TESTS OF LORENTZ VIOLATION: A 2013 UPDATE

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THE QUANTUM GRAVITY PROBLEM

- **Why we need a theory of Quantum Gravity?**
  - Philosophy: reductionism in physics
  - Lack of predictability of current theories
    (e.g. Singularities, Time Machines, spacetime topology and signature...)

- **To eventually understand QG, we will need to**
  - observe phenomena that depend on QG
  - extract reliable predictions from candidate theories & compare them with observations

But we have been facing a BIG problem...
Quantum gravity phenomenology is broader than Lorentz violations tests.

- Primordial gravitons from the vacuum
- Loss of quantum coherence or state collapse
- QG imprint on initial cosmological perturbations
- Cosmological variations of couplings
- Extra dimensions and low-scale QG (LHC BH) \( M_p^2 = R^n M_{p(4+n)^{n+2}} \)
- Modified Uncertainty principle
- Violation of global internal symmetries
- Violation of discrete symmetries
- Violation of spacetime symmetries

We shall focus here on the last item. More precisely on the possibility that Local Lorentz invariance can be violated...
WHY TESTING LORENTZ INVARIANCE?

• Lorentz invariance is assumed to be a fundamental symmetry of nature. It is rooted via the equivalence principle in GR and it is a fundamental pillar of the SM.

The more fundamental is an ingredient of your theory the more needs to be tested observationally.

• Several ideas related to quantum gravity have suggested violations of Lorentz invariance.

• This is one of the few cases in which our sensitivity can constraints new physics set at the Planck scale, so test of Lorentz invariance can be used to rule out QG models: Lorentz violations tests are so far the best example of QG phenomenology.
Suggestions for Lorentz violation searches (at low or high energies) came from several QG models:

- String theory tensor VEVs (Kostelecky-Samuel 1989, ...)
- Cosmological varying moduli (Damour-Polyakov 1994)
- Some semiclassical spin-network calculations in Loop QG (Gambini-Pullin 1999)
- Einstein-Aether Gravity (Jacobson-Mattingly 2000, ...)
- Some non-commutative geometry calculations (Carroll et al. 2001)
- Some brane-world backgrounds (Burgess et al. 2002)
- Ghost condensate in EFT (Cheng, Luty, Mukohyama, Thaler 2006)
- Horava-Lifshitz Gravity (Horava 2009, ...)

As well as from long standing problems in BH physics and QG:


- A possible get around for Weinberg-Witten theorem

- Power counting renormalizability of canonical quantum gravity:
WHAT KIND OF DEPARTURE FROM SPECIAL RELATIVITY?

Lorentz breaking is not one to one with relativity breaking.

W. von Ignatowsky theorem (1911):
- Principle of relativity $\rightarrow$ group structure
- Homogeneity $\rightarrow$ linearity of the transformations
- Isotropy $\rightarrow$ rotational invariance and Riemannian structure
- Precausality $\rightarrow$ observer independence of co-local time ordering

Lorentz transformations with unfixed limit speed $C$
- $C = \infty$ $\rightarrow$ Galileo
- $C = c_{\text{light}}$ $\rightarrow$ Lorentz

Experiments determine $C$!

Breaking scenarios (please one breaking at a time)

- Break Principle of relativity $\rightarrow$ Preferred frame, (see also Baccetti, Tate, Visser: arXiv:1112.1466)

- Break Homogeneity $\rightarrow$ Non-Linear transformations $\rightarrow$ Alternative relativity? (seems to require non-locality)

- Break kinematical isotropy $\rightarrow$ Finsler geometries. E.g. Very Special Relativity (Glashow).

- Break Precausality $\rightarrow$ Too messy to touch this, how knows!

Even breaking Lorentz invariance can be done in different ways
- IR Lorentz $\rightarrow$ UV Galileo
- but also IR Lorentz $\rightarrow$ UV Lorentz with different limit speed
AN EXAMPLE OF LORENTZ TO LORENTZ TRANSITION: RELATIVISTIC BEC

Bose–Einstein condensation, may occur also for relativistic bosons. So far only theoretical model.

The associated dispersion relation has a gapped and gapless mode

\[
\omega_\pm^2 = c^2 \left\{ k^2 + 2 \left( \frac{\mu}{\hbar c} \right)^2 \left[ 1 + \left( \frac{m c c_0}{\mu} \right)^2 \right] \pm 2 \left( \frac{\mu}{\hbar c} \right) \sqrt{k^2 + \left( \frac{\mu}{\hbar c} \right)^2 \left[ 1 + \left( \frac{m c c_0}{\mu} \right)^2 \right] } \right\}
\]

where \( c_0 = \frac{\hbar^2}{2m} \rho U''(\rho, \lambda) \), \( \mu = \) relativistic chemical potential.

The gapless/massless mode is the interesting one as it admits an effective metric in the phononic regime

In the limit of very relativistic atoms \( b \equiv \frac{m c c_0}{\mu} \gg 1 \)

Long wavelengths limit \( k \ll \frac{2mc_0}{\hbar} \implies \omega_-^2 = c_s^2 k^2 \) with \( g_{\mu\nu} = \rho \frac{c}{c_s} \left[ \eta_{\mu\nu} + \left( 1 - \frac{c_s^2}{c^2} \right) \frac{v_{\mu} v_{\nu}}{c^2} \right] \)

\( c_s^2 = c^2 b/(1 + b) \)

\[ \text{LLI+Gravity} \]

Short wavelengths limit \( k \gg \frac{2mc_0}{\hbar} \implies \omega_-^2 = c^2 k^2 \) with \( g_{\mu\nu} = \eta_{\mu\nu} \)

No LLI+Rainbow metric/Finsler?

