Gauge Theories from Geometry

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Outline

1. Overview
2. Geometry of ADE singularities
    - Surface singularities
    - Singular curves in Calabi-Yau threefolds
3. Gauge Theory Description
    - Enhanced Gauge Symmetries
    - Adjoint Breaking

OVERVIEW.

- Codimension 2 *ADE* singularities give rise to non-abelian gauge symmetries in F-theory and type IIA string theory
    - IIA: Curve of ADE singularities in a Calabi-Yau threefold
    - F-theory: singular fibers over a surface in the threefold base of an elliptically fibered Calabi-Yau fourfold
        - Gauge theory realized by stacks of D7 branes wrapping the singular locus
- Worsening of the singularities in codimension 3 gives rise to charged matter

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- When we consider worldvolume theories, obtain analogous results for enhanced gauge symmetries and matter, but details are different

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EQUATIONS OF ADE SURFACE SINGULARITIES.

$$\begin{array}{cc} A_n & xy + z^{n+1} = 0 \\ D_n & x^2 + y^2 z + z^{n-1} = 0 \\ E_6 & x^2 + y^3 + z^4 = 0 \\ E_7 & x^2 + y^3 + yz^3 = 0 \\ E_8 & x^2 + y^3 + z^5 = 0 \end{array}$$

RESOLUTIONS OF ADE SINGULARITIES.

Kähler deformations.

- ADE surface singularities S can be resolved by surfaces $\tilde{S}$
- Exceptional $\mathbf{P}^1$'s intersect according to the dual graph of the corresponding Dynkin diagram with n vertices

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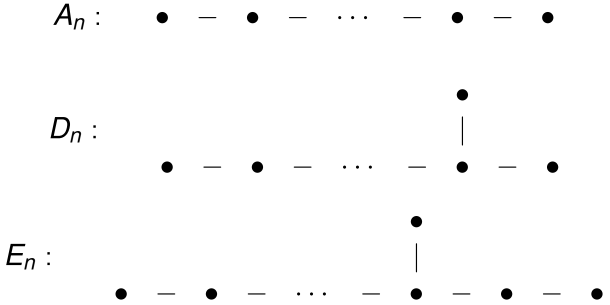

DEFORMATIONS.

- The versal deformation space Res of the resolved singularity $\tilde{S}$ can be identified with the root space of the corresponding ADE root system
- The versal deformation space Def of the singularity S can be identified with the quotient of Res by the corresponding Weyl group
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- Consider the A_n case $xy + z^{n+1} = 0$
- The exceptional curves $C_i \subset \tilde{S}$ are parametrized by $z^i/x,\ i = 1, \ldots, n$.
- Now $\tilde{S}$ can be deformed.

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VERSAL DEFORMATION OF $\tilde{S}$, THE A_n CASE.

- Introduce deformation parameters of $\mathrm{Res}(A_n)$: $(t_1, \ldots, t_{n+1}),\ \sum t_i = 0$
- Deform singularity to S_t: $xy + \prod_{i=1}^{n+1}(z + t_i) = 0$
- If $t_i = t_j$, then S_t is singular at $(x, y, z) = (0, 0, -t_i)$, generically A_1.
- Can blow up to resolve singularities

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THE A_n ROOT SYSTEM.

- A_n Cartan subalgebra $h = \{(t_1, \ldots, t_{n+1}) \mid \sum t_i = 0\}$
- Roots in h^*: $e_i^* - e_j^*$ for $i \neq j$
  - This root is orthogonal to the hyperplane $t_i = t_j$ in h.
- Positive simple roots $v_i = e_i^* - e_{i+1}^*$.

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DEFORMATIONS OF $\tilde{S}$, A_n CASE.

- Over $t_i = t_{i+1}$ (the hyperplane orthogonal to v_i), the deformed C_i is parametrized by $(z+t_1)(z+t_2)\cdots(z+t_i)/x$.
- Over $t_i = t_j$ (the hyperplane orthogonal to $v_i + \ldots + v_{j-1}$), the curve $C_i + C_{i+1} + \ldots + C_{j-1}$ deforms

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VERSAL DEFORMATION OF S, THE A_n CASE.

- Rewrite S_t as $xy + z^{n+1} + \sum_{i=2}^{n+1} \sigma_i z^{n+1-i}$
- The σ_i are the elementary symmetric functions of the $t_1, \ldots, t_{n+1}$.
- $(\sigma_2, \ldots, \sigma_{n+1})$ are coordinates on $\mathrm{Def}(A_n)$
- $\mathrm{Def}(A_n)$ is the quotient of $\mathrm{Res}(A_n)$ by the Weyl group S_{n+1} of A_n; parametrizes deformations of S.

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GENERAL ADE SURFACE SINGULARITIES.

- Positive simple roots correspond to components of exceptional curves in $\tilde{S}$
- Positive roots correspond to exceptional divisors (not necessarily irreducible) in $\tilde{S}$
- The complex structure deformation space Res of the resolved ADE is parametrized by the root space of the corresponding root system
    - Exceptional divisors persist over the hyperplane orthogonal to the corresponding root
- The deformation space Def of the singularity is parametrized by the quotient of Res by the corresponding Weyl group
    - Singularities persist over the hypersurfaces covered by the above hyperplanes

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Outline

THE LOCAL A_n CASE.

