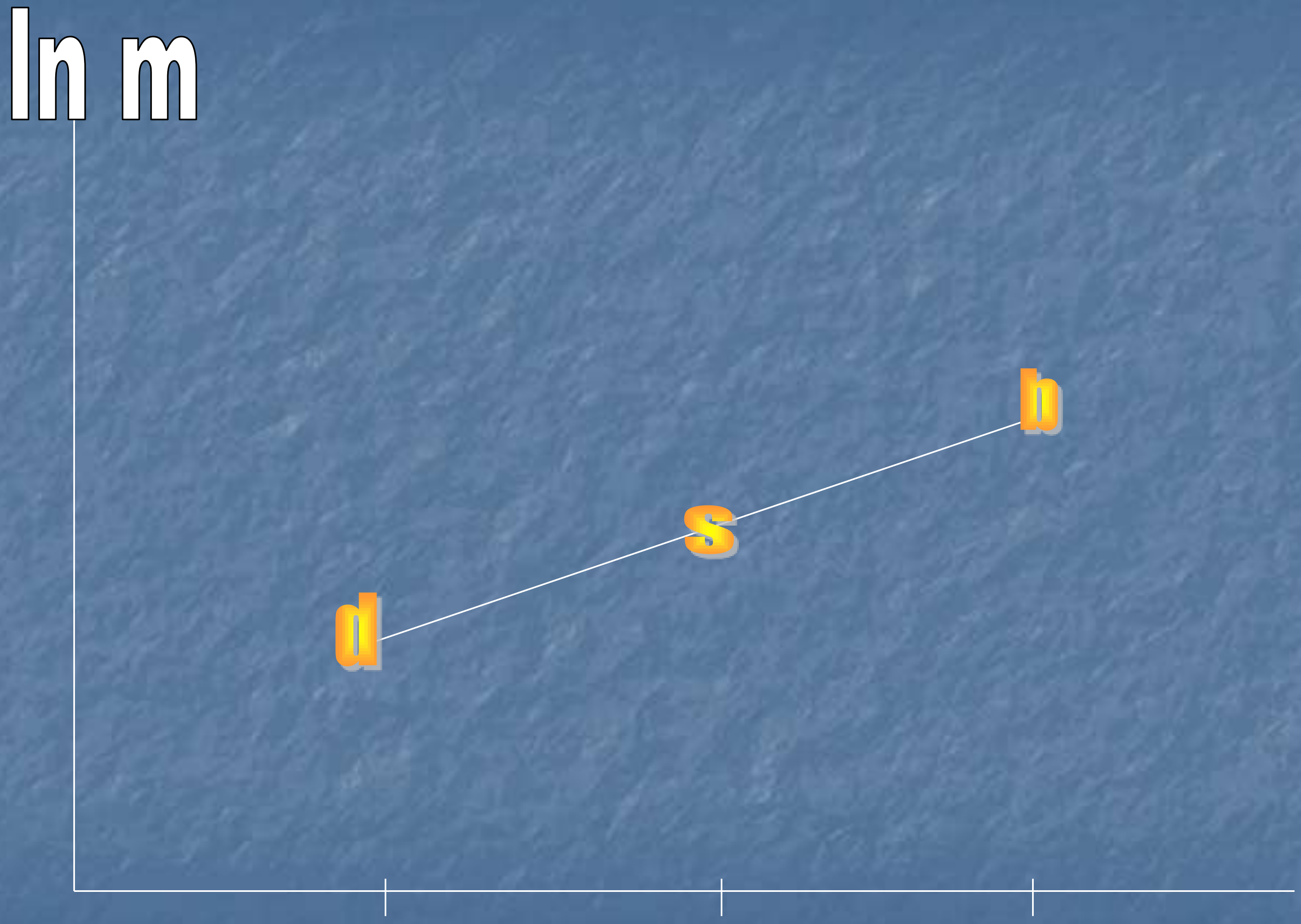
$$m(c) : m(t) = m(u):m(c)$$

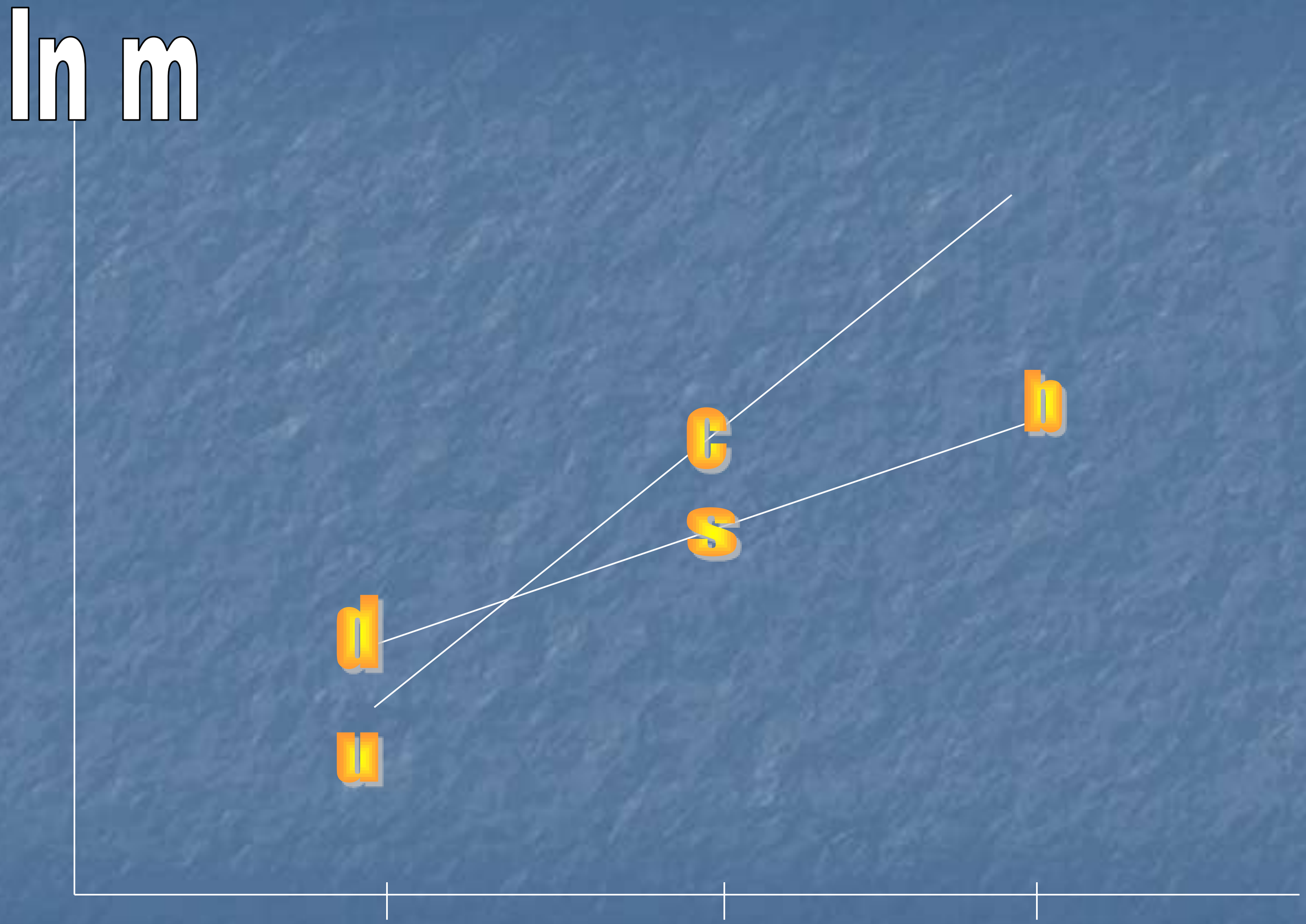
1/207 1/207

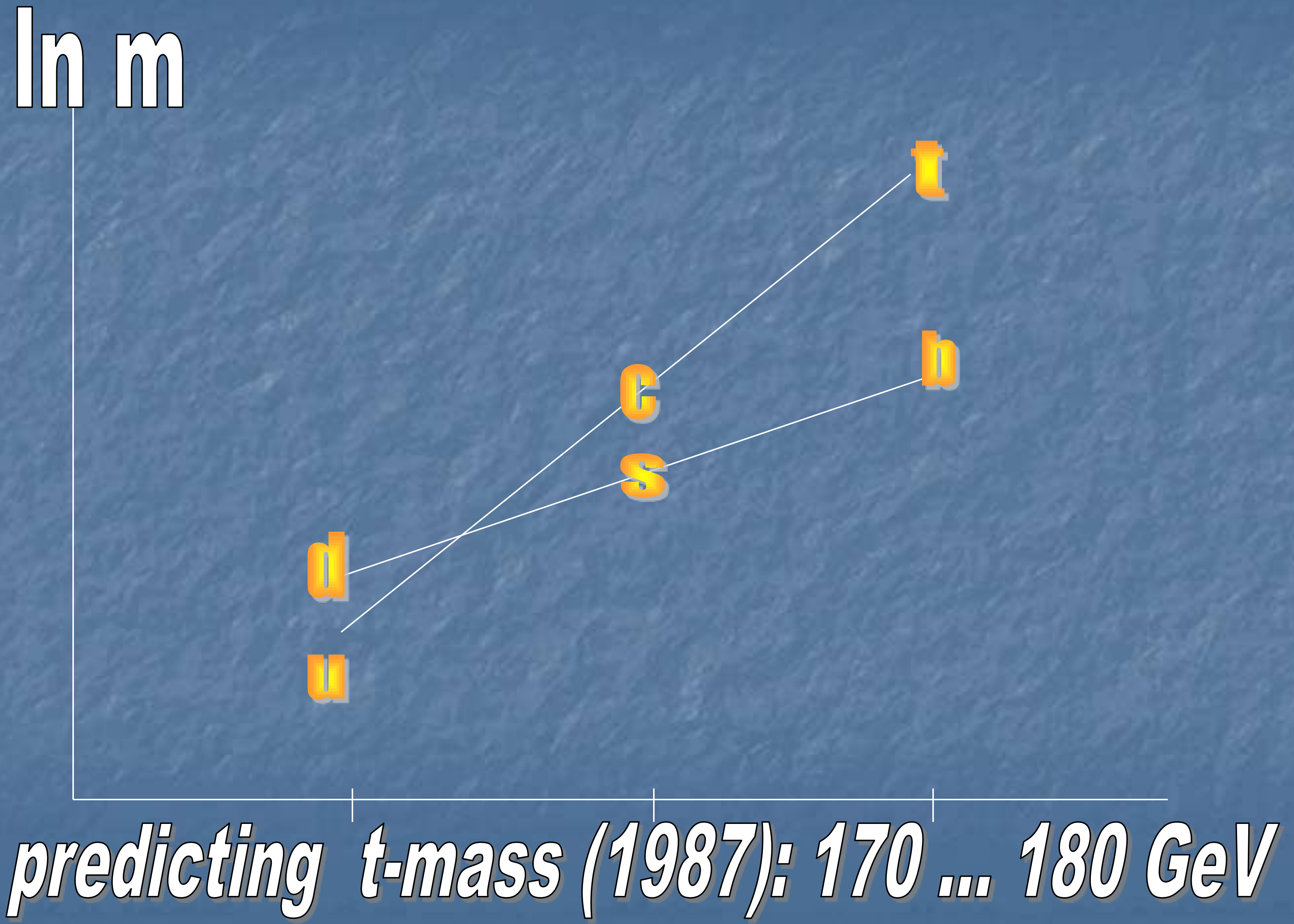
$$m(s):m(b) = m(d):m(s)$$

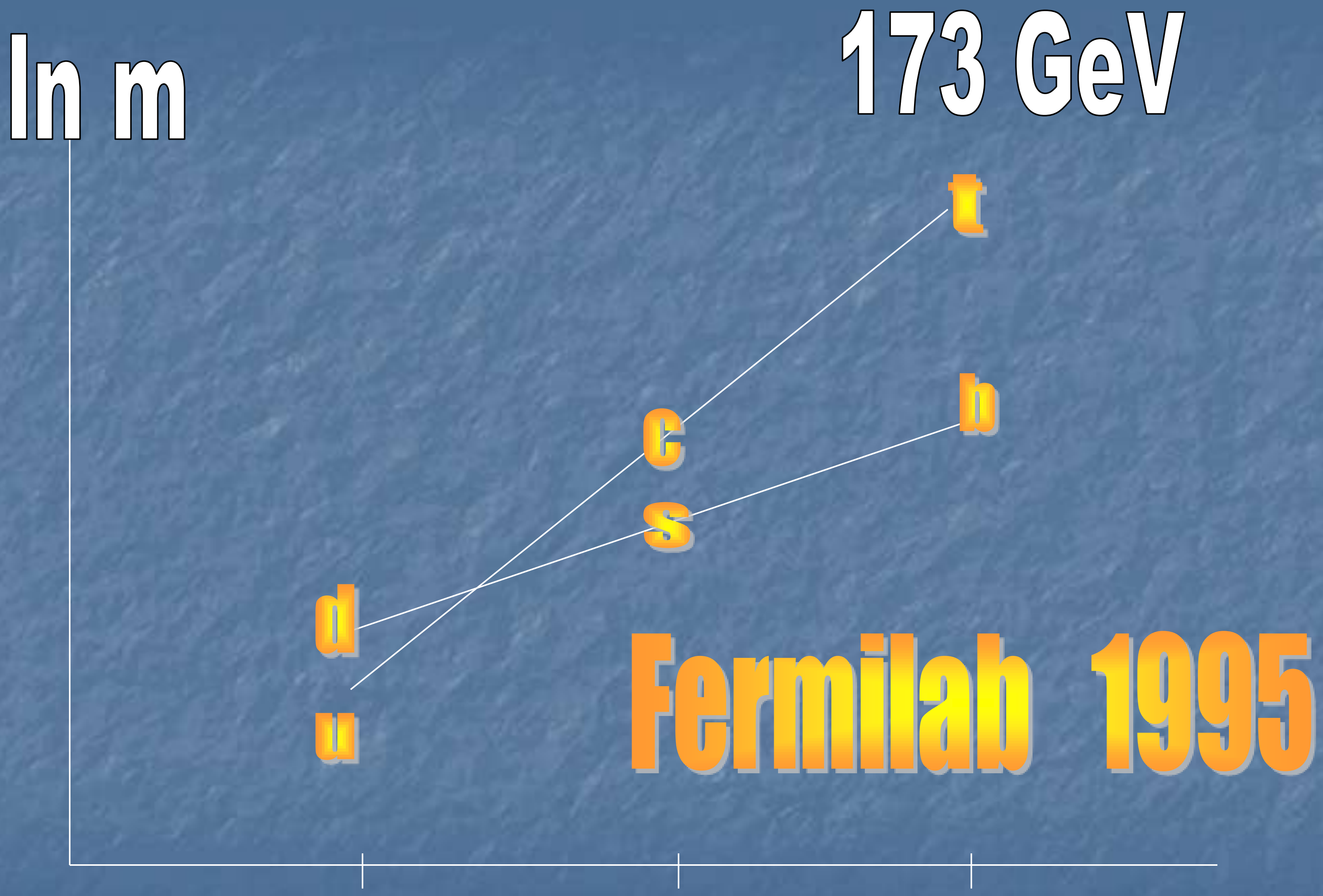
1/23 1/23

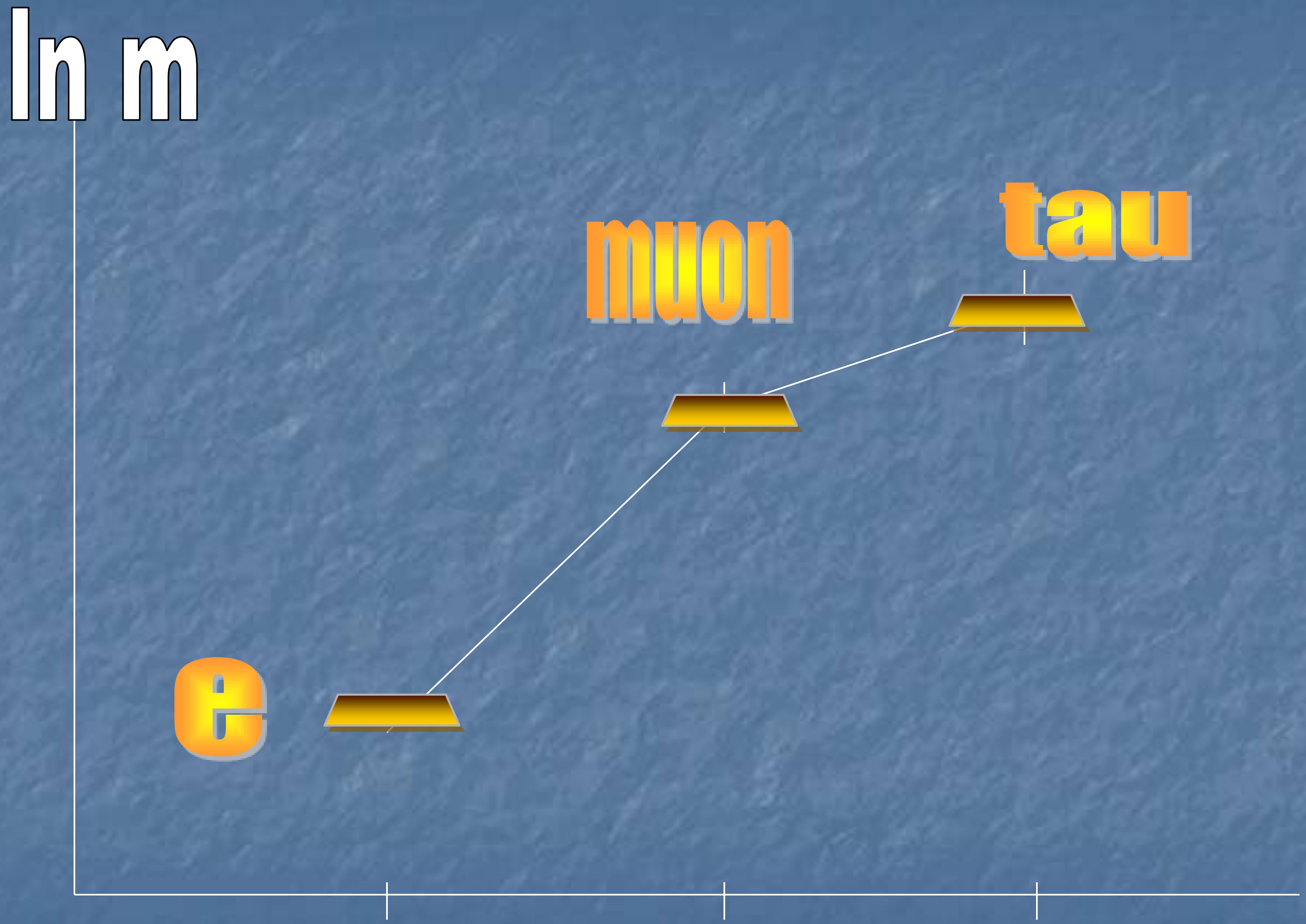

ln m
d
s
b


ln m
b
s
d

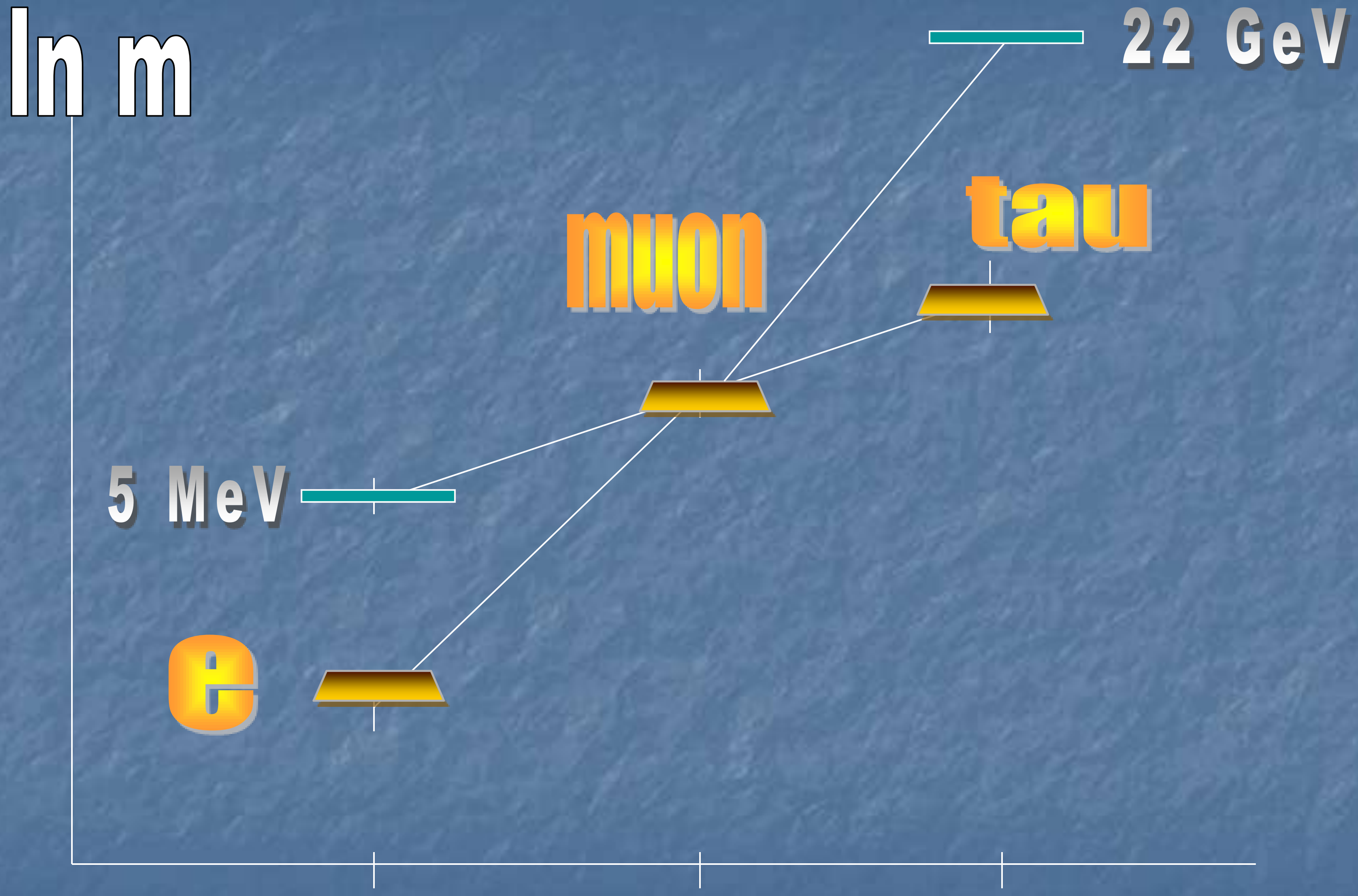

ln m
c
b
s
d
u


ln m
t
b
c
s
d
u
predicting t-mass (1987): 170 ... 180 GeV


ln m
173 GeV
t
b
c
s
d
u
Fermilab 1995



ln m
muon
tau
e


ln m
22 GeV
tau
muon
5 MeV
e

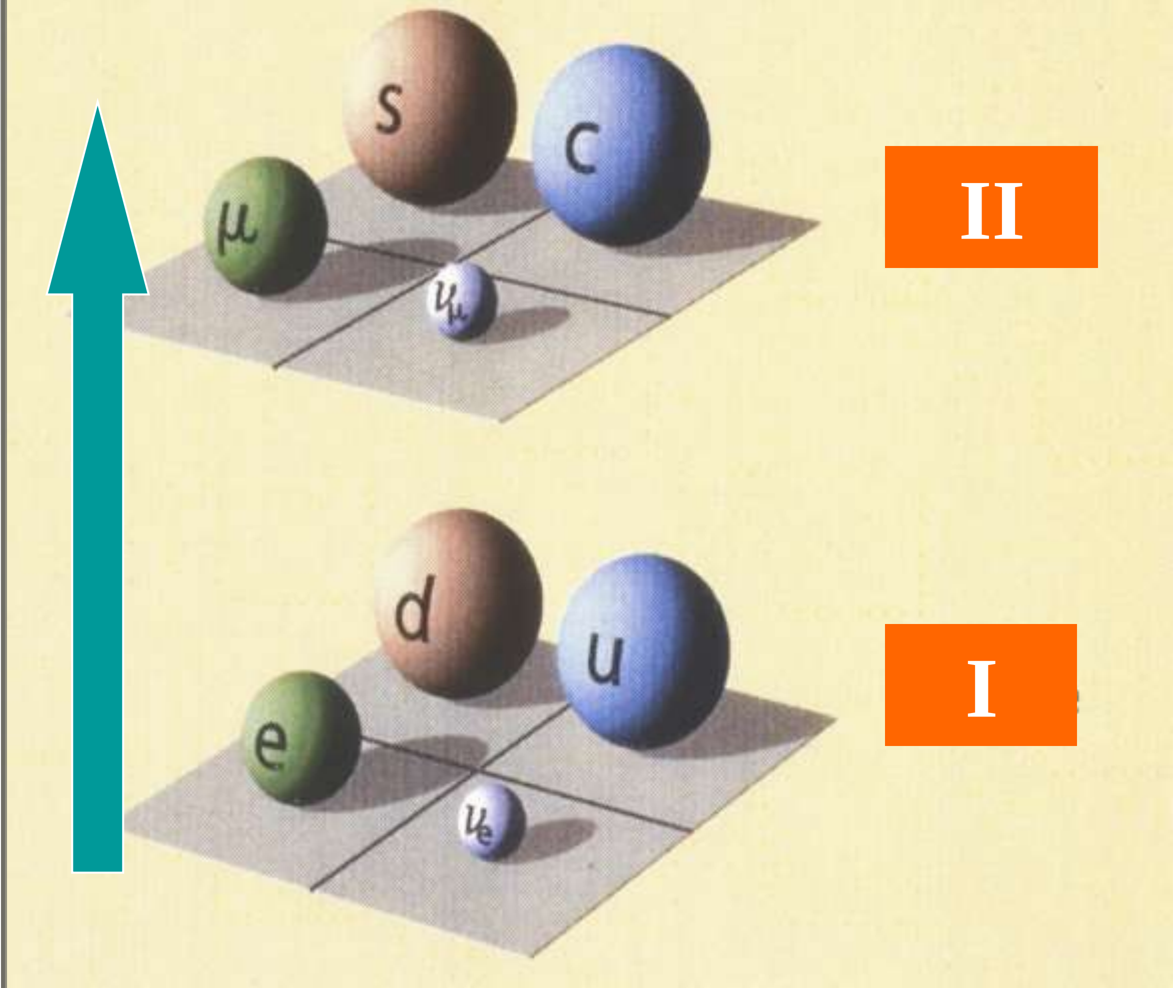
flavor mixing angles

↕

fermion masses

flavor mixing

2 families



$$\left\langle \frac{u}{d\cos\theta_c + s\sin\theta_c} \right\rangle ==== \left\langle \frac{c}{-d\sin\theta_c + s\cos\theta_c} \right\rangle$$