Intermediate \( \omega_-^2 = c_s^2 \left[ k^2 + \frac{k^4}{4b \left( \frac{\mu}{hc} \right)^2 (1 + b)^2} \right] \)
HISTORY OF A HERESY

- Is there an Aether? (Dirac, 1951)
- Dispersion & LV (Pavlopoulos, 1967)
- Vector-tensor gravity (Nordvedt & Will, 1972)
- Emergent LI in gauge theory? (Nielsen & Picek, 1983)
- LV modification of general relativity (Gasperini, 1987)
- Spontaneous LV in string theory (Kostelecky & Samuel, 1988)
- LV Chem-Simons in Electrodynamics (Carroll, Field & Jackiw, 1990)
- LV & BH trans-Planckian question (Jacobson, 1990)
- Non-critical string spacetime foam models (Ellis, Mavromatos & Nanopoulos, 1992)
- Possibilities of LV phenomenology (Gonzalez-Mestres, 1995)
- “Minimal Standard model extension” & experimental limits (Colladay & Kostelecky, 1997 & many experimenters)
- GRB photon dispersion limits at the Planck scale (Amelino-Camelia et al, 1997)
- Coleman-Glashow test theory (manageable subcase of SME) (Coleman & Glashow, 1997-8)
- TeV gamma ray crisis? (Protheroe & Mayer 2000)
- Einstein-Aether gravity (Jacobson-Mattingly 2000)
- Doubly/Deformed Special Relativity (Amelino-Camelia 2002)
- Horava-Lifshitz Gravity (Horava 2009, ...)
- (Flawed) Detection of superluminal neutrinos (Opera collaboration, 2011)
MODIFIED DISPERSION RELATIONS

Many of the aforementioned QG models have been shown to lead to modified dispersion relations

Let’s take a purely phenomenological point of view and encode the general form of Lorentz invariance violation (LIV) into the dispersion relations

\[ E^2 = p^2 + m^2 + \Delta(p, M, \mu) \]

\[ \mu = \text{some particle mass scale} \]

\[ M \equiv \text{spacetime structure scale, generally assumed } \approx M_{\text{Planck}} = 10^{19} \text{ GeV} \]

Generally assumed rotational invariance
• simpler
• cutoff idea only implies boosts are broken, rotations maybe not
• boost violation constraints likely also boost + rotation violation constraints

Then one can perform a momentum expansion…

\[ E^2 = p^2 + m^2 + M \eta^{(1)} |p| + \eta^{(2)} p^2 + \eta^{(3)} |p|^3 / M \]

Where \( \eta^{(i)} \) are dimensionless coefficients possibly containing the small ratio \( (\mu / M)^m \)

The lowest order \( (p, p^2) \) terms encode a (better small!) low energy LIV violation
The highest \( (p^3 \text{ and higher}) \) encode high energy LIV
PICKING UP A FRAMEWORK...

Missing a definitive QG candidate able to provide definitive sub-Planckian predictions, different general dynamical framework have been proposed. Many of the aforementioned QG models have been shown to lead to modified dispersion relations but we need also a dynamical framework.

- Lorentz symmetry violation
- New relativity Groups
- Non-critical Strings
- Spacetime foam models
- DSR-Relative Locality
- Non-commutative spacetimes
- Very special relativity-Finsler Geometries
- New Measurement theory at E_{pl}

EFT+LV
Renormalizable, or higher dimension operators

Minimal Standard Model Extension
Renormalizable ops. (IR LIV- LI SSB)
E.g. QED, rot. Inv. dim 3,4 operators

- Electrons: $E^2 = m^2 + p^2 + f_{e}^{(1)} p + f_{e}^{(2)} p^2$
- Photons: $\omega^2 = (1 + f_{\gamma}^{(2)}) k^2$

(Colladay-Kosteleky 1998)

EFT with LIV
Non-renormalizable (no anisotropic scaling) ops,
(UV LIV – Emergent LI)
E.g. QED, dim 5 operators

- Electrons: $E^2 = m^2 + p^2 + \eta_{\pm}^{(3)} (E^3/M_{Pl})$
- Photons: $\omega^2 = k^2 \pm \xi (\omega^3/M_{Pl})$

(Myers-Pospelov 2003)
LIV PHENOMENOLOGY TOOLKIT

Terrestrial tests (low energy):
- Penning traps
- Clock comparison experiments
- Cavity experiments
- Spin polarized torsion balance
- Neutral mesons
- Slow atoms recoils

Astrophysical tests (high energy):
- Cosmological variation of couplings
- Cumulative effects in astrophysics
- Anomalous threshold reactions
- Shift of standard thresholds reactions with new threshold phenomenology
- LIV induced decays not characterized by a threshold
- Reactions affected by “speeds limits”

This wealth of tests already severely constraints the Minimal Standard Model extension (dim 3,4 ops, boost and rot breaking):

**QED:** up to $O(10^{-22})$ on dim 4,
**Hadronic sector:** up to $O(10^{-46})$ on dim 3, $O(10^{-27})$ on dim 4.
**Neutrinos:** up to $O(10^{-28})$ on dim 4 from neutrino oscillations

Hence we shall in what follow consider the higher order LIV operators mass dimension 5 and 6 and hence mainly Astrophysical/Cosmological constraints...

