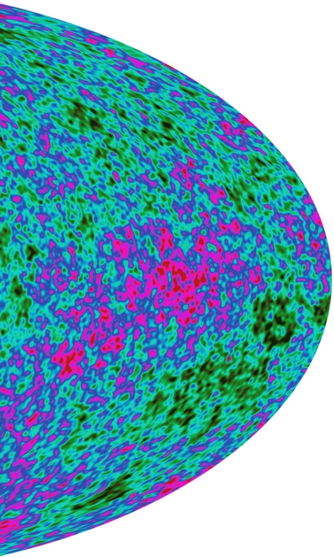

# Holography for Cosmology

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# GRAPPA

A new initiative formed by three physics institutes:

- Institute for Theoretical Physics (ITFA)
  Affiliate members: J. de Boer, E. Verlinde, K. Skenderis, M. Taylor, P. van der Schaar
- Institute for High Energy Physics (IHEF)
  Affiliate members: S. Bentvelsen, E. de Wolf, E. Laenen, P. Kooijman, P. de Jong
- Anton Pannekoek Institute (API) for astronomy
  Affiliate members: R. Wijers, A. Watts, M. van der Klis, S. Markoff, R. Wijnands

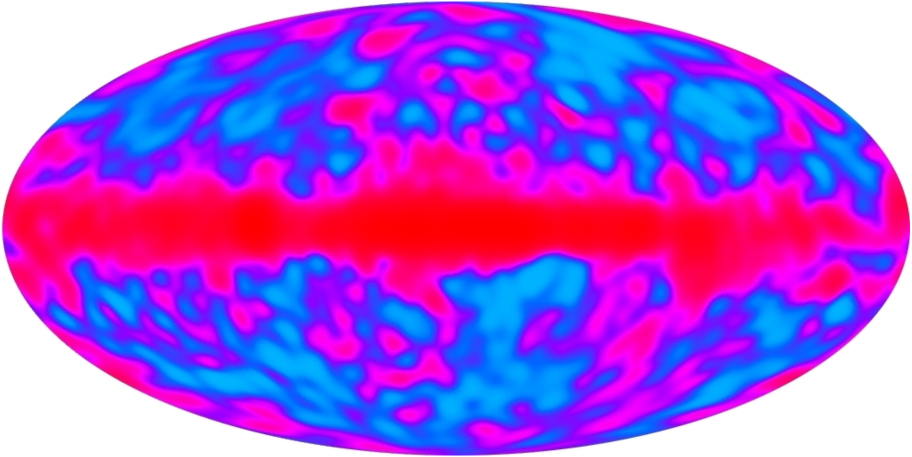
We are currently searching for at least four new faculty members in senior and junior ranks, tenured or tenure-track.

# Outline

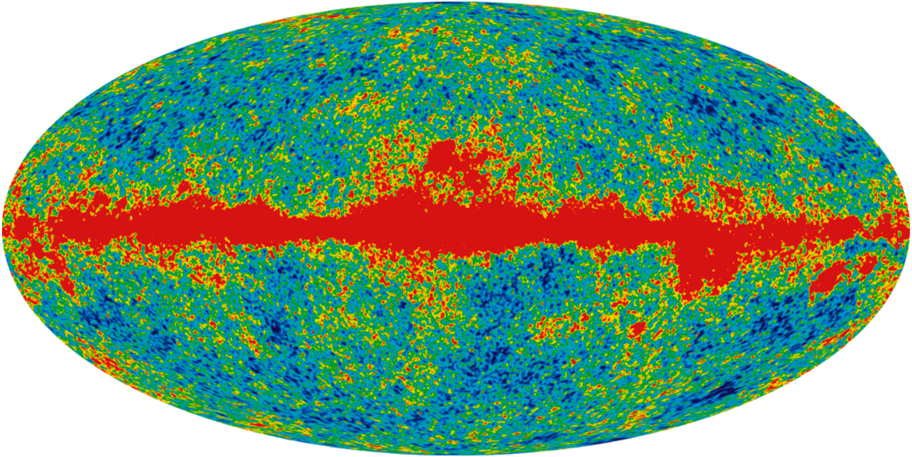
# Introduction

Over the last two decades, striking new observations have transformed cosmology from a *qualitative* to a *quantitative* science.