mixing of mass eigenstates by weak interaction

(Cabibbo angle)

$$\theta_c \approx 13^0$$

mass matrices:

texture 0

u,c - d,s

$$\begin{pmatrix} 0 & a \\ a^{\otimes} & b \end{pmatrix}$$

(beyond Standard Model)

H. Fritzsch
S. Weinberg
1978

texture zero:

$SU(2)_L \times SU(2)_R$

Reflection symmetries

(~parity)

===> Grand Unification -SO(10)

Grand unification

SU(3)xSU(2)xU(1)

==>SO(10)

(Fritzsch - Minkowski; Georgi - 1975)

SO(10)

$$SO(6) \times SO(4)$$

$$SU(4) \times SU(2,L) \times SU(2,R)$$

$$SU(3) \times SU(2,L) \times U(1)$$

SO(10)

SO(6) x SO(4)

SU(4) x SU(2,L) x SU(2,R)

SU(3) x SU(2,L) x U(1)

In SO(10):

lefthanded and righthanded neutrinos

Electroweak theory:

$SU(2)_L \times SU(2)_R \times U(1)$

New energy scale for righthanded SU(2)

related to neutrino masses? - consistent with LEP unlike SU(5)

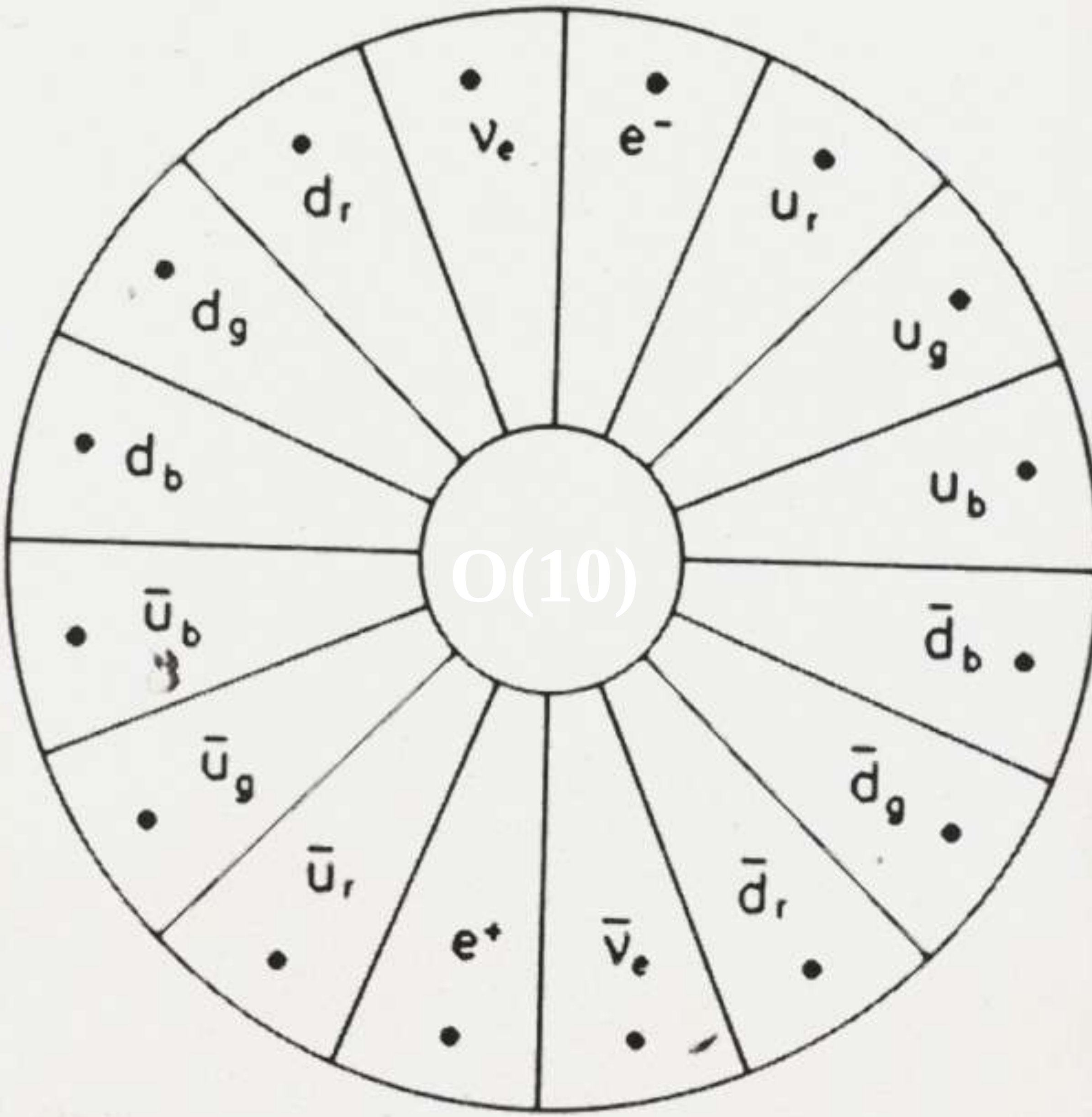
SO(10)

Fermions in 16-plet

(incl. righthanded neutrinos)

SO(10)

leptons
and
quarks
in one
multiplet



Neutrinos are massive

Relations between masses and mixing angles

$$\begin{pmatrix} 0 & a \\ a^{\otimes} & b \end{pmatrix} \succ \begin{pmatrix} -m_d & 0 \\ 0 & m_s \end{pmatrix}$$

$$\tan 2\theta_d = \frac{2\sqrt{m_d m_s}}{m_s - m_d} \qquad \theta_d \approx \sqrt{\frac{m_d}{m_s}}$$

Cabibbo angle 13^o

$$\theta_c \cong \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right|$$

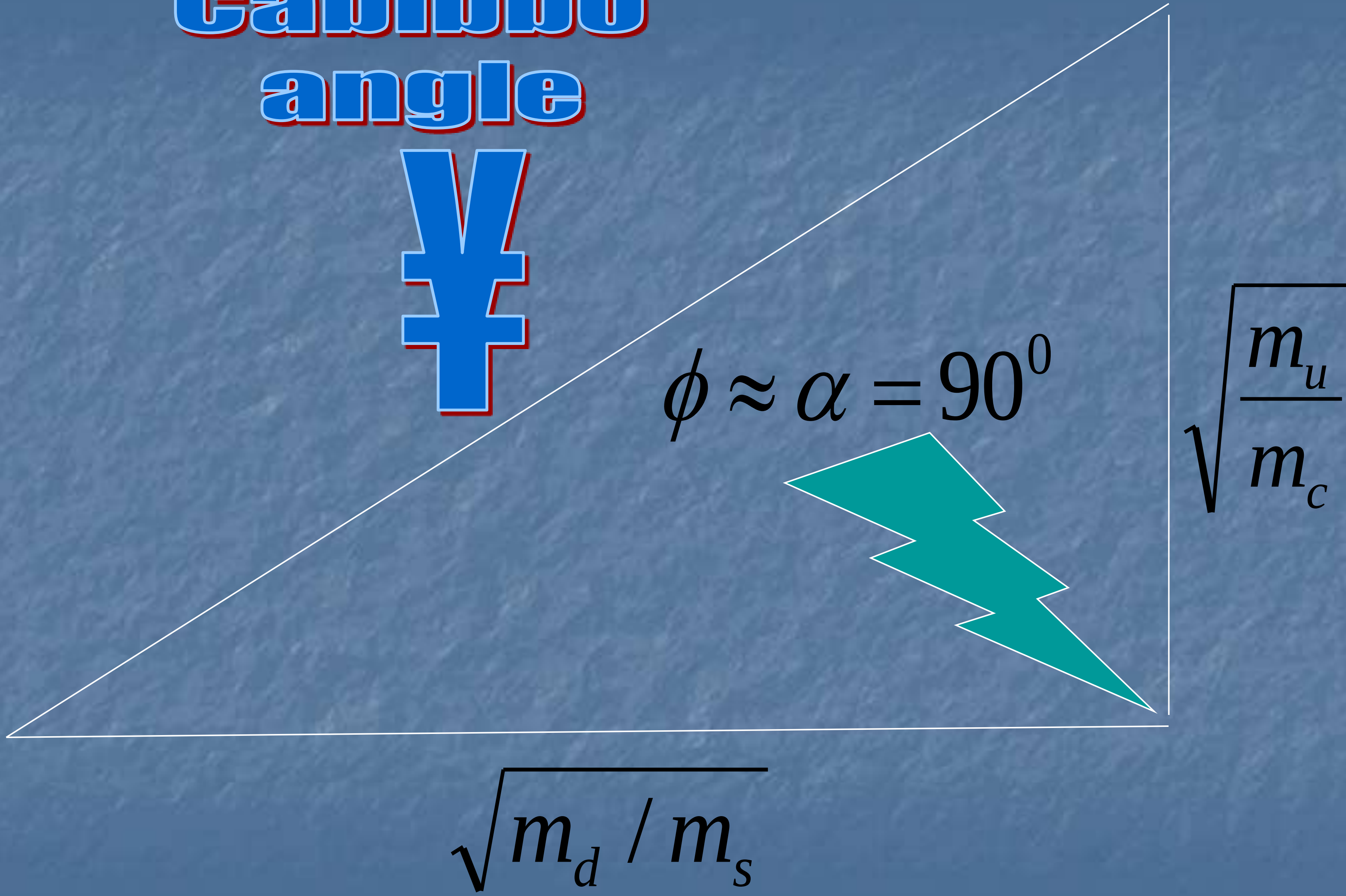
$$\sqrt{\frac{m_d}{m_s}} \approx 0.21 \qquad \sqrt{\frac{m_u}{m_c}} \approx 0.07$$

Exp.: phase 90 degrees

(symmetry !)