Furthermore generally assumed rotational invariance
- simpler
- cutoff idea only implies boosts are broken, rotations maybe not
- boost violation constraints likely also boost + rotation violation constraints
Photon helicities have opposite LIV coefficients. Electron helicities have independent LIV coefficients. Moreover electron and positron have exchanged and opposite positive and negatives helicities LIV coefficients (Jacobson, SL, Mattingly, Stecker. 2003).

Let’s consider all the Lorentz-violating dimension 5 CPT odd terms that are quadratic in fields, gauge & rotation invariant, not reducible to lower order terms (Myers-Pospelov, 2003).

For $E \gg m$ this ansatz leads to the following dispersion relations:

- Electrons: $E^2 = m^2 + p^2 + \eta_{\pm}(p^3/M_{P1})$
- Photons: $\omega^2 = k^2 \pm \xi(k^3/M_{P1})$

where $\xi$ is the dual of $F$ and $\xi, \zeta_{1,2}$ are dimensionless parameters.

For $E \gg m$ this ansatz leads to the following dispersion relations:

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Moreover electron and positron have exchanged and opposite positive and negatives helicities LIV coefficients (Jacobson, SL, Mattingly, Stecker. 2003).

**Note**: RG studies show that the running of LIV coefficients is only logarithmic: so if LIV is $O(1)$ at $M_{pl}$ we expect it to remain so at TeV scales (Bolokhov & Pospelov, hep-ph/0703291)
Note: no birefringence

Electron and positron have exchanged and opposite positive and negative helicities LIV coefficients but without minus sign.

<table>
<thead>
<tr>
<th></th>
<th>Positive helicity</th>
<th>Negative helicity</th>
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</thead>
<tbody>
<tr>
<td>Electron</td>
<td>$\eta_+$</td>
<td>$\eta_-$</td>
</tr>
<tr>
<td>Positron</td>
<td>$\eta_-$</td>
<td>$\eta_+$</td>
</tr>
</tbody>
</table>
AN OPEN PROBLEM: UN-NATURALNESS OF SMALL LV IN EFT

[Collins et al. PRL93 (2004), Lifshitz theories (anisotropic scaling): Iengo, Russo, Serone (2009)]

Dim 3,4 operators are tightly constrained: \(O(10^{-46}), O(10^{-27})\). This is why much attention was focused on dim 5 and higher operators (which are already Planck suppressed).

However

- if one postulates classically a dispersion relation with only naively (no anisotropic scaling) non-renormalizable operators (i.e. terms \(\eta(n)p^n/M_{Pl}^{n-2}\) with \(n \geq 3\) and \(\eta(n) \approx O(1)\) in disp.rel.)
- then radiative (loop) corrections involve integration up to the natural cutoff \(M_{Pl}\) will generate the terms associated to renormalizable operators \((\eta(1)pM_{Pl}, \eta(2)p^2)\) which are unacceptable observationally if \(\eta(1,2) \approx O(1)\).
- Roughly the generated coefficients \(\eta(1), \eta(2)\) are of order one because the \(M_{Pl}^{n-2}\) suppression is cancelled by the integration cutoff which is again \(M_{Pl}\)

This is THE problem with UV Lorentz breaking EFT!

Two Ways out

- **Custodial symmetry**
  - One needs another scale other from \(E_{UV}\) (which we have so far assumed \(O(M_{Pl})\).
  - So far main candidate SUSY but needs \(E_{SUSY}\) not too high.
  - Eg. gr-qc/0402028 (Myers-Pospelov) or hep-ph/0404271 (Niblink-Pospelov) or gr-qc/0504019 (Jain-Ralston),
  - See also Pujolas-Sibiryakov (arXiv:1109.4495) for SUSY Einstein-Aether gravity.

- **Gravitational confinement**
  - Assume only gravity UV with \(M_{UV} < M_{Pl}\)
  - then percolation into the (constrained) matter sector is suppressed by smallness of coupling constant \(G_N\).
  - Eg. Horava gravity coupled to U(1) Standard Model: Pospelov & Shang arXiv.org/1010.5249v2
  - But let’s see what we can say “order by order” for the moment…
Constraint on the photon LIV coefficient $\xi$ by using the fact that different colors will travel at different speeds. Given current data we can cast constrains only on $O(E/M)$ LIV...

\[ v_\gamma = \frac{\partial E}{\partial p} = 1 + \xi \frac{E}{E_{Pl}} \]

\[ \Delta t = \Delta vT = \frac{\xi (E_2 - E_1)}{M} \]

\[ \Delta t \approx 10 \text{ msec} \xi d_{Gpc} E_{GeV} \]

Actually for cosmological distances this generalizes to:

\[ \Delta t \approx \xi \frac{\Delta E}{M} \frac{1}{H_0} \int_0^z \frac{dz}{\sqrt{\Omega_A + (1+z)^3 \Omega_M}} \]

Constraints of $\sim O(10^{-1})$ on $O(E/M)$ LIV have been cast using time of arrival measurements on beams of light from distant sources like GRBs and AGN (FERMI,MAGIC,HESS).

Problem: there is strong evidence that most GRB and AGN are not “good” objects for TOF constraints because of intrinsic time lags (different energies emitted at different times) not well understood.

