RPMU talk II Sakura

Plan: From semi-local to global

1) Global consistency constraints
2) Construction of a 3-fold base B_3
3) consistent semi-local model w/ 3 generations realized in B_3.

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1) Global constraints

Input: Physics requirements: SU(5) GUT

- $3 \times 10_M$, $3 \times \bar{5}_M$, 5_H, $\bar{5}_H$
- $W \supseteq 10\, 10\, 5 + \bar{5}\, \bar{5}\, 10$
- ~~GUT~~ & 3/2 splitting w/ $U(1)_Y$ flux. $F_Y|_M = 0$, $F_Y|_H = 1$, $F_Y|_{\bar{H}} = -1$

Realized in $E_2 \to X_4$, $K_X = 0$.

$$\downarrow$$

$B_3 \supseteq S_{gut}$.

Geometrical

- S_{gut} dP_n sfc.
- A_4 sing over S_{gut}.
  - $\to A_5$: $\bar{5}_M$, 5_H, $\bar{5}_H$
  - $\to D_5$: 10_M.
- D_6 E_6 for Yukawas.

~~D3 tadpole~~ **global constraints**

- $U(1)_Y$ GUT breaking: $\nearrow \int_{\mathbb{R}^{3,1}\times S} C_4 \wedge Tr(F_{3,1} \wedge F_Y)$

  F_Y be the connection for A_Y.

  gauge boson of $U(1)_Y$ remains massless $\Longleftrightarrow$ $[F_Y]$ dual is a trivial cycle in B_3

  $H_2(S_{GUT}, \mathbb{Z}) \xrightarrow{i} H_2(B_3, \mathbb{Z})$

  $\text{Ker}\, i$.

- D7-brane tadpole : $K_X = 0$

$$12 K_B + \sum a_i D_i = 0$$

$\uparrow$ a_i = degree of degeneration over comp of discriminant D_i.

$$\Rightarrow K_B \neq 6.$$

- Flux: quantization:

$$[G_4] - \frac{c_2(X_4)}{2} \in H^4(X_4, \mathbb{Z})$$

J-brane tadpole $\Longleftarrow$ $J \wedge G = 0$

- D3-brane tadpole :

$$\frac{\chi(X_4)}{24} = \frac{1}{2} \int G_4 \wedge G_4 + N_{D3}$$

$\chi = \chi^* + \chi_F - \chi_{E_8}$.

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(2) 3-fold base B_3

* $K_B \neq 0$ in fact $-K_B \geq 0$. ← Gen. cond.

& $U(1)_Y$ constraint: $H_2(S_{GUT}) \to H_2(B_3)$

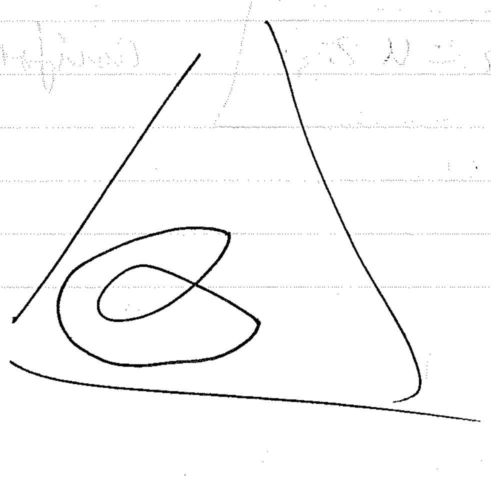
↳ has nontrivial kernel.

Construction: inspiration from Mori's classif. of Fano 3-folds where sim. kind of contractions appear when Pic drops.