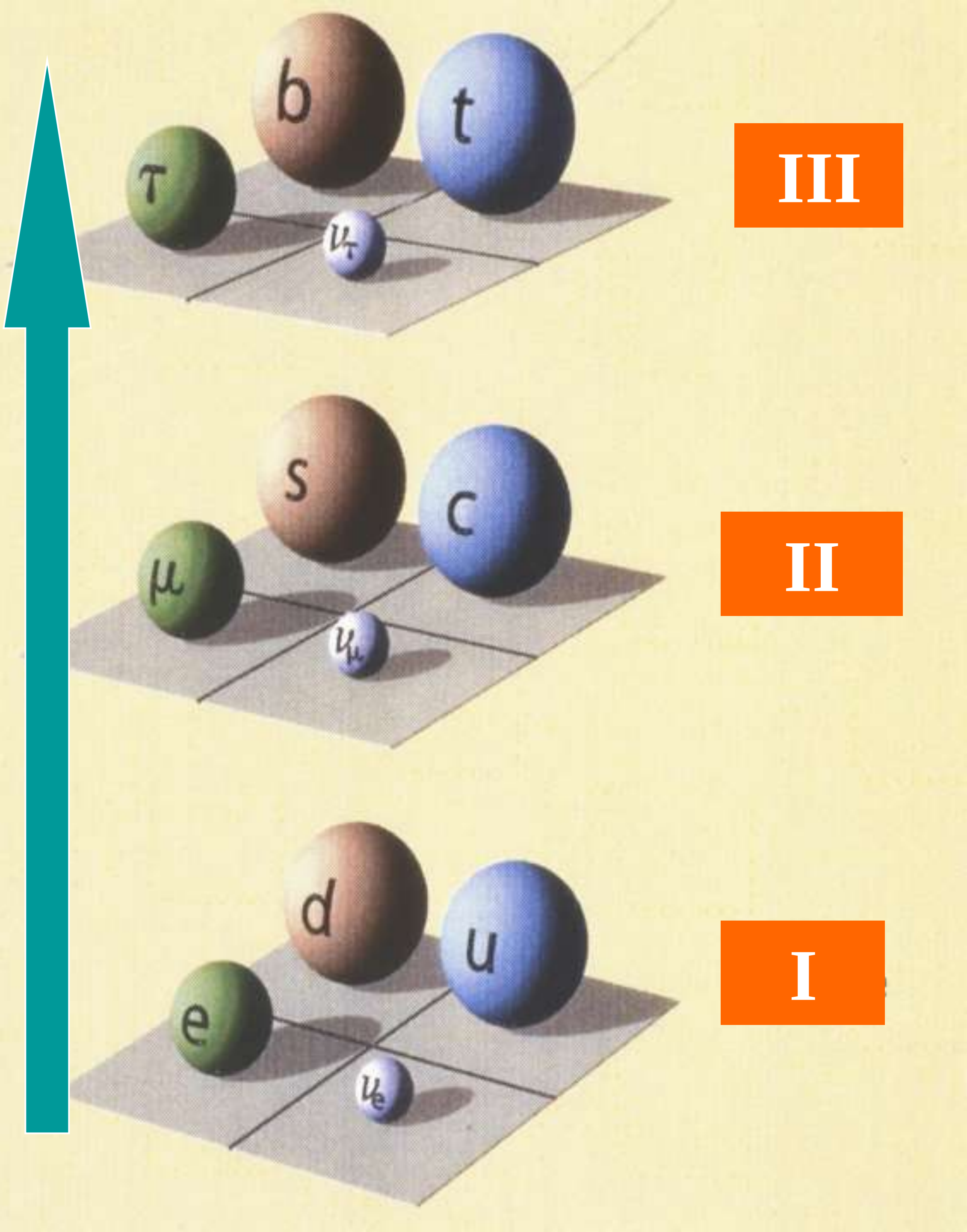
Cabibbo angle

¥



3 families

flavor mixing



texture zeros:

$$\begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}$$

3 texture zeros

flavor mixing

CKM - matrix:

three angles - one phase

$$(|V_{ij}|) = \begin{bmatrix} 0{,}97459 & 0{,}2257 & 0{,}00359 \\ 0{,}2256 & 0{,}97334 & 0{,}0415 \\ 0{,}00874 & 0{,}0407 & 0{,}999133 \end{bmatrix}$$

CKM matrix

standard parametrization

$$angles: \theta_{12}, \theta_{23}, \theta_{13}$$

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \bullet \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \bullet \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

New parametrization:

$$V = \begin{bmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \bullet \begin{bmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

H. F. - Z. Xing

Theory:

$$\begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}$$

3 texture zeros

$$\tan\theta_d = \sqrt{m_d} / \sqrt{m_s}$$

$$\tan\theta_u = \sqrt{m_u} / \sqrt{m_c}$$

The angles

θ_u and θ_d

can be measured separately.

SLAC

DESY

KEK

CERN

CORNELL

theory:

$$\tan\theta_d = \sqrt{m_d} / \sqrt{m_s}$$

$$\theta_d \approx 13.0 \pm 0.4^o$$

theory:

$$\theta_d \approx 13.0 \pm 0.4^o$$

$$Exp: 11.7^o \pm 2.6^o$$

theory:

$$\tan\theta_u = \sqrt{m_u} / \sqrt{m_c}$$

$$\theta_u \approx 5.0^o \pm 0.7^o$$

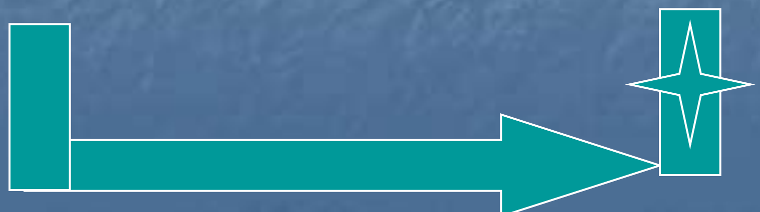
theory:

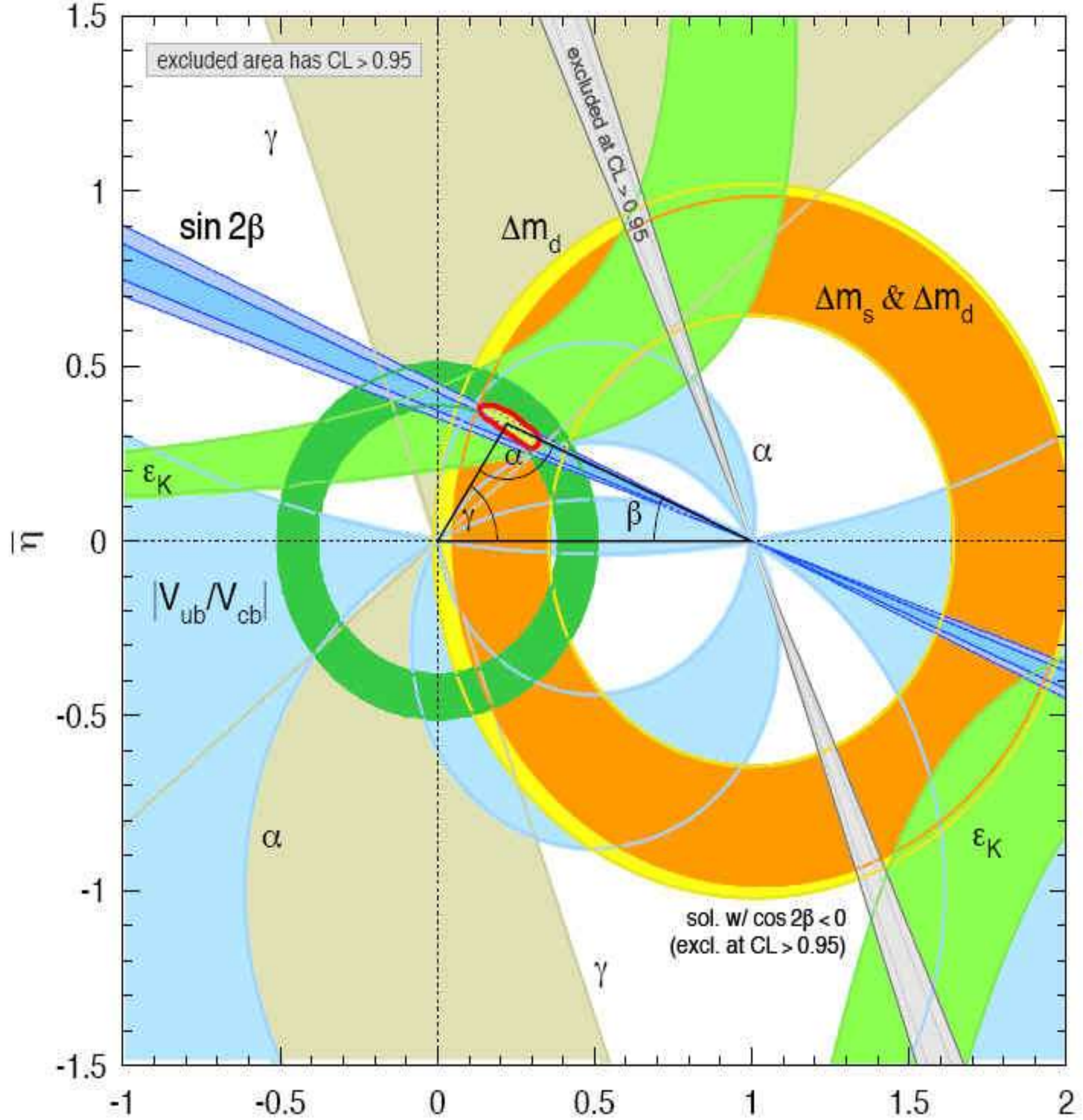
$$\theta_u \approx 5.0^o \pm 0.7^o$$

$$Exp: 5.4^o \pm 1.1^o$$

unitarity triangle

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cu} & V_{cs} & V_{cb} \\ V_{tu} & V_{ts} & V_{tb} \end{pmatrix}$$

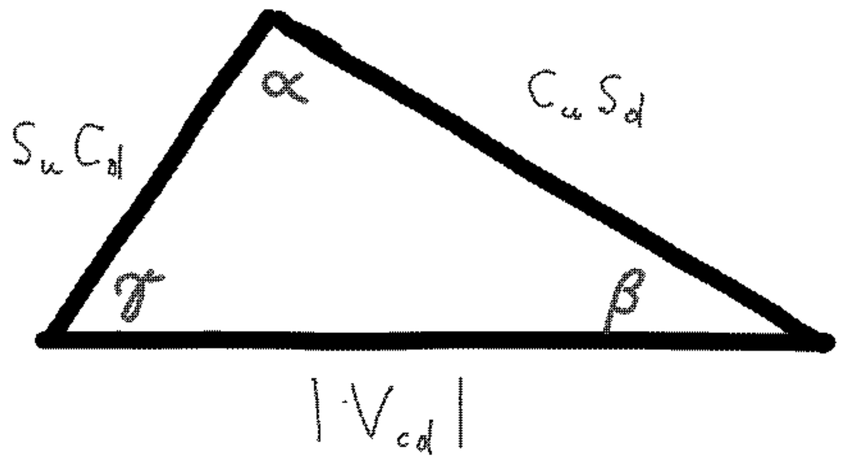




alpha: $86^0 \ldots 95^0$

Unitarity Triangles

Quarks:



$$\alpha \cong 90^\circ$$

(~rectangular triangle)

Cabibbo angle

$$\phi \approx \alpha = 90^0$$

Unitarity triangle

(rectangular)