Ellis et al (2005): careful statistical analysis on large sample of sources of the delay-redshift correlation leads to conservative limit $|\xi| < 10^3$

**The EFT tackle**

We have seen that QED with $O(E/M)$ LIV has birefringence photons. In this case unpolarized light beams will have both helicities and the net effect of slow and fast modes can cancel the above TOF effect. Indeed one gets only a beam intensity LV induced modulation (SL, Maccione, 2009).

However, being sure both photon polarization are present in the pulse, one could use the fact that opposite coefficients for photon helicities imply larger dispersion $2|\xi| p/M$ at the same energy rather than that due to different energies $\xi (p_2 - p_1)/M$.

This would remove problem of source delays and roughly cut in half the current constraints but implies separate detection of opposite helicities and no spurious helicity dependent mechanism.
The birefringence constraint arises from the fact that for CPT violating LV operators (e.g. dim 5 O(E/M) ) the LV parameters for left and right circular polarized photons are opposite.

Linear polarization is therefore rotated through an energy dependent angle as a signal propagates, which depolarizes an initially linearly polarized signal comprised of a range of wavevectors. For a monochromatic plane wave with wave-vector \( k \) over a propagation time \( t \)

\[
\theta(t) = [\omega_+ - \omega_-(k)] t/2 = \xi k^2 t/2M
\]

The difference in rotation angles for wave-vectors \( k_1 \) and \( k_2 \) is thus

\[
\Delta \theta = \xi \left( k_2^2 - k_1^2 \right) d/2M, \quad \text{(where } d = \text{distance source-detector)}
\]

The polarization is strongly reduced if this angle becomes \( \Delta \theta \ll \pi/2 \) and this condition can be used to cast a constraint.

Alternatively a more accurate way is to match the theoretical polarization \( \Pi(\xi) \) (Stokes parameters) to the observed one.

\[
\Pi(\xi) = \sqrt{\langle \cos(2\theta) \rangle_P^2 + \langle \sin(2\theta) \rangle_P^2},
\]
Key point: the effect of the non LI dispersion relations can be important at energies well below the fundamental scale.

\[ E^2 = c^2 p^2 \left( 1 + \frac{m^2 c^2}{p^2} + \eta \frac{p^{n-2}}{M^{n-2}} \right) \]

Corrections start to be relevant when the last term is of the same order as the second. If \( \eta \) is order unity, then

\[ \frac{m^2}{p^2} \approx \frac{p^{n-2}}{M^{n-2}} \Rightarrow p_{\text{crit}} \approx \eta \sqrt{m^2 M^{n-2}} \]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( p_{\text{crit}} \text{ for } \nu_e )</th>
<th>( p_{\text{crit}} \text{ for } e^- )</th>
<th>( p_{\text{crit}} \text{ for } p^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( p \approx m_\nu \sim 1 \text{ eV} )</td>
<td>( p \approx m_e = 0.5 \text{ MeV} )</td>
<td>( p \approx m_e = 0.938 \text{ GeV} )</td>
</tr>
<tr>
<td>3</td>
<td>( \sim 1 \text{ GeV} )</td>
<td>( \sim 10 \text{ TeV} )</td>
<td>( \sim 1 \text{ PeV} )</td>
</tr>
<tr>
<td>4</td>
<td>( \sim 100 \text{ TeV} )</td>
<td>( \sim 100 \text{ PeV} )</td>
<td>( \sim 3 \text{ EeV} )</td>
</tr>
</tbody>
</table>

Eg. for \( n=3 \) and \( m = m_{\text{electron}} \)

\[ m^2 \approx \eta p^3 / M \leftrightarrow p \approx (m^2 M / \eta)^{1/3} \approx 10 \text{ TeV} \eta^{-1/3} \]

\( \eta \) constraint \( \propto \frac{1}{P_{\text{max}}^3} \)
NOVELTIES IN THRESHOLD REACTIONS: WHY

It is still true that threshold happens when incoming particles are head on and outgoing particles are parallel (Mattingly, SL Jacobson, 2003)

Upper thresholds

The range of available energies of the incoming particles for which the reactions happen is changed.

ΔE_f = \frac{\partial}{\partial p_2} E^2_0 \frac{\partial}{\partial p_2} p_1 = \Delta p (\Delta p)

Sufficient condition for asymmetric threshold.

If LI holds there is never an upper threshold. However, the presence of different coefficients for different particles allows \( E_i \) to intersect two or more times \( E_f \) switching on and off the reaction!

It is still true that threshold happens when incoming particles are head on and outgoing particles are parallel (Mattingly, SL Jacobson, 2003)

See also generalization by Baccetti, Tate, Visser, arXiv:1111.6340.
THRESHOLD REACTIONS IN LIV EFT

- **New threshold reactions**
  - Vacuum Cherenkov: $e^- \rightarrow e^- \gamma$
    - Moreover now possible Cherenkov with emission of an hard photon
  - Gamma decay: $\gamma \rightarrow e^+ e^-$
    - Moreover now possible asymmetric pair production of electron-positron pair
  - Helicity decay: $e_L \rightarrow e_R \gamma$
    - No real threshold but effective one due to suppression of phase space at low energies with effective threshold below but comparable to the Cherenkov one.
  - Photon splitting: $\gamma \rightarrow n \gamma$. Rate maybe important if photon effective mass larger than electron one. $m^2_{\gamma} \equiv \frac{\xi E_n}{M^4_{\text{Pl}}} \ll m_e$
  - Electron pair production: $e^- \rightarrow e^- e^+ e^-$. Similar to vacuum Cherenkov, threshold slightly higher.