$\mathbb{P}^3$: $[z_1 z_2 z_3 z_4]$

$K_{\mathbb{P}^3} = -4H$

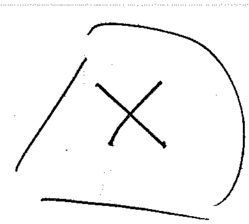
Nodal curve $\mathcal{C}$: $\begin{cases} z_1 z_2 z_4 + (z_1+z_2)^3 = 0 \\ z_3 = 0 \end{cases}$



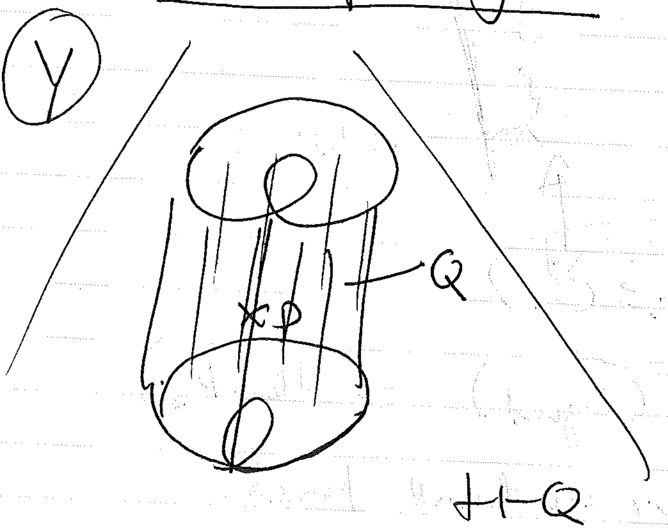
sing. point: $[0,0,0,1]$

vicinity: $z_1 z_2 = 0$

$z_3 = 0$.



Blowup along C : $Y \xrightarrow{\psi} \mathbb{P}^3$



$\mathbb{C}^3 \times \mathbb{P}^1$ (in patch $z_4 = 1$)
$\quad \| $
$[v_0, v_1]$.

$$v_0 (z_1 z_2 + (z_1+z_2)^3) = v_1 z_3 .$$

Sing. point: $p = \{(z_1, z_2, z_3) [v_0 v_1]\} = \{(0,0,0), [1,0]\}$

$$K_Y = -4H + Q$$

$$\psi^*(H) = Q + (H-Q) = H .$$

Patch: $v_0 \neq 0$ of $\mathbb{P}^1$: $\quad u = \frac{v_1}{v_0}$

$$\Rightarrow \quad z_1 z_2 + (z_1+z_2)^3 = u z_3 .$$

close to p:
$\Rightarrow$

$$\boxed{z_1 z_2 = u z_3}$$ conifold sing.

Blowup p :