Other angles of un. triangle

$$\tan\beta = \frac{\sin\theta_u \cos\theta_d}{\cos\theta_u \sin\theta_d}$$

==>

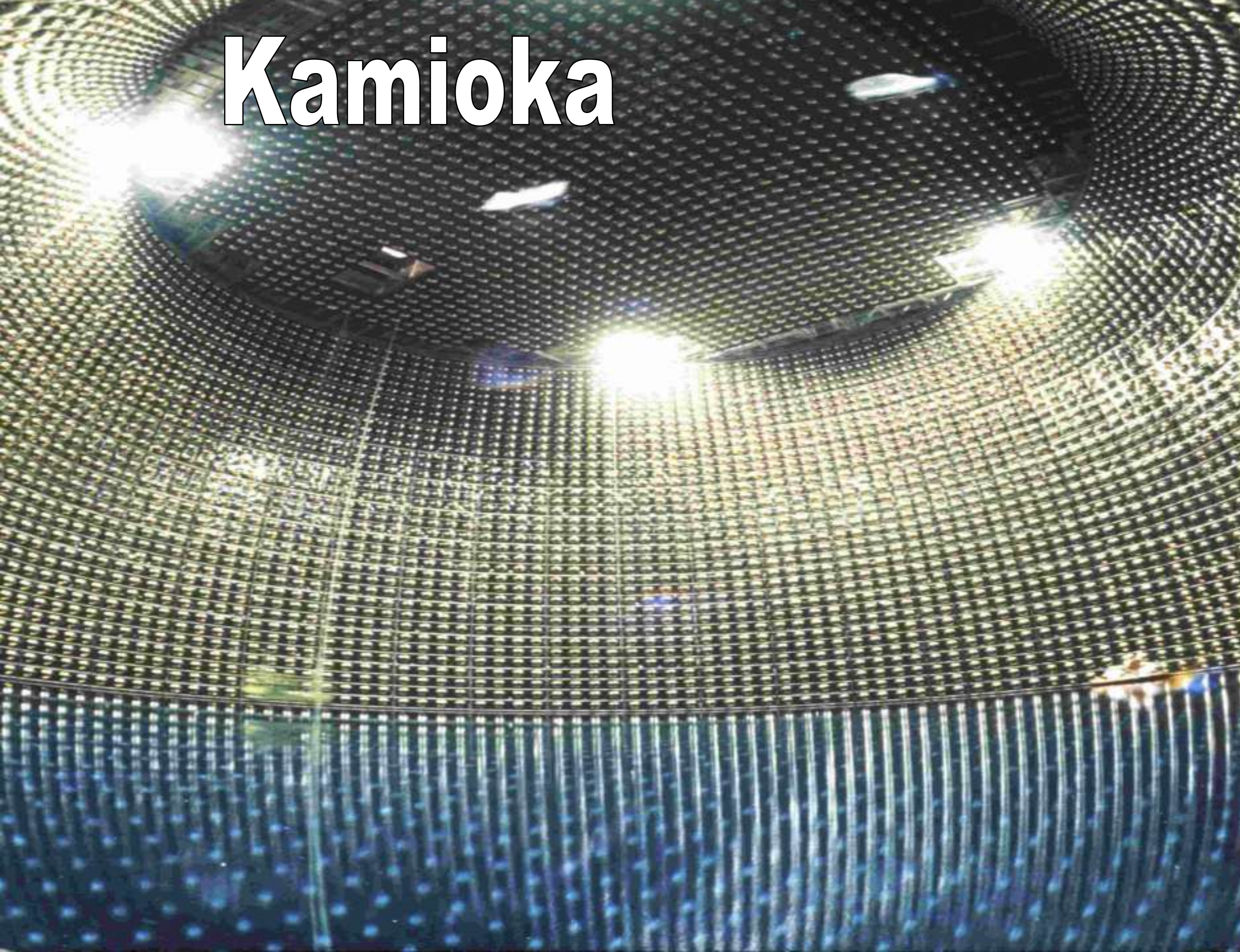
$$\sin 2\beta \cong 0.663$$

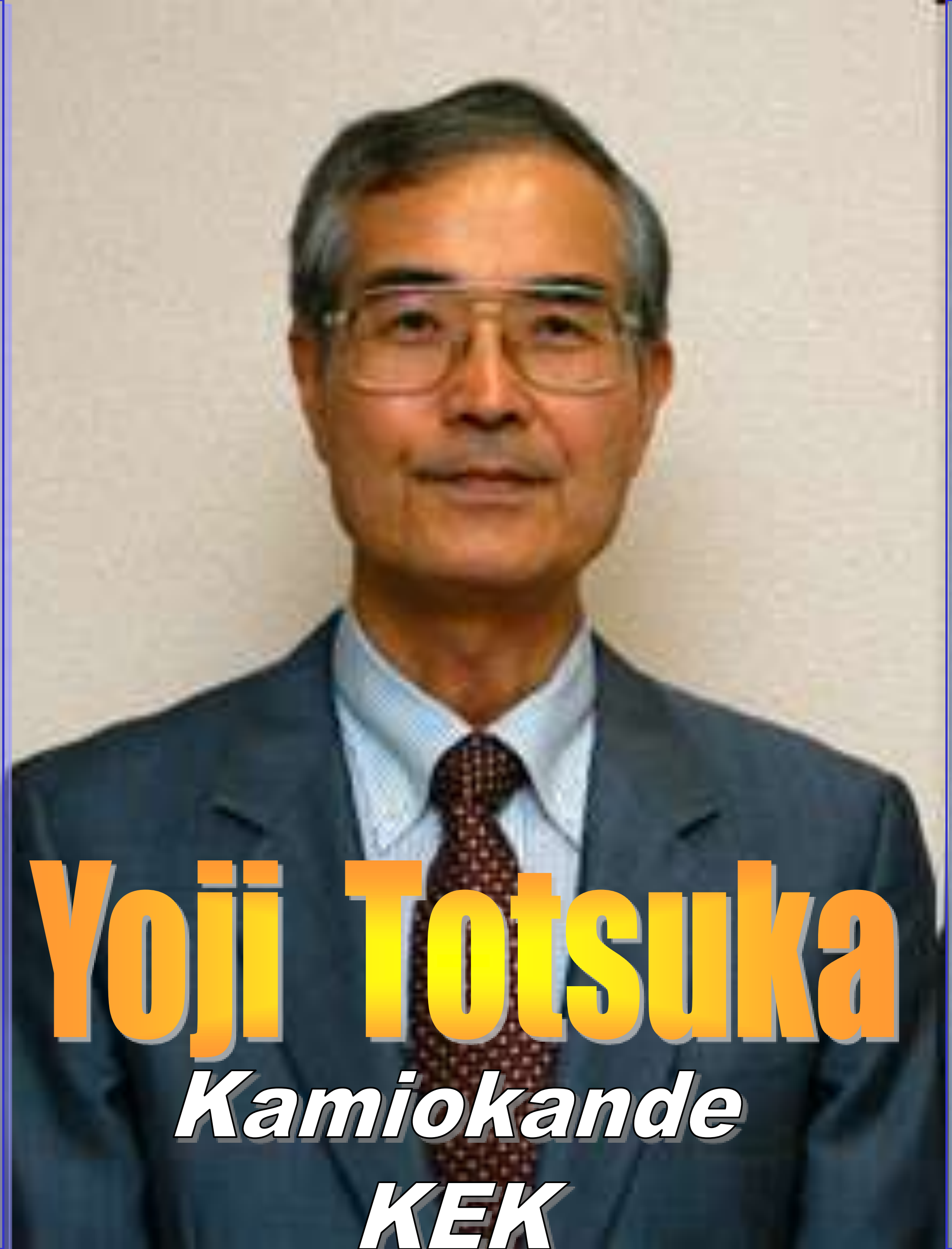
$$Exp : \sin 2\beta = 0.681 \pm 0.025$$

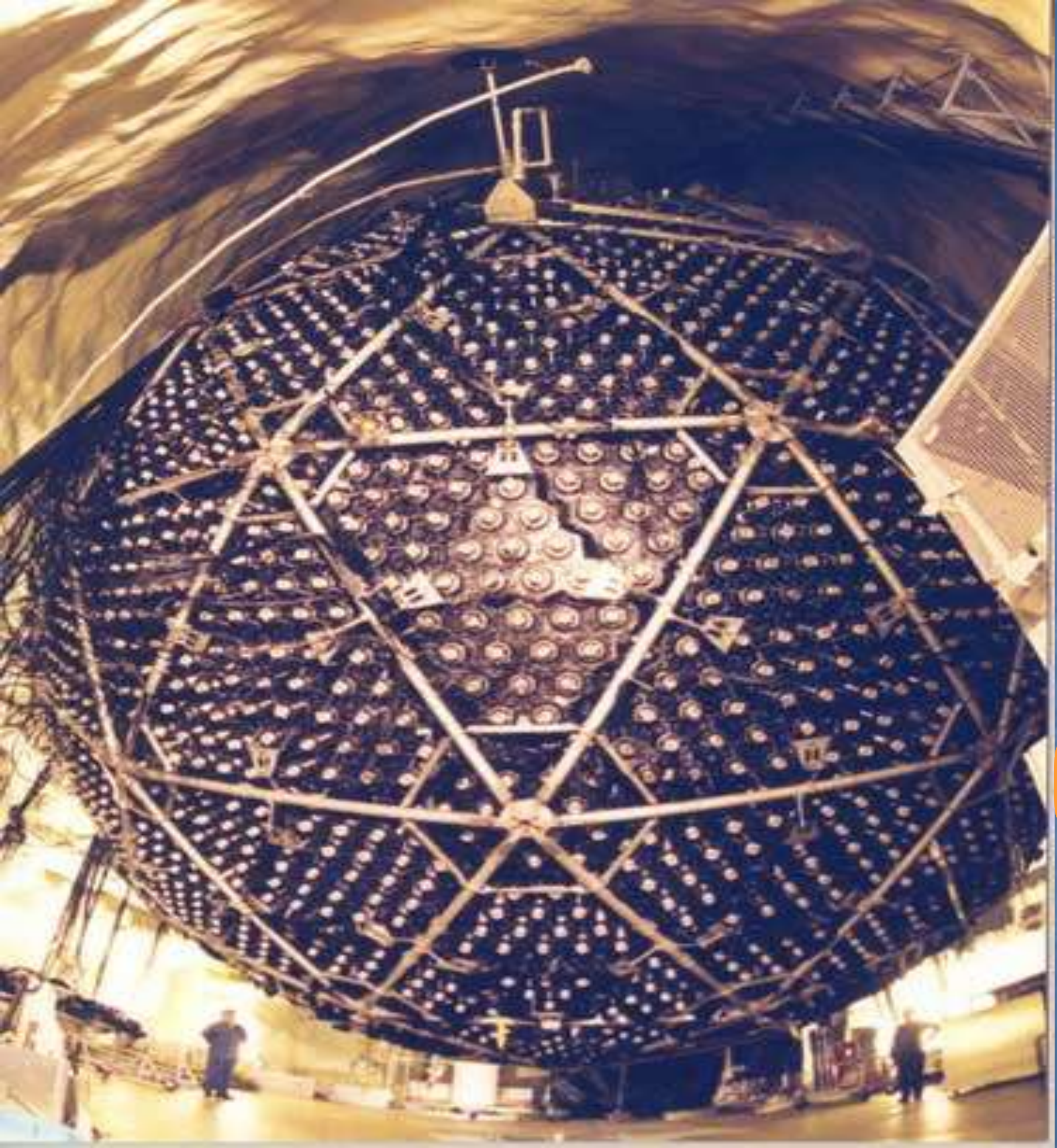
$$\phi \approx \alpha = 90^0$$

maximal CP-violation

Neutrinos


Kamioka


Yoji Totsuka
Kamiokande
KEK



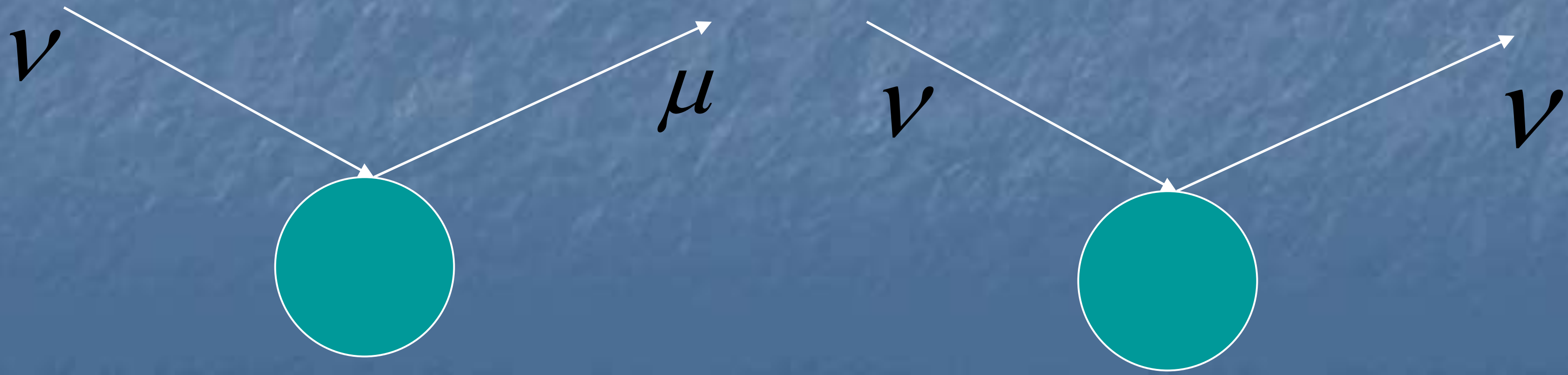
S N O

Sudbury Neutrino Observatory

Canada

S N O

charged current and neutral current



Kamiokande, SNO

neutrino mass differences

$$\Delta m_{21}^{2} \approx 8^{+0.6}_{-0.4} \cdot 10^{-5} eV^{2}$$

$$\Delta m^{2}_{32} \approx 2.4^{+0.6}_{-0.5} \cdot 10^{-3} eV^{2}$$

(absolute masses unknown)