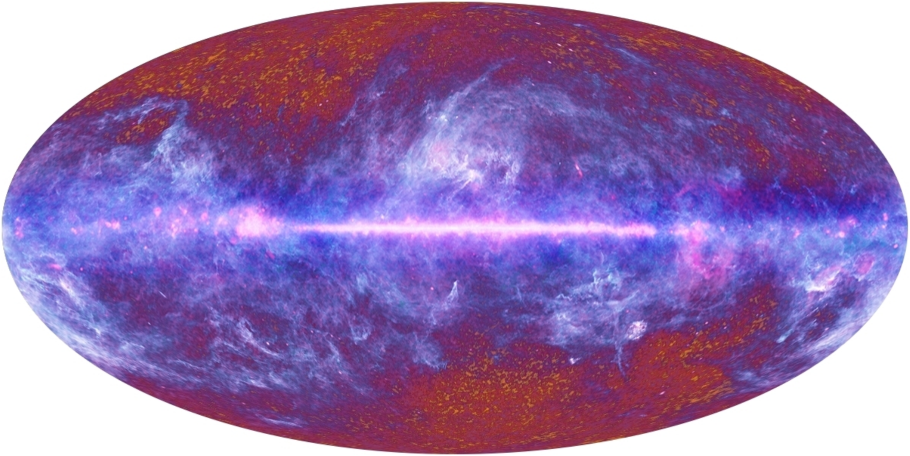
# COBE (1989)



# WMAP (2001)



# Planck (2009)



# Primordial perturbations

The primordial perturbations offer some of our best clues as to the fundamental physics underlying the big bang. Their form appears to be very simple:

- Small amplitude: $\delta T/T \sim 10^{-5}$.
- *Nearly* Gaussian.
- *Nearly* scale-invariant.
- Adiabatic.

Any proposed cosmological model must be able to account for these basic features, and any predicted deviations (*e.g.* from Gaussianity) are likely to prove critical in distinguishing different models.

# The primordial power spectrum

A Gaussian distribution is fully characterised by its 2-point function or power spectrum. From observations, the power spectrum takes the form:

$$\Delta_S^2(q) = \Delta_S^2(q_0)\,(q/q_0)^{n_S(q)-1}$$

The WMAP data yield (for $q_0 = 0.002\mathrm{Mpc}^{-1}$)

$$\Delta_S^2(q_0) = (2.445 \pm 0.096) \times 10^{-9}, \qquad n_S - 1 = -0.040 \pm 0.013,$$

*i.e.*, the scalar perturbations have small amplitude and are nearly scale invariant.

- These two small numbers should appear naturally in any theory that explains the data.

# Non-Gaussianity

Non-Gaussianity implies non-zero higher-point correlation functions. The lowest order is the 3-point function, or bispectrum, of curvature perturbations $\zeta$:

$$\langle \zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle = (2\pi)^3\delta(\sum q_i)B(q_i)$$

Non-Gaussianity arises from nonlinearities in cosmological evolution. The three main sources are:

1. Nonlinearities (interactions) in inflationary dynamics.
2. Nonlinear evolution of perturbations in radiation/matter era.
3. Nonlinearities in relationship between metric perturbations and CMB temperature fluctuations. (To linear order, $\Delta T/T = (1/3)\Phi$).

# Non-Gaussianity

- Non-Gaussianity is important as it potentially provides a *very strong* test of inflationary models. The amplitude of the bispectrum is parametrised by $f_{NL}$:

$$B(q_i) = f_{NL} \times (\text{shape function})$$

- Different inflationary models give different predictions for $f_{NL}$ and shape function.

# The shape of non-Gaussianity

- Local form [Gangui etal (1994)]; [Verde etal(2000)] ; Komatsu & Spergel (2001)]

$$B_{\rm local}(q_1,q_2,q_3) = f_{NL}^{\rm local} \frac{6A^2}{5q_1^3q_2^3q_3^3}\sum_{i=1}^{3} q_i^3, \qquad A = 2\pi^2\Delta_S^2(q)$$

  $\rightarrow$ WMAP7: $f_{NL}^{\rm local} = 32 \pm 21 (68\% CL)$
  $\rightarrow$ Single scalar slow-roll inflation: $f_{NL}^{\rm local} \sim O(\epsilon,\eta) \sim 0.01$
- Equilateral form [Creminelli etal, astro-ph/0509029]]

$$B_{\rm equil}(q_1,q_2,q_3) = f_{NL}^{\rm equil} \frac{18A^2}{5q_1^3q_2^3q_3^3}\left(-2q_1q_2q_3 - \sum_{i=1}^{3} q_i^3 + (q_1q_2^2 + 5\ perm)\right)$$

  $\rightarrow$ WMAP7: $f_{NL}^{\rm equil} = 26 \pm 140 (68\% CL)$
- The Planck data (expected next year) should be sensitive to just

$$f_{NL} \sim 5.$$

# Holographic Universe

In this talk I will present new holographic models for the very early universe:

- In these models, the very early universe is non-geometric and has a weakly coupled description in terms of a three dimensional QFT.
- They provide a new mechanism for a scale invariant spectrum.
- They are compatible with current observations, yet they have different phenomenology than conventional inflation.