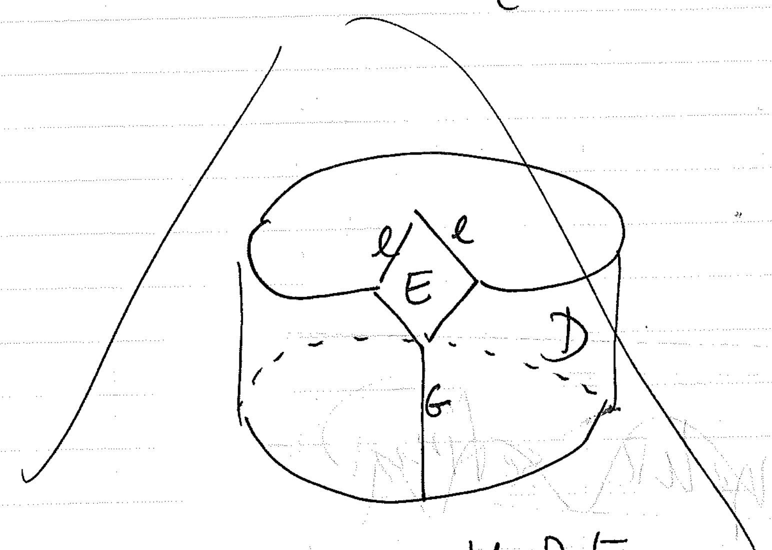
$\phi : \mathcal{B} \to Y$

Let $\mathbb{P}^3_W : [W_1\ W_2\ W_3\ W_4]$ w/ condition

$$W_1 W_2 = W_3 W_4$$

In patch $v_0 := 1$

$$\mathcal{B} = \left\{ (z_1 z_2 z_3\ v_1 ; W_1 \ldots W_4) \in \mathbb{C}^4 \times \mathbb{P}^3_W : \begin{cases} (z_1 \ldots u) \in [W_1, \ldots, W_4] \\ z_1 z_2 = z_3 u \\ W_1 W_2 = W_3 W_4 \end{cases} \right\}.$$



$$K_{\mathcal{B}} = -4H + (D+E) + E$$

$(\phi^* D = D + E)$.

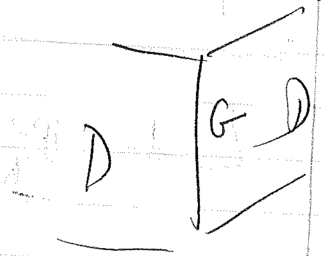
~~Typical to see~~:

E : exceptional div. of blowup;

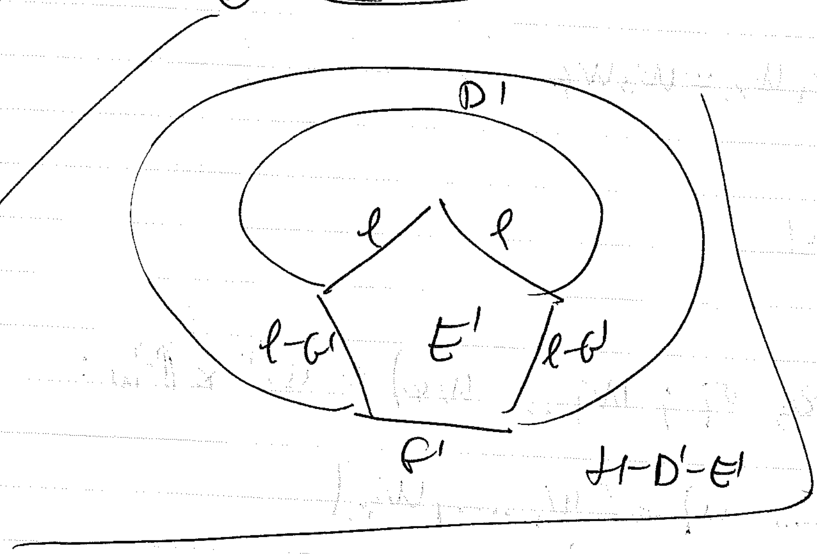
$\mathbb{P}^1_{(1)} : W_2 = W_4 = 0$ $\qquad$ $\mathbb{P}^1_{(2)} : W_2 = W_3 = 0$

have same $\cap$ w/ all divisors $\Rightarrow$ homologous.

$G = \mathbb{P}^1 \times \mathbb{P}^1$ $\quad H = dP_3$ $\quad H - D - G = \mathbb{P}^2$



↓ flop G $\quad (G \cdot D = -2)$.



$K_{\tilde{B}} = -4H + D' + 2E'$

$E' \cong dP_2$.

$H = dP_3$.

$D' = \mathbb{F}_4$.

$H - D' - E' = dP_1$.

~~[illegible] setting~~

	H	E'	$H-D'-E'$
H	l_0	0	$l_0-3(l-G')$
E'	0	$-2l+G'$	G'
$H-D'-E'$	$l_0-3l+3G'$	G'	$-2l_0+6l-5G'$