- **Anomalous thresholds** (modification of standard threshold reactions)
  - Shift of lower thresholds (Coleman-Glashow, JLM, Konopka-Major, etc...)
  - Emergence of upper thresholds (Klusniak, JLM)
  - Asymmetric pair production (JLM, Konopka-Major)

So far mainly considered
- Photon pair creation: $\gamma + \gamma_{\text{CMB},\text{FIRB}} \rightarrow e^+ + e^-$
- For proton-pions GZK reaction: $p^+ + \gamma_{\text{CMB}} \rightarrow p^+ + \pi^0$
ASTROPHYSICAL CONSTRAINTS: SYNCHROTRON RADIATION

**Li synchrotron critical frequency:**
\[ \omega_{cL}^{LI} = \frac{3}{2} \frac{eB\gamma^2}{m} \]

- $e$ - electron charge
- $m$ - electron mass
- $B$ - magnetic field

However, a proper analysis requires a detailed re-derivation of the synchrotron effect with LIV based on EFT. Let’s take QED with $O(E/M)$ LIV.

This leads to a modified formula for the peak frequency:
\[ \omega_{cL}^{LIV} = \frac{3}{2} \frac{eB\gamma^3}{E} \]

While the rate of energy loss differs from the LV one only nearby the VC threshold...

Now:
\[ \gamma = (1 - v^2)^{-1/2} \approx \left( \frac{m^2}{E^2} - 2\eta \frac{E}{M_{QG}} \right)^{-1/2} \]

- $\eta < 0$
  - $\gamma$ is a bounded function of $E$. There is now a maximum achievable synchrotron frequency $\omega_{\text{max}}$ for ALL electrons!
- $\eta > 0$
  - $\gamma$ diverges as $p_{\text{th}}$ is approached. This is unphysical as also the energy loss rates diverges in this limit, however signifies a rapid decay of the electron energy and a violent phase of synchrotron radiation which becomes vacuum Cherenkov.

So one gets a constraints from asking $\omega_{\text{max}} \geq (\omega_{\text{max}})_{\text{observed}}$.

What is then the best studied synching astrophysical object?
The Crab nebula a supernova remnant (1054 A.D.) distance ~1.9 kpc from Earth. Spectrum (and other SNR) well explained by synchrotron self-Compton (SSC)

1. Electrons are accelerated to very high energies at pulsar: in LI QED \( \gamma_e \approx 10^9 \div 10^{10} \)
2. High energy electrons emit synchrotron radiation
3. Synchrotron photons undergo inverse Compton with the high energy electrons

Currently the best two tests come from the measurement of the spectrum and polarization of Crab synchrotron emission.

The synchrotron spectrum is strongly affected by LIV: maximum gamma factor for subliminal leptons and vacuum Cherekov limit for superluminal ones (there are both electrons and positrons and they have opposite \( \eta \)). Spectrum very well known via EGRET, now AGILE+FERMI

The polarization of the synchrotron spectrum is strongly affected by LIV: there is a rotation of the angle of linear polarization with different rates at different energies. Strong, LIV induced, depolarization effect.

\[ \Delta \theta = \xi (k_2^2 - k_1^2) d/2M, \quad \text{(where } d = \text{distance source-d detector)} \]

Polarization recently accurately measured by INTEGRAL mission: 40\(\pm\)3\% linear polarization in the 100 keV - 1 MeV band + angle \( \theta_{\text{obs}} = (123 \pm 1.5) \) from the North.
In LI theory UHE gamma rays are attenuated mainly by pair production: $\gamma\gamma \rightarrow e^+e^-$ onto CMB and URB (Universal radio Background) leading to a theoretically expected photon fraction $<1\%$ at $10^{19}$ eV and $<10\%$ at $10^{20}$ eV.

- Present limits on photon fraction: $2.0\%, 5.1\%, 31\%, 36\%$ (95% CL) at 10, 20, 40, 100 EeV from AUGER

- LIV strongly affects the threshold of this process: lower and also upper thresholds.
- If $k_{up} < 10^{20}$ eV then photon fraction in UHECR much larger than present upper limits

- LIV also introduces competitive processes: $\gamma$-decay
- If photons above $10^{19}$ eV are detected then $\gamma$-decay threshold $> 10^{19}$ eV

\[ \omega^2 = k^2 + \xi k^4/M_{Pl}^2 \]
\[ E_{\pm}^2 = p^2 + m_e^2 + \eta \pm p^4/M_{Pl}^2 \]

where $\pm$ = opposite helicity states

GZK photons are pair produced by decay of $\pi^0$s produced in GZK process:

\[ p + \gamma \rightarrow p + \pi^0(n + \pi^+) \]

\[ E_{th} = \frac{2m_p m_{\pi} + m^2_{\pi}}{4e} \sim 4 \cdot 10^{19} \text{ eV} \]

The Greisen-Zatsepin-Kuzmin effect: secondary production

[Diagram showing log-log plot with curves and labels]
Going further…

**Hadronic sector dim 6 LIV (CPT even) ops constraints using UHECR**

Theoretical reconstruction of Ultra High Energy Cosmic Rays spectrum in a EFT with dim 6 operators and confrontation with data

\[
-10^{-3} \lesssim \eta_p \lesssim 10^{-6} \\
-10^{-3} \lesssim \eta_\pi \lesssim 10^{-1} \quad (\eta_p > 0) \\
\lesssim 10^{-6} \quad (\eta_p < 0).
\]