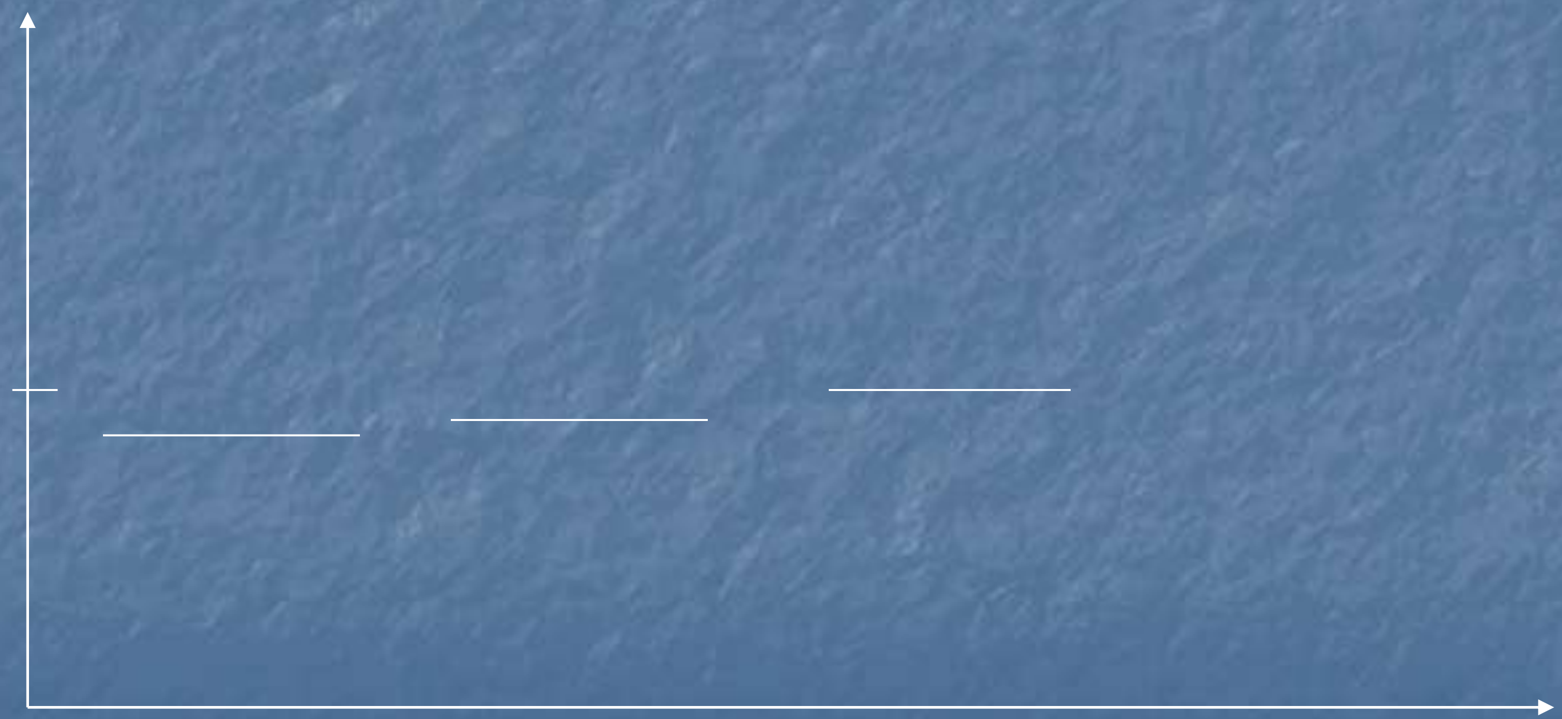
Neutrino masses:

A: masses about 1 eV

m(1) = 0.94 eV

m(2) = 0.95 eV

m(3) = 1 eV (nearly degenerate)



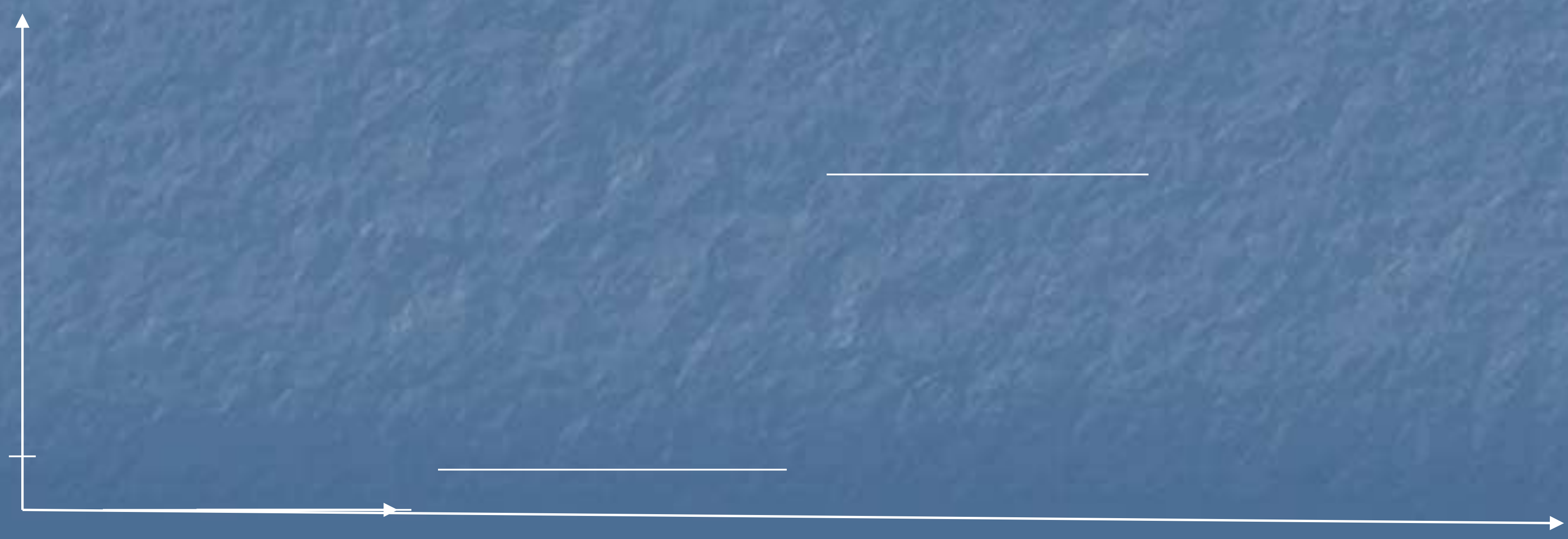
Neutrino masses:

B: masses much smaller than 1 eV

m(1) = 0 eV

m(2) = 0.009 eV

m(3) = 0.06 eV (strong hierarchy)



Neutrino masses:

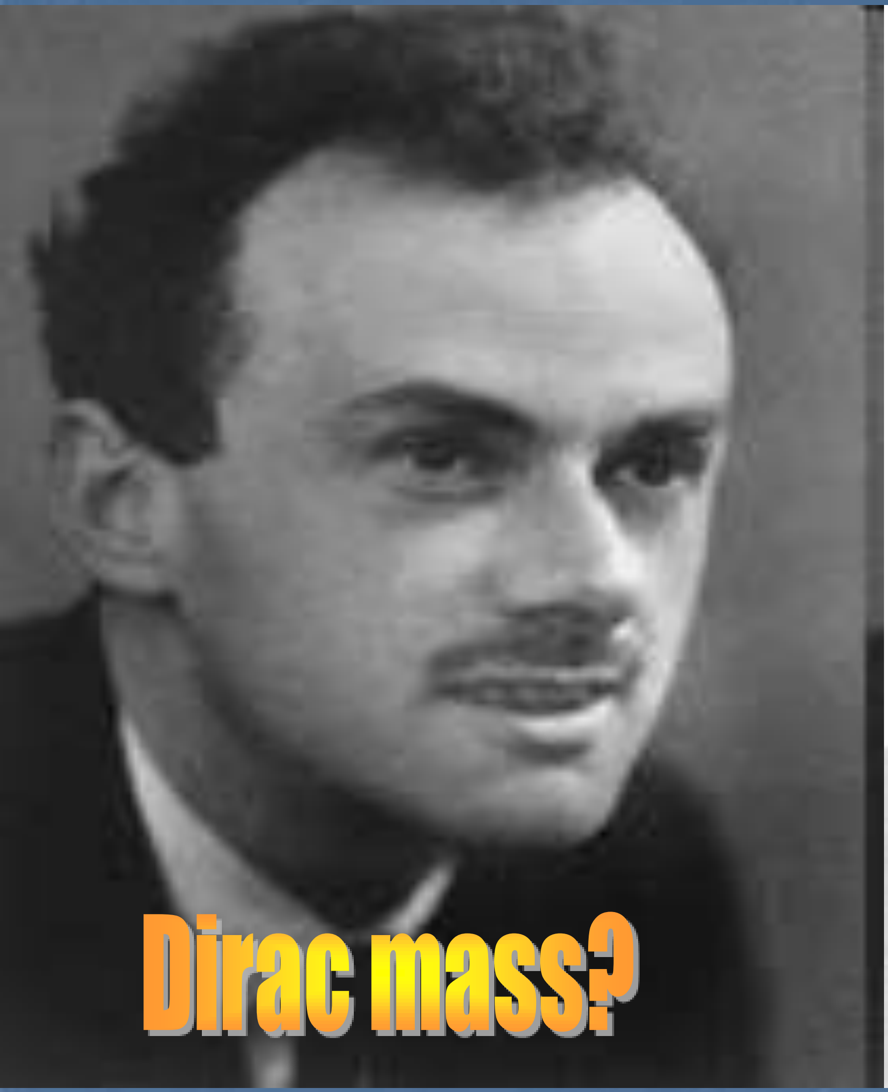
C: neither hierarchy nor degeneracy

m(1) = 0.10 eV
m(2) = 0.14 eV
m(3) = 0.29 eV

reality

neutrino masses

What type of mass term?


Dirac mass?


Majorana mass?

Superposition of Dirac mass and Majorana mass:

See-Saw Mechanism

$$M_\nu = \begin{bmatrix} 0 & D \\ D & M \end{bmatrix}$$

D: Dirac mass

M: Majorana mass

$$m_\nu = \frac{D^2}{M}$$

Minkowski
Yanagida
Gell-Mann, Ramond, Slansky
1978

Neutrino Masses:

Mass terms for charged leptons and neutrinos are not parallel ➔

Neutrino Mixing

(Pontecorvo ,1957... ==>)

Bruno Pontecorvo

1913 - 1993

neutrino mixing matrix:

(like CKM Matrix)

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

$$\nu_e = V_{1e}\nu_1 + V_{2e}\nu_2 + V_{3e}\nu_3$$

$$\nu_\mu = V_{1\mu}\nu_1 + V_{2\mu}\nu_2 + V_{3\mu}\nu_3$$

$$\nu_\tau = V_{1\tau}\nu_1 + V_{2\tau}\nu_2 + V_{3\tau}\nu_3$$

V = U x P

$$P = \begin{bmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \cos\theta_l & \sin\theta_l & 0 \\ -\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \bullet \begin{bmatrix} \cos\theta_\nu & -\sin\theta_\nu & 0 \\ \sin\theta_\nu & \cos\theta_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\theta \approx \theta_{at}$ $\theta_\nu \approx \theta_{sun}$ $\theta_l \approx reactor - angle$ $(unknown)$

standard parametrization:

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \cos\theta_l & \sin\theta_l & 0 \\ -\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \bullet$$

$$\begin{bmatrix} \cos\theta_\nu & -\sin\theta_\nu & 0 \\ \sin\theta_\nu & \cos\theta_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(F. , Xing)