	H	E'	$H-D'-E'$
l_0	1	0	$+1$
l	0	-1	0
G'	0	-1	-1

$$H^3 = 1$$

$$E'^3 = 1$$

$$D'^3 = -6$$

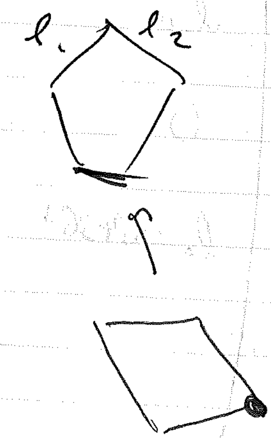
$$D'^2 H = -3$$

$$D'^2 E' = -2.$$

Holomph. Sections

Z_4	H	0
Z_1, Z_2	$(H-E)+E = H$	0
Z_3	$(H-D-E)+(D+E) = H$	0
$W_{1,2,3}$	$H-E$	h
W_4	$3H-D-2E$	$2h-e_1-e_2$
V_1	$3H-D-2E+E = 3H-D-E$	$(2h-e_1-e_2)-h$
V_0	$H-D-E$	$h-e_1-e_2.$

$S_{gut} = E' = dP_2$



$l_1 \simeq l_2$

$l_1^2 = l_2^2 = l_i \cdot G' = 0$

$l_1 \cdot l_2 = 1$

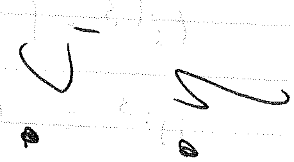
$G'^2 = -1$

$l_1 = h - e_1$

$l_2 = h - e_2$ in dP_2 basis.

$G' = h - e_1 - e_2$

in $S_{gut} = E'$	E'
H	0
D'	$e_1 + e_2$
E'	$-h$
$H-D-E$	$h - e_1 - e_2$.



$K_B \cdot G' = 0 \Rightarrow$ not Fano but weak Fano

$$\frac{\chi_{\text{geom}}}{24} = 582$$

$$= 12\int_B c_1 (c_2 + 30 c_1^2)$$

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③ Realizing 3-generation SU(5) GUT on B_3

Recall semi-local model:

$$y^2 = x^3 + b_5 xy + b_4 x^2 z + b_3 y z^2 + b_2 x z^3 + b_0 z^5$$

Data: $\eta = 6c_1 - t$ $\qquad$ $t = -c_1(N_{S_{gut}})$

$c_1 = c_1(S_{gut})$

Local sing	GUT interpret.	Locus
A_4	gauge fields.	$b_m \neq 0$ then
A_5	$5, \bar{5}$	$P = b_0 b_5^2 - b_2 b_3 b_5 + b_3^2 b_4 = 0$
D_5	10	$b_5 = 0$
D_6	$\bar{5}\,\bar{5}\,10$	$b_5 = b_3 = 0$
E_6	$5\,10\,10$	$b_5 = b_4 = 0$
E_8		$b_m = 0 \ \forall\, m \neq 0.$

$E_8 \to SU(5)_{gut} \times SU(5)_\perp$

b_m = elem. symm. polys in t_i ← wt of $SU(5)_\perp$.

e.g. $b_5 = \prod t_i$ $\qquad \sum t_i = 0$

Spectral cover : sfc. in $\mathbb{P}(\mathcal{O}_{S_{gut}} \oplus K_{S_{gut}})$

$b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 = 0$

$[U, V]$ $\mathbb{P}^1$.

U ← $\mathcal{O}(1) \otimes K_S$, V ← $\mathcal{O}(1)$

$s = U/V$: (# factors of $\mathcal{C}$) − (# indep. $U(1)$'s) − 1

Recall :
- Minimal SU(5) GUT
- No μ-term
- $U(1)_Y$ GUT

$\Rightarrow \mathcal{C} = \mathcal{C}^{(4)} \cdot \mathcal{C}^{(1)}$

$U(1)_X$	−1	3	2	−2
↑	10_M	$\bar{5}_M$	5_H	$\bar{5}_H$

↓ S_{gut}

Problems

- No $U(1)_{PQ}$
- No ν scen. realized.
- also: as in all other models:

  gauge coupl. unif.