Maccione, Taylor, Mattingly, SL: **JCAP 0904 (2009) 022**

**Neutrinos dim 6 LIV ops constraints using cosmogenic neutrinos**

\[\nu\text{-splitting}: \nu \rightarrow \nu \nu \bar{\nu}.\]

For positive $O(1)$ coefficients no neutrino will survive above $10^{19}$ eV. The existence of this cutoff generates a bump in the neutrino spectrum at energies of $10^{17}$ eV and depression at UHE. Experiments in construction or being planned have the potential to cast limits as strong as $\eta < 10^{-7}$ on the neutrino LV parameter, depending on how LV is distributed among neutrino mass states.

Mattingly, Maccione, Galavemi, SL, Sigl: **JCAP 1002 (2010) 007**

Liberati, Maccione, Mattingly, (2012)
NEUTRINOS THRESHOLD REACTIONS

- Vacuum Cherenkov: $\nu \rightarrow \nu \gamma$
  - Too suppressed: relevant only above $\sim 10^{19}$ eV
    \[
    \tau_{\nu \gamma} \approx \left( \frac{M}{M_{\text{Pl}}} \right)^2 \left( \frac{E}{1 \text{ PeV}} \right)^{-(2n+1)} \times 10^{26n-86} \text{ s}
    \]

- Neutrino splitting:
  \[
  \nu_{\text{I}} \rightarrow \nu_{\text{I}} \nu_{\text{J}} \nu_{\text{J}}
  \]

- Neutrino decay by pair creation:
  \[
  \nu_{\text{I}} \rightarrow \nu_{\text{I}} e^+ e^-
  \]
  (Idea and $n=2$ worked out in Cohen-Glashow 2011)

- Dirac neutrinos: no rotational invariant LIV operators can be constructed for Majorana (but Majorana might acquire LIV from Higgs like mechanism)

- Rests on a few assumptions:
  - A preferred frame exists
  - Energy-momentum conservation laws are linear
  - Hamiltonian dynamics is preserved
  - Only neutrinos are superluminal
  - The asymptotic electron states are free and on-shell in vacuum

- Neglect electron-positron LIV (much more constrained than neutrino)

- Used to “disprove” OPERA claim of superluminal neutrino

- For flavor blind LIV:
  \[
  \tau_{\nu-\text{pair}} \approx \frac{m_Z^4 \cos^2 \theta_W}{g^4 E^5} \left( \frac{M}{E} \right)^{3(n-2)}
  \]
  \[
  E_{\text{th}} = (m_\nu M_{\text{Pl}}^{-2})^{1/n}
  \]
  \[
  \pi^+ \rightarrow \nu_\mu + \mu^+
  \]

See also constraints from pion decay
Hep-ph/1109.6667, 1206.0713

$\nu_{\mu} \rightarrow \nu_{\mu} + 2 \nu_{\mu} + 2 \nu_{\mu}$
Cohen and Glashow used the fact that superluminal neutrinos should emit electron-positron pairs to argue that the OPERA results were not even self-consistent. The argument was formally correct but did not worry about adjusting for the finite size of the baseline: a finite baseline can be of the same order as the energy loss length of neutrinos undergoing pair production. This allows for some neutrinos to undergo only one or a few Cherenkov emissions within their time of flight. Therefore the most energetic neutrinos of the injection beam can still reach the end of the baseline with an energy larger than $E_T$.

It is then necessary, in order to cast a robust constraint on LIV by using long baseline experiments, to run a full Monte Carlo simulation of the propagation of neutrinos aimed at computing the neutrino spectrum on arrival in the presence of this energy loss process.
QG phenomenology of Lorentz and CPT violations is a success story in physics. We have gone in few years (1997->2010) from almost no tests to tight, robust constraints on EFT models.

Chances are high that improving observations in HE astrophysics will strengthen these constraints in a near future…

If there is Lorentz violation, and it is described by the same modified dispersion relation at all energies then its scales seems required to be well beyond the Planck scale…

<table>
<thead>
<tr>
<th>Order</th>
<th>photon</th>
<th>$e^-/e^+$</th>
<th>Hadrons</th>
<th>Neutrinos(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=2</td>
<td>N.A.</td>
<td>$O(10^{-16})$</td>
<td>$O(10^{-27})$</td>
<td>$O(10^{-8})$</td>
</tr>
<tr>
<td>n=3</td>
<td>$O(10^{-15})$ (GRB)</td>
<td>$O(10^{-16})$ (CR)</td>
<td>$O(10^{-14})$ (CR)</td>
<td>$O(30)$</td>
</tr>
<tr>
<td>n=4</td>
<td>$O(10^{-8})$ (CR)</td>
<td>$O(10^{-8})$ (CR)</td>
<td>$O(10^{-6})$ (CR)</td>
<td>$O(10^{-7})^* (CR)$</td>
</tr>
</tbody>
</table>

**Table 2.** Summary of typical strengths of the available constrains on the SME at different orders. GRB=gamma rays burst, CR=cosmic rays. \(^a\) From neutrino oscillations we have constraints on the difference of LV coefficients of different flavors up to $O(10^{-28})$ on dim 4, $O(10^{-8})$ and expected up to $O(10^{-14})$ on dim 5 (ICE3), expected up to $O(10^{-4})$ on dim 6 op. * Expected constraint from future experiments.