Kamiokande, SNO

$$31.7^o \le \theta_{sun} \le 36.3^o$$

$$38^o \le \theta_{at} \le 52^o$$

$$\Delta m_{21}^{\ 2} \approx 7.6 \cdot 10^{-5} eV^2$$

$$\Delta m^2{}_{32} \approx 2.4 \cdot 10^{-3} eV^2$$

Theory:

$$\begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}$$

3 texture zeros

(charged leptons and neutrinos)

$$U=\begin{bmatrix}\cos\theta_l & \sin\theta_l & 0\\ -\sin\theta_l & \cos\theta_l & 0\\ 0 & 0 & 1\end{bmatrix}\bullet\begin{bmatrix}e^{-i\varphi} & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}\bullet\begin{bmatrix}\cos\theta_\nu & -\sin\theta_\nu & 0\\ \sin\theta_\nu & \cos\theta_\nu & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\tan 2\theta_l=\frac{2\sqrt{m_e m_\mu}}{m_\mu-m_e}\cong 0.0695$$

$$\tan 2\theta_\nu=\frac{2\sqrt{m_1 m_2}}{m_2-m_1}$$

Observation:

$$\theta_\nu \approx 33^o \;\_\_\_\; \theta \approx 45^o$$

$$:$$

$$==> m_1 / m_2 \approx 0.42_{-0.04}^{+0.12}$$

weak mass hierarchy
for neutrinos

$$\Delta m_{21}^{2} \approx 8 \cdot 10^{-5} eV^{2}$$

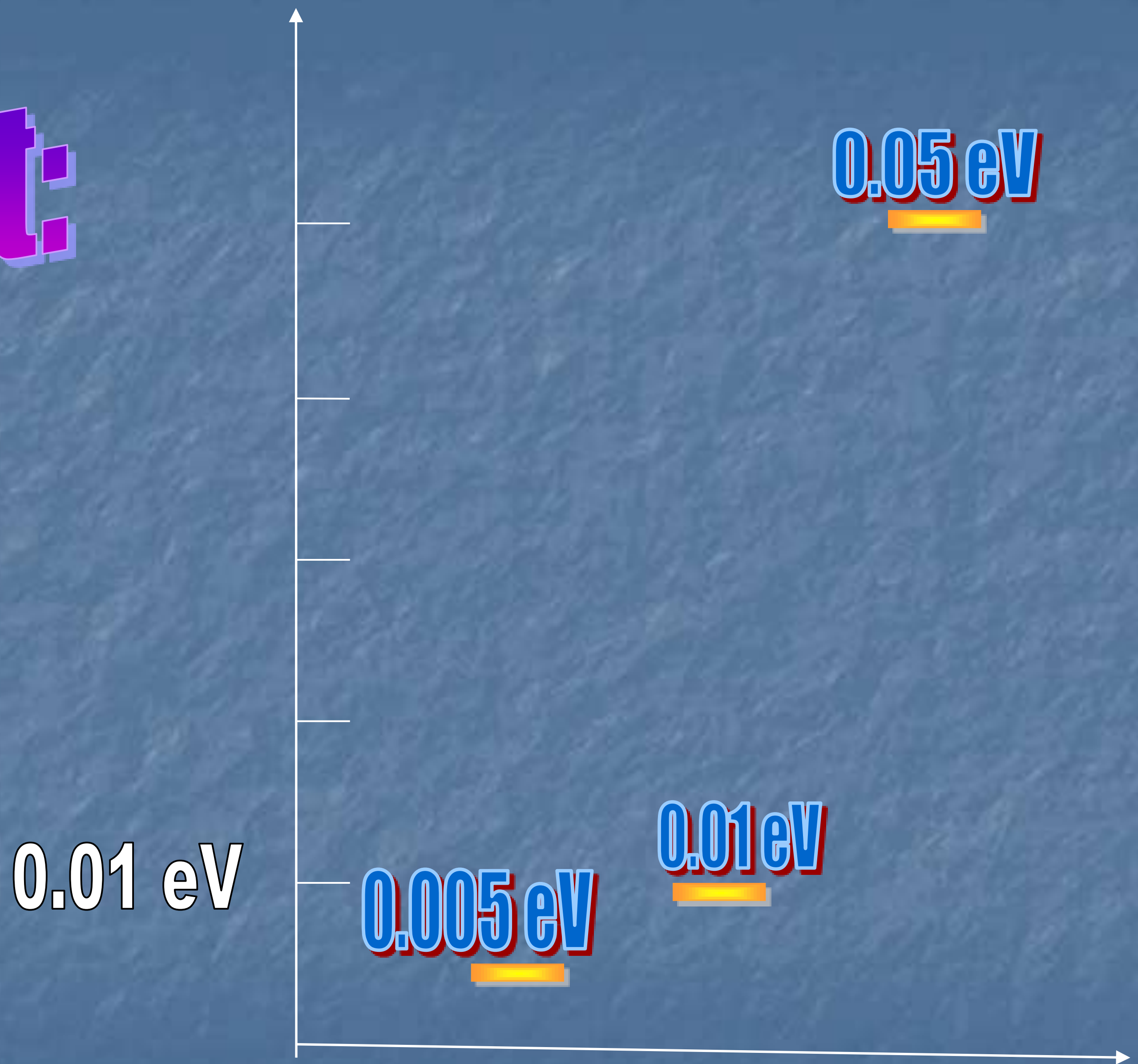
$$\Delta m^{2}_{32} \approx 2.4 \cdot 10^{-3} eV^{2}$$

$$m_1 / m_2 \approx 0.42$$

:

==> neutrino masses fixed

result:



m(1) = (0.004 +/- 0.0012) eV

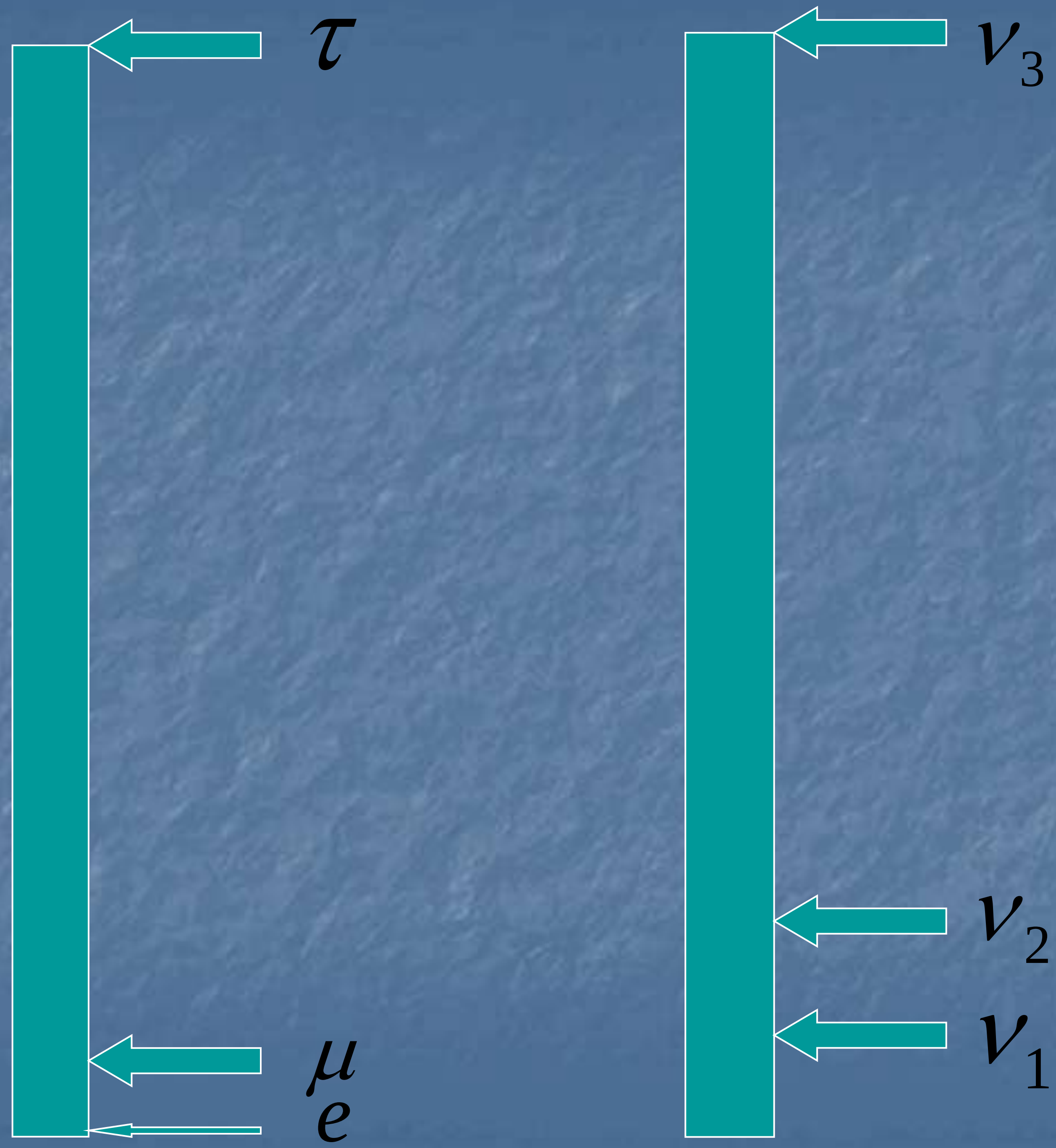
m(2) = (0.011 +/- 0.002) eV

m(3) = (0.051 +/- 0.007)eV

normal mass hierarchy
(no inversion)

Masses

(relative)



weak mass hierarchy for neutrinos

⇒ large mixing angles

Leptons: mass matrices

$$M = \begin{bmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & D \end{bmatrix}$$

4 texture zeros

Reflection symmetry in SO(10) theory

atmospheric mixing angle

$$\theta_{at} = \theta_1 + \theta_2$$

$$\theta_1 \approx \sqrt{\frac{m_\mu}{m_\tau}} \approx 14^o \qquad \theta_2 \approx \sqrt{\frac{m_2}{m_3}} \approx 26^o$$

$$=====>\theta \approx 40^o$$

$$\sin^2 2\theta \approx 0.97$$

(agrees with exp.)

$$M = \begin{bmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & D \end{bmatrix}$$

A mass matrix of this type does not work for the quarks.

m(top) < 90 GeV

quarks

$$M = \begin{bmatrix} 0 & A & 0 \\ A^* & \mathbf{X} & B \\ 0 & B^* & D \end{bmatrix}$$

Alternative approach

(with Z. Xing,
to appear in Phys. Lett. B)

$$M_l = \begin{bmatrix} 0 & A & 0 \\ A^* & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

$$M_\nu = \begin{bmatrix} 0 & X & -X \\ X & Y & Z \\ -X & Z & Y \end{bmatrix}$$

==>

$$U = \begin{pmatrix} \sqrt{\frac{m_\mu}{m_e + m_\mu}} & \sqrt{\frac{m_e}{m_e + m_\mu}} & 0 \\ -\sqrt{\frac{m_e}{m_e + m_\mu}} & \sqrt{\frac{m_\mu}{m_e + m_\mu}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & \sqrt{1/2} & \sqrt{1/2} \\ 0 & -\sqrt{1/2} & \sqrt{1/2} \end{pmatrix} \bullet \begin{pmatrix} \sqrt{\frac{m_2}{m_1 + m_2}} & -\sqrt{\frac{m_1}{m_1 + m_1}} & 0 \\ \sqrt{\frac{m_1}{m_1 + m_2}} & \sqrt{\frac{m_2}{m_1 + m_2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

same relations for mixing angles

plus: $\theta \approx \theta_{at}$ = 45 degrees

Neutrino Mixing Matrix:

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

<====== *not 0*

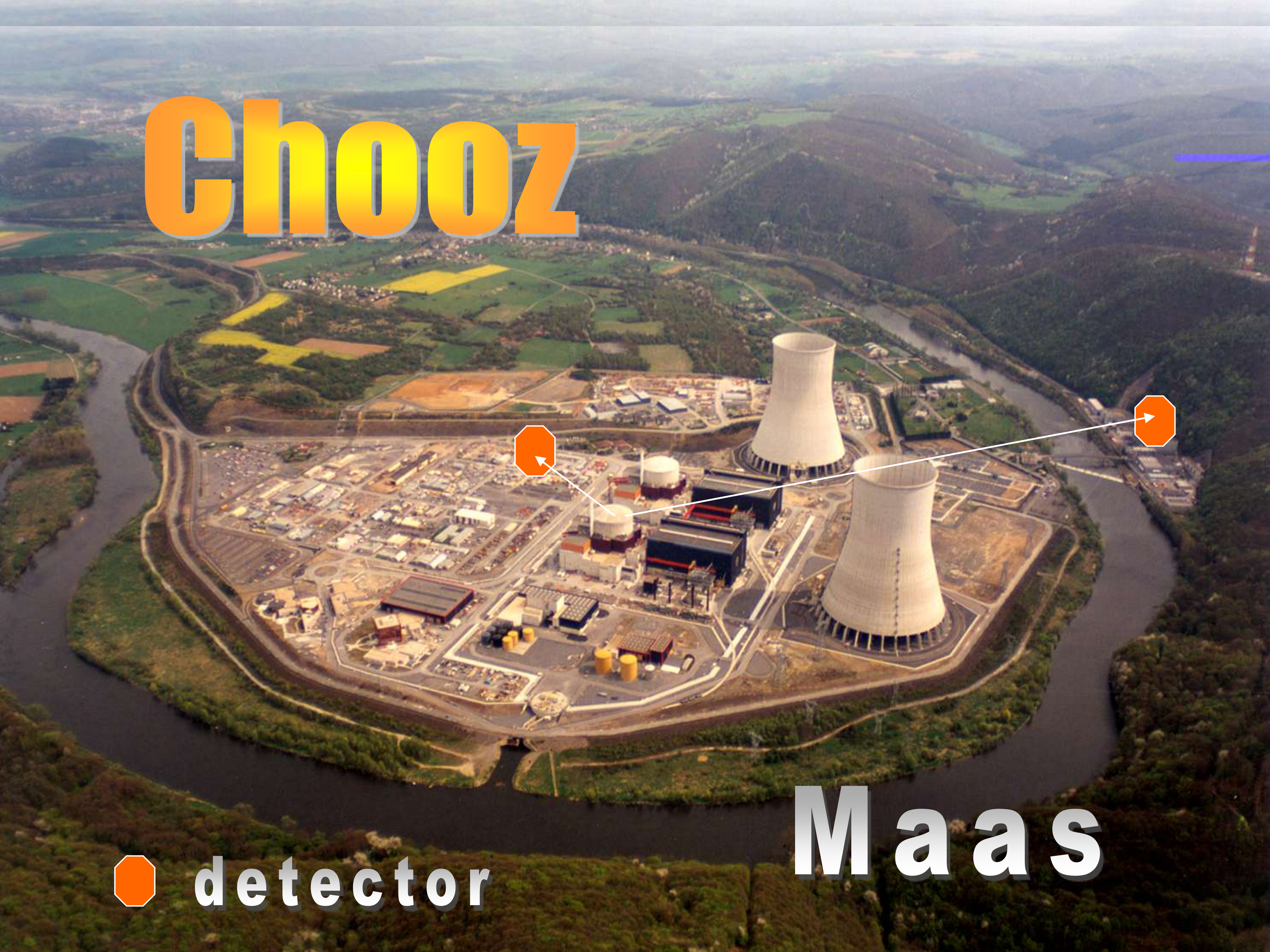
$$V_{e3} = \sin\theta_l \sin\theta \qquad \tan\theta_l = \sqrt{\frac{m_e}{m_\mu}} \cong 0.0695$$