(*) $\alpha_1^{-1}(M_{gut}) - \frac{3}{5}\alpha_2^{-1}(M_{gut}) - \frac{2}{5}\alpha_3^{-1}(M_{gut}) = 0$

- dim 5 proton decay $\int \frac{QQQL}{\Lambda}$ ok. ↯

⇒ Can resolve all issues by relaxing minimality:

Simplest model : 3 + 2

$\equiv \}\, C^{(1)}$

$= \}\, C^{(2)}$

↓

— S_{gut}.

- $U(1)_{PQ}$
- $F_Y \Big|_{\Sigma \text{ Matter}} \neq 0 \rightarrow$ non GUT exotics.

  ⇒ not compatible w/ (*)

Monodromy gp

$$S_3 \times \mathbb{Z}_2 \supseteq G^{(1)} \times G^{(2)}.$$

$$\mathcal{C} \to (a_0 U^3 + a_1 U^2 V + a_2 U V^2 + a_3 V^3)(e_0 U^2 + e_1 UV + e_2 V^2) = 0.$$

& $b_1 = 0 \iff$ ($\sum t_i = 0 \iff$)

$a_1 e_0 + a_0 e_1 = 0$

$\Rightarrow$ solve by $a_0 = \alpha e_0$

$a_1 = -\alpha e_1$

$X_{\mathcal{C}}$ — $P(\mathcal{O} \oplus K) \xrightarrow{\pi} S$

$$[\mathcal{C}^{(1)}] = 3\sigma + \pi^*(\eta - 2c_1 - \xi)$$

$$[\mathcal{C}^{(2)}] = 2\sigma + \pi^*(2c_1 + \xi)$$ ← free dims.

↓

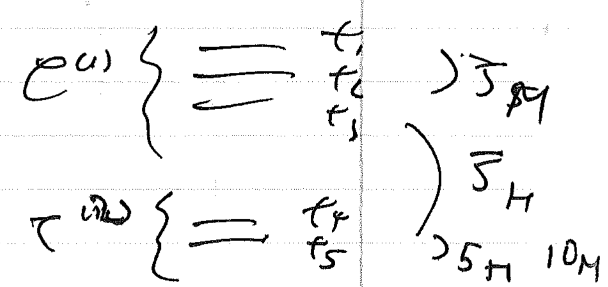
$$[\mathcal{C}] = 5\sigma + \pi^*(\eta)$$

Explicit sections

	divisors in X_c
u	σ
v	$\sigma + \pi^* c_1$
b_m	$\pi^*(\eta - m c_1)$
e_2	$\pi^* \xi$
e_1	$\pi^*(c_1 + \xi)$
e_0	$\pi^*(2c_1 + \xi)$
a_m	$\pi^*(\eta - (m+2)c_1 - \xi)$
α	$\pi^*(\eta - 4c_1 - 2\xi)$

↖ we realize these explicitly later in B_3.

How to assign matter curves s.t. Yukawas o.k.:



10_{other} $10^{(1)}$: t_1, t_2, t_3

10_M ? $10^{(2)}$: t_4, t_5

$\bar{5}_M$ $\bar{5}^{(1)}$: $t_i + t_j$ $\quad i,j = 1..3$

5_H $\bar{5}^{(2)}$: $t_4 + t_5$

$\bar{5}_H$ $\bar{5}^{(1)(2)}$: $t_i + t_a$.

	10_M	5_H	$\bar{5}_H$	$\bar{5}_M$	10_{vec}
$U(1)_{PQ}$	3	-6	t_1	-4	-2

μ forbidden.

Neutrinos

$t_i - t_j \triangleq$ singlet.

$N_0 \quad N_R : \quad H_u L N_R$

$-t_4 - t_5 \quad t_1 + t_2 \quad \zeta$

But Dirac:

$$\frac{1}{\Lambda}\int d^4\theta \;\; \langle H_d^\dagger \rangle L \;\; \underline{1}$$

$-(t_1+t_4) \quad t_1+t_2 \quad t_4-t_2$ ✓.