Should we conclude that we have deviations from Special Relativity enough? **Mission Accomplished?**

Not quite...
CAVEAT: A POTENTIAL PROBLEM WITH THE UHECR DATA?

• With increased statistics the composition of UHECR beyond $10^{19}$ eV seems more and more dominated by iron ions rather than protons at AUGER. But Telescope Array (TA) in Utah is instead Ok with purely proton composition. Are we seeing the GZK?

• With improved statistic the correlated AUGER UHECR-AGN events have decreased from 70% to 40%; large deflections?

• Also no evidence at the TA for AGN correlation. But some hint of correlation with LLS for $E>57$ EeV

• Ions do photodisintegration rather than the GZK reaction, this may generate much less protons which are able to create pions via GZK and hence UHE photons.

  • **Shaky $n=4$ constraints?**

  **However...**

  *Astro-ph [HE]:1007.1306*, D. Hooper, A. Taylor, S. Sarkar
  They find the flux of UHE-photons is just suppressed by one order of magnitude. LIV effects would increase the flux by about four orders...perhaps we are safe?

  *Astro-ph [HE]:1101.2903*, A. Saveliev, L. Maccione, G. Sigl
  Assuming UHECR are heavy nuclcy and they are not loosing energy by LV spontaneous decay and vacuum Cherenkov the get the following tentative constraints

<table>
<thead>
<tr>
<th></th>
<th>$E_{\text{max}} = 10^{19.6}$ eV</th>
<th>$E_{\text{max}} = 10^{20}$ eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>$-3 \times 10^{-3} \lesssim \eta \lesssim 4 \times 10^{-3}$</td>
<td>$-7 \times 10^{-5} \lesssim \eta \lesssim 1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>$-7 \times 10^{-2} \lesssim \eta \lesssim 1$</td>
<td>$-2 \times 10^{-3} \lesssim \eta \lesssim 3 \times 10^{-2}$</td>
</tr>
<tr>
<td>$^{56}$Fe</td>
<td>$-1 \lesssim \eta \lesssim 200$</td>
<td>$-3 \times 10^{-2} \lesssim \eta \lesssim 4$</td>
</tr>
</tbody>
</table>

$\eta =$ generic LIV coefficient of dim 6 ops for single nucleon
So far much attention has been focussed on constraining LIV EFT. This was reasonable as a first approach but it is now time to do more. We need QG models to provide observables and low energy predictions that can be tested.

Generically QG models can predict

- Lorentz invariance and/or CPT breaking sometime not within EFT
  - Non-Localy effects
  - Running coupling constants
  - Modified uncertainty principle
  - ...

For the moment let’s consider then specific QG models predicting LIV and show what kind of implications can have the sort of constraints we saw so far...
UHE PHOTONS AND LV IN SPACE-TIME FOAM

Spacetime foam models


QG medium as oscillators that absorb and emit photons. Oscillators are D0 branes-D-particles flashing in the space-time Photon absorption and re-emission: The D-particle recoils. D-particles are neutral: charged particles do not feel their presence.

Consequences:
LV only for on-shell photons (and Majorana neutrinos)
Photons are delayed and acquire an effective modified dispersion relation. Note: no birefringence, no gamma decay...

\[ E_\gamma^2 = p^2 - \xi \frac{p^3}{M} \quad \text{with} \quad \xi > 0 \]
In case D-particles have a bulk recoiling motion which does not average to zero, the background metric is modified and energy non-conservation during interaction is possible: one can effectively “encode” this by introducing a new parameter $\xi_I$ associated to deviation from exact energy conservation in an interaction.

\[
E_1 + \omega = E_2 + E_3 + \delta E_D \\
p_1 - \omega = p_2 + p_3
\]

\[
\xi_I + \frac{\xi}{2} \frac{\omega}{M} = \frac{m_e^2}{M^2} + \cdots = 0
\]

Hence pair production is modified by LV even in the case of space time foam models (including redshift dependence of D-particles) and we can again cast a constraint by the absence of an upper threshold...

\[
\frac{\xi}{2} + \xi_I \lesssim 10^{-12}
\]

Note however that this constraint can be evaded by alternative spacetime foam models, See Ellis et al, arXiv:1004.4167
Testing LIV in Horava-Lifshitz Gravity

Let's consider the general form of the action for Horava-Lifshitz gravity before assuming any restrictive requirements.

\[ S_{HL} = \frac{M_{P}^2}{2} \int dt d^3x \sqrt{h} \left( L_2 + \frac{1}{M_*^2} L_4 + \frac{1}{M_*^4} L_6 \right), \]

where \( h \) is the determinant of the induced metric \( h_{ij} \) on the spacelike hypersurfaces, and \( L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi R + \eta a_i a^i \) with \( K \) is the trace of the extrinsic curvature. \( K_{ij}, \xi R \) is the Ricci scalar of \( h_{ij} \). \( N \) is the lapse function, and \( a_i = \partial_i \log N \).

\( L_4 \) and \( L_6 \) denote a collection of 4th and 6th order operators respectively and \( M_* \) is the scale that suppresses these operators.

These Infrared (IR) Lorentz violations are controlled by three dimensionless parameters that take the values \( \lambda = 1, \xi = 1, \eta = 0 \) in General Relativity (GR).