$$38^o \le \theta_{at} \le 52^o$$

prediction:

$$\left|V_{e3}\right| = \sin\theta_{13} \cong 0.049 \pm 0.006$$

$$\sin^2 2\theta_{13} = 0.0096 \pm 0.0022$$


Chooz
Maas
detector

present limit from CHOOZ:

$$V(e3) < 0.2$$

expected: 0.05

Double Chooz:

2012

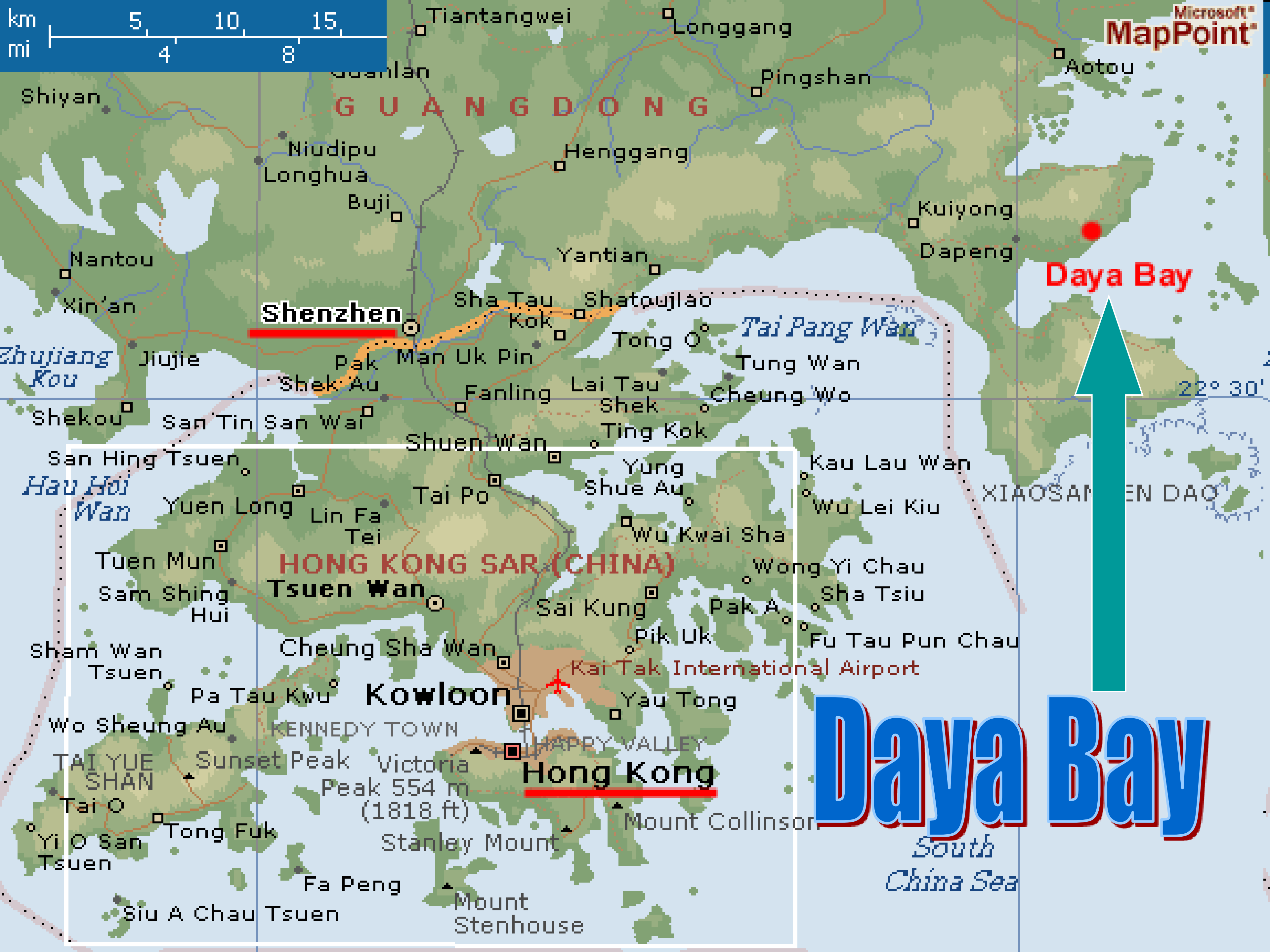
limit $\sin^2 2\theta_{13} : \_\ 0.03$

expected: 0.01

Minos, 2009:

$V(e3) = 0.1 +/- 0.06$

expected: 0.05

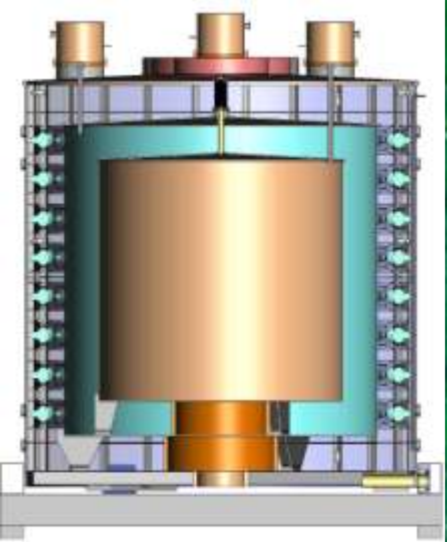


km 5 10 15
mi 4 8
Microsoft® MapPoint®
Tiantangwei
Longgang
Aotou
Guanlan
Pingshan
Shiyan
G U A N G D O N G
Niudipu
Longhua
Henggang
Buji
Kuiyong
Dapeng
Nantou
Yantian
Daya Bay
Xin'an
Shenzhen
Sha Tau Kok
Shatoujiao
Tai Pang Wan
Zhujiang Kou
Jiujie
Pak Shek Au
Man Uk Pin
Tong O
Tung Wan
Fanling
Lai Tau Shek
Cheung Wo
22° 30'
Shekou
San Tin San Wai
Ting Kok
Shuen Wan
San Hing Tsuen
Hau Hoi Wan
Yung Shue Au
Kau Lau Wan
Yuen Long
Lin Fa Tei
Tai Po
Wu Lei Kiu
XIAOSANMEN DAO
Wu Kwai Sha
Tuen Mun
HONG KONG SAR (CHINA)
Wong Yi Chau
Sam Shing Hui
Tsuen Wan
Sai Kung
Sha Tsiu
Pak A
Pik Uk
Sham Wan Tsuen
Cheung Sha Wan
Fu Tau Pun Chau
Kai Tak International Airport
Pa Tau Kwu
Kowloon
Yau Tong
Wo Sheung Au
KENNEDY TOWN
HAPPY VALLEY
TAI YUE SHAN
Sunset Peak
Victoria Peak 554 m (1818 ft)
Hong Kong
Tai O
Tong Fuk
Mount Collinson
Yi O San Tsuen
Stanley Mount
South China Sea
Fa Peng
Siu A Chau Tsuen
Mount Stenhouse
Daya Bay


Daya Bay
4 reactors


far detector
near detector
LingAo II
LingAo cores
near detector
Daya Bay cores
Daya Bay

Daya Bay



Total Tunnel length ~3000 m

- Multiple detectors per site cross-check detector efficiency
- Two near sites sample flux from reactor groups

prediction:

$$\sin^2 2\theta_{13} = 0.0096 \pm 0.0022$$

Daya Bay:

limit ~ 0.008

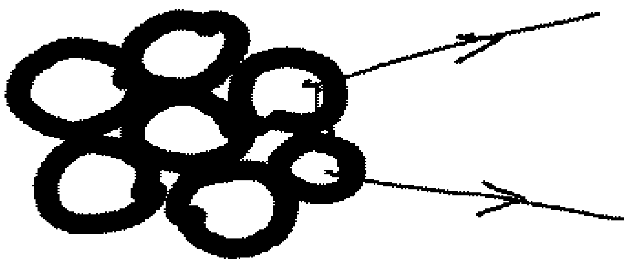
(observation in 3 years?)

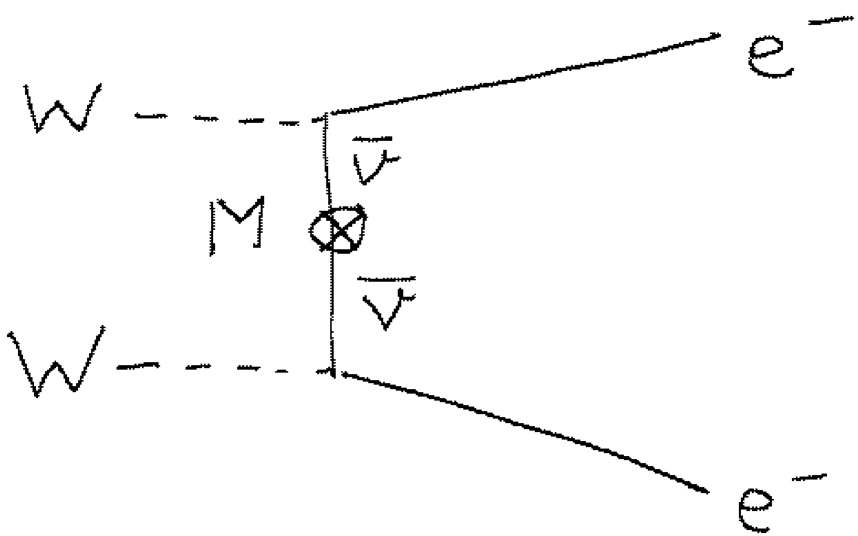
Majorana masses:

no fermion number

neutrino = antineutrino

Neutrinoless double β-decay





(decay via Majorana mass term)

Neutrinoless Double Beta Decay

Present limit about 0.23 ev

Cuoricino Exp.: Te (130)

Gran Sasso Lab.

expected:

$$m.e.:(0.05)\ (0.05)\ \approx 0.0025$$

$$m_3 \qquad V_{e3}$$

factor 100 improvement !?

CP violation of leptons:

maximal CP violation !

==> reactor neutrinos

Conclude:

Fermion masses remain a mystery. Neutrino masses might be Dirac masses or Majorana masses.

Mixing angles for quarks are fixed by ratios of quark masses. Very good agreement with experiment.

mass matrices of quarks and leptons:

Structure:

$$\begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}$$

3 texture zeros

This works also well for leptons. One finds, using the observed mass differences:

m(1)/m(2)=0.42,   m(2)/m(3)=0.19

m(1): 0.0041 eV
m(2): 0.0097 eV
m(3): 0.051 eV

Atmospheric angle about 40 degrees

Neutrinoless double beta decay: difficult

reactor neutrinos

$V_{3e} \sim 0.05$

==> Daya Bay

Observation in 2012 ?