Embedding in global B_3 model

$$c_1(dP_2 = S_{GUT}) = 3h - e_1 - e_2$$

$$t = -c_1(N_{S_{GUT}|B_3}) = h$$

$$\eta = 6c_1 - t = 17h - 6e_1 - 6e_2$$

Choose $\xi = h - e_1$.

$$(F_Y) = e_2 - e_1.$$

	Sect.		$\mathcal{L}_Y$
10_M	e_2	$h - e_1 = \xi$	-1
5_H	p_1	$3h - e_1 - e_2 + \xi = 4h$	-1
$\bar{5}_M$	P_2	$8h - 3e_1 - 3e_2$	0
$\bar{5}_H$	P_3	$10h - 4e_1 - 4e_2 - \xi$	$+1$
10_{oth}	a_3	$2h - e_1 - e_2 - \xi$	$+1$

$$P = \prod P_i$$

$$P_2 \hat{=} \mathcal{C}^1 \wedge \tau \mathcal{C}^1$$

$$P_3 \hat{=} \mathcal{C}^1 \wedge \tau \mathcal{C}^2$$

$$P_1 \hat{=} \mathcal{C}^2 \wedge \tau \mathcal{C}^2.$$

Fluxes

$G = \sum \omega_i \wedge F_{(i)}$

ω_i: (1,1) form on $\mathcal{C}_8$ fib dual to t_i; $F_{(i)}$: $U(1) \subset SU(5)_\perp$.

Universal fluxes:

$$\int_{\mathcal{C}_5} \omega_j = \delta_{ij}.$$

$$c_1(p_{1*}\mathcal{L}_1) + c_1(p_{2*}\mathcal{L}_2) = 0$$

$$c_1(\mathcal{L}_i) \in H^{1,1}(\mathcal{C}^{(i)}, \mathbb{Z}).$$

Univ. fluxes

Int. w/ divisors in $X_{\mathcal{C}}$ w/ $\mathcal{C}$:

- $\gamma_i = \sigma \cdot \mathcal{C}^{(1)}$

or

$p_1^* \Sigma = \pi^*(\Sigma) \cdot \mathcal{C}^{(1)}$ ($\Sigma \in S_{GUT}$).

Tracelessness: $\tilde{\gamma}_{12} = 3\gamma_{12} - p_{12}^* p_{12*} \gamma_{12}$

- $\rho = 2p_1^* \rho - 3p_2^* \rho$.

- $\delta_1 = 2\sigma \cdot \mathcal{C}^{(1)} - p_2^* p_{1*}(\sigma \cdot \mathcal{C}^{(1)})$ etc.

Nonuniv. fl.

tuning of cx. str. allows other fluxes.
(not int. w/ div.).

e.g. $\psi_{\pm}$: $V \pm gU = 0$ $\psi = 0$.

$\psi_1 = (3 - p_1 * p_{1*}) \psi_+$

2 2 2 2 −

$\Delta = \psi_+ - \psi_-$

$\Rightarrow$ Use flux to constrain

↙ ↘

<u>spectrum</u>

get nontrivial F_Y on $\underline{10_M}$

$\Rightarrow F_Y|_{10_M} = -1$

$F_Y|_{10_{other}} = 1$

$\Rightarrow \Gamma|_{10_M} = M$ $\Rightarrow$ engineer models

$\Gamma|_{10_{oth}} = -M$ w/ additional

$(3,2)_{1/6} + (\bar{3},1)_{-2/3} \to 10_M$

$(\bar{3},2)_{-1/6} + (3,1)_{2/3} \to 10_{other}$.

don't overshoot D3-bound.

$N_{D3_1}^{flux} = \frac{1}{2} \int G \wedge G$

$= -\frac{1}{2} \Gamma^2$.

$\Rightarrow \exists$ models satisfying both 3 gen + min extras.
& D3