Unfortunately \( L_4 \) and \( L_6 \) contain a very large number of operators (~10^2) and so have been proposed several restrictions to the theory to limit them. In particular, projectability; \( N = N(t) \) | Detailed balance.

There is still debate about these constraints, we shall not deal with them here and our conclusions are general and does not hinge on the exact form of \( L_4 \) and \( L_6 \).
How much can be $M_*$? It is indeed bounded from below and above

$\mathcal{M}_{\text{obs}} < M_* < 10^{16}$ GeV

$\mathcal{M}_{\text{obs}} \approx \text{few meV}$ (from sub mm tests)

Due to the reduced symmetry with respect to GR, the theory propagates an extra scalar mode. If one chooses to restore diffeomorphism invariance, then this mode manifests as a foliation-defining scalar.

The condition $M_* < 10^{16}$ GeV

is a consequence of the need to protect perturbative renormalizability by assuring that the mass scale of the Horava scalar mode $M_{sc} > M_*$ (i.e. strong coupling when UV terms become non negligible)

Plus Solar System constraints on L2 that generically imply $M_{sc} < 10^{16}$ GeV.

However we have already seen that LIV cannot be confined to gravity!

- Higher order operators will always induce lower order ones by radiative corrections! (Collins et al. PRL93 (2004), Iengo, Russo, Serone 2009)
- So in general even starting with a Lorentz invariant matter sector at tree level one expects that matter LIV operators will be generated via graviton radiative corrections
- Let us assume that some protective mechanism can be envisaged to protect the lowest order operators (universal coefficient of $p^2$ in MDR c=1), i.e. Horava gravity IR viable.
- Then the symmetries of the LIV operators in Hořava-Lifshitz action naturally leads to the expectation for matter MDR (we assume no LIV at three level in matter and that CPT,P even nature of LIV in gravity sector is maintained in the LIV terms induced in matter)

\[ E^2 = m^2 + p^2 + \eta \frac{p^4}{M_{\text{LV}}^2} + O \left( \frac{p^6}{M_{LV}^4} \right). \]  

Now: Is $M_{\text{LIV}} \sim M_*$ or $M_{\text{LIV}} \gg M_*$?

Using time delay from GRB one can infer $M_{\text{LIV}} > 10^{11}$ GeV. Can we improve this without using UHECR?
Synchrotron radiation constraint for HH gravity

Crab Nebula spectrum for the LI case (blue, solid curve), for the LV case $n=4$, with $M_{LV} = 10^{15}$ GeV and $\eta > 0$ (red, dashed curve), and for the case with same parameters but $\eta < 0$ (magenta, dot-dashed curve). While, as discussed, the $\eta < 0$ case would lead to premature fall off of the synchrotron spectrum, we see here that for $\eta > 0$ there is a sudden surge of emission at high frequencies, followed by a dramatic drop due to the onset of vacuum Čerenkov emission at the characteristic threshold energy $E_{th} \approx [mM_{LV}]^{1/2}/\eta^{1/4}$.

Dependence of the reduced $\chi^2$ on $M_{LV}$. By considering the offset from the minimum of the reduced $\chi^2$ we set exclusion limits at 90%, 95% and 99% Confidence Level (CL). Mass scales $M_{LV} \approx 2 \times 10^{16}$ GeV are excluded at 95% CL. The window for $M_{LV} \sim M_*$ is closed. Therefore a mechanism, suppressing the percolation of LV in the matter sector, must be present in HL models, and such mechanism should not only protect lower order operators.
It is indeed possible to constrain QG models predicting LIV with high energy astrophysics observations.

The LIV Naturalness problem is a crucial open issue, custodial symmetry? Microphysics (beyond EFT) explanation?

It is time to be more specific about the models we are trying to constrain: We have here considered the specific case of Horava-Lifshitz gravity.

We have shown that the scale of induced Lorentz violations in matter cannot be of the same order of the scale setting LIV effects in the gravity sector.

Ways out: tune $\lambda$, $\xi$, $\eta$ so that $M_{sc}$ gets higher (Blas, Pujolas Sibiryakov, JHEP 1104, 2011) Or do not assume P invariance.

A probably more appealing option is offered by the mechanism proposed by (Pospelov & Shang arXiv.org/1010.5249v2) of "gravitational confinement".

CONCLUSIONS

Are we at the dawn of a real QG phenomenology?
It is indeed possible to constrain QG models predicting LIV with high energy astrophysics observations. The LIV Naturalness problem is a crucial open issue, custodial symmetry? Microphysics (beyond EFT) explanation? It is time to be more specific about the models we are trying to constrain: We have here considered the specific case of Horava-Lifshitz gravity. We have shown that the scale of induced Lorentz violations in matter cannot be of the same order of the scale setting LIV effects in the gravity sector. Ways out: tune $\lambda$, $\xi$, $\eta$ so that $M_{\text{sc}}$ gets higher (Blas, Pujolas Sibiryakov, JHEP 1104, 2011) Or do not assume P invariance. A probably more appealing option is offered by the mechanism proposed by (Pospelov & Shang arXiv.org/1010.5249v2) of "gravitational confinement". Can it be that a large LIV is indeed hidden in the UV of the gravitational sector?

CONCLUSIONS

- The QG phenomenology community is watching you....
- So keep on doing your fancy theories but remember: The QG phenomenology community is watching you....