- Fiber the ADE geometry over a smooth curve B of genus g
- $xy + z^{n+1} = 0, \qquad x \in L, \qquad y \in K_B^{n+1} \otimes L^{\otimes -1}, \qquad z \in K_B$
- Singularity can be resolved by n exceptional divisors E_i, each fibered over B by curves C_i
- Can deform this threefold before or after the resolution of singularities

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DEFORMATIONS OF LOCAL A_n.

- $xy + z^{n+1} + \sum_{i=2}^{n+1} \sigma_i z^{n+1-i} = 0, \qquad \sigma_i \in H^0(B, K_B^{\otimes i})$ deforms the threefold
  - Number of parameters $\sum_{i=2}^{n-1}(2i-1)(g-1) = (n^2-1)(g-1)$
- When the deformation is of the form $xy + \prod_{i=1}^{n+1}(z+\omega_i) = 0, \qquad \omega_i \in H^0(B, K_B)$, the deformation can be resolved to become a deformation of the resolved geometry
  - Unlike the surface case, the existence of a factorization is not generic — only *ng* parameters.

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GLOBAL CASE.

- Can have compact models of Calabi-Yau threefold X containing a curve B of ADE singularities
- Many global examples arise as Calabi-Yau hypersurfaces in singular toric varieties (e.g. weighted projective spaces).

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GAUGE SYMMETRIES AND MATTER IN IIA STRING THEORY.

- Compactification of IIA on Calabi-Yau threefold X yields $N = 2$ theory in 4 dimensions
- Kähler moduli of X contained in vector multiplets
  - Effective $U(1)^{h^{1,1}(X)}$ gauge theory at generic points of Kähler moduli
  - $U(1)$ factors associated with elements $D \in H^2(X, \mathbf{Z})$
- Complex structure moduli of X contained in neutral hypermultiplets

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MATTER ASSOCIATED WITH CONTRACTING CURVES.

- D2-branes wrapping holomorphic curves C appear as solitons in 4d, part of charges given by intersections $D \cdot C$
  - Hypermultiplets become massless when area of C goes to zero
- $U(1)^n$ factor associated with exceptional divisors of resolution of ADE singularity at generic points in moduli
- D2-branes wrapping fibers C_i charged under $U(1)^n$, parametrized by curve B.
  - Effectively a twisted gauge theory on $B \times \mathbf{R}^4$
  - Even cohomology of B gives vectors in 4d
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MULTIPLET STRUCTURE OF THE MATTER.

- The $U(1)^n$ gauge factor is enhanced to the full ADE group.
  - The n vectors associated with the Kähler moduli of the exceptional divisors combine with the vectors associated with the branes (corresponding to the nontrivial roots) to realize a non-abelian gauge theory with the corresponding ADE group.
- The effective theory contains g adjoint hypermultiplets
  - The ng hypermultiplets corresponding to the complex moduli of the resolution combine with the hypermultiplets associated with the branes (corresponding to the nontrivial roots) to form g adjoint hypermultiplets.
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- Rewrite in $N = 1$ superfield notation
- $N = 2$ vector is an $N = 1$ vector and an $N = 1$ chiral
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- The effective Lagrangian is

$$\mathcal{L} = \mathrm{Im}\left[\mathrm{Tr}\int d^4\theta\left(M_i^\dagger e^V M^i + \tilde{M}^{\dagger i} e^V \tilde{M}_i + \Phi^\dagger e^V \Phi\right)\right.$$
$$\left. + \frac{\tau}{2}\int d^2\theta \mathrm{Tr} W^2 + i\int d^2\theta \mathcal{W}\right]$$

  where $\mathcal{W} = \mathrm{Tr}\tilde{M}^i[\Phi, M_i]$ is the superpotential
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$$\mathrm{Tr}\left[[m_i, m^{\dagger i}]^2 + [\tilde{m}^i, \tilde{m}_i^\dagger]^2 + [\phi, \phi^\dagger]^2\right.$$
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  - $[\phi, \phi^\dagger] = 0$ implies $\phi = \mathrm{diag}(\phi_1, \ldots, \phi_{n+1})$ up to gauge, $\sum \phi_i = 0$
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- A suggestive way to parametrize g adjoint hypermultiplets in the A_n case is as a traceless $n \times n$ matrix of holomorphic 1-forms on B, up to conjugation by a scalar matrix.
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Outline

1. Overview
2. Geometry of ADE singularities
   - Surface singularities
   - Singular curves in Calabi-Yau threefolds
3. Gauge Theory Description
   - Enhanced Gauge Symmetries
   - Adjoint Breaking

MATTER FROM GEOMETRY.

- Additional charged matter localizes at points of B where ADE singularity gets worse
- Matter arises from adjoint breaking mechanism $G \to H \times U(1)$ K and Vafa hep-th/96006086
  - *Remark:* The terminology refers to the breaking of a 6d gauge symmetry. In 4d, this turns into the decomposition of hypermultiplets in the adjoint representation of the larger 6d gauge group into irreducible representations of the 4d gauge group. There is no broken gauge symmetry in 4d.

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- Suppose singularity gets worse over $t = 0 \in B$, corresponding to gauge group G in 6d.
- D-branes wrap cycles corresponding to roots of G
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- Singular at $(x, y, z) = (0, t^3, -t^2)$
- D_5 for $t = 0$
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