# References

The talk is based on

- work with Paul McFadden
  Holography for Cosmology, arXiv:0907.5542
  The Holographic Universe, arXiv:1007.2007
  Observational signatures of holographic models of inflation, arXiv:1010.0244
  Holographic Non-Gaussianity, arXiv:1010.????
- on-going work with Adam Bzowski and Paul McFadden

# Holography for cosmology

Any holographic proposal for cosmology should specify

1. what the dual QFT is
2. how it can be used to compute cosmological observables *(the holographic dictionary)*

Having defined the duality,

- the new description should recover established results in the regime where the weakly coupled gravitational description is valid
- new results should follow by using the duality in the regime where gravity is strongly coupled.

# Main results

Our two main results are:

Standard inflation is holographic.

There are holographic models that have different phenomenology than slow-roll inflation but they are nevertheless consistent with current observations. The Planck data has the power to comfortably refute or confirm these models.

# Plan

In the first part, I will explain the sense in which inflation is holographic.

- Review standard inflationary computations.
- Review how to compute strong coupling QFT results using standard gauge/gravity duality.
- Show that the inflationary results can be fully expressed in terms of correlators of strongly coupled QFTs.

In the second part, I will discuss the new holographic models. While standard inflation is linked to strongly coupled QFTs, the new models are based on weakly coupled three dimensional QFT.

# Outline

# Outline

# Cosmological Perturbations

We start by reviewing standard inflationary cosmology.

- We will discuss (for simplicity) single field four dimensional inflationary models,

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}(R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi))$$

- We assume a spatially flat background (for simplicity)

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t) dx^i dx^i \\ \Phi &= \varphi(t) \end{aligned}$$

- The physical degrees of freedom are a scalar field $\zeta$ and a transverse traceless metric $\gamma_{ij}$.

# Power spectrum

In the inflationary paradigm, cosmological perturbations are assumed to originate at sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\begin{aligned} \langle \zeta(t,\vec{q})\zeta(t,-\vec{q})\rangle &= |\zeta_q(t)|^2 \\ \langle \gamma_{ij}(t,\vec{q})\gamma_{kl}(t,-\vec{q})\rangle &= 2|\gamma_q(t)|^2\Pi_{ijkl}, \end{aligned}$$

  where $\Pi_{ijkl}$ is the transverse traceless projection operator and $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are obtained by

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}|\zeta_q(0)|^2, \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}|\gamma_q(0)|^2,$$

  where $\gamma_q(0)$ and $\zeta_q(0)$ are the constant late-time values of the cosmological mode functions.

# Non-gaussianity

- Non-Gaussianity is related to higher-point functions. In this talk we focus on the three-point function of $\zeta$. This is computed using the in-in formalism as

$$\langle \zeta^3(t) \rangle = -i \int_{t_0}^{t} dt' \langle [\zeta^3(t), H_{int}(t')] \rangle$$

  where $H_{int}$ is obtained by expanding the action to cubic order.
- This leads to

$$\langle \zeta_{q_1} \zeta_{q_2} \zeta_{q_3} \rangle = (2\pi)^3 \delta(q_1 + q_2 + q_3) B(q_1, q_2, q_3)$$

  Different models are characterized by different $B(q_1, q_2, q_3)$.

# Response functions

Let us now reformulate these results in terms of response functions.

- The response functions, $\Omega_2, \Omega_3, E_2, \ldots$, are defined by

$$\Pi(\vec{x}_1) = \int d^3x_2 \Omega_2(\vec{x}_1 - \vec{x}_2)\zeta(\vec{x}_2) + \int d^3x_2 d^3x_3 \Omega_3(\vec{x}_2 - \vec{x}_1, \vec{x}_3 - \vec{x}_1)\zeta(\vec{x}_2)\zeta(\vec{x}_2) + \cdots$$

$$\Pi^\gamma_{ij}(\vec{x}_1) = \int d^3x_2 E_2(\vec{x}_1 - \vec{x}_2)\gamma_{ij}(\vec{x}_2) + \cdots ,$$

where $\Pi$ and $\Pi^\gamma_{ij}$ are the canonical momenta of $\zeta$ and $\gamma_{ij}$ and the dots indicate other terms that are quadratic and higher order in fluctuations.

# Perturbation equations

The field equations can be written in terms of response functions and we present here the ones associated with $\zeta$:

$$
\begin{aligned}
0 &= \dot{\Omega}_2(q) + \frac{1}{2a^3\epsilon}\Omega_2^2(q) - 2a\epsilon q^2, \\
0 &= \dot{\Omega}_3(q_i) + \frac{1}{2a^3\epsilon}\big(\Omega_2(q_1) + \Omega_2(q_2) + \Omega_2(q_3)\big)\Omega_3(q_i) + \mathcal{X}(q_i),
\end{aligned}
$$

where $\mathcal{X}(q_i)$ depends on the interactions, $\epsilon = 2(H'/H)^2$ and $H$ is the Hubble function.

- The cubic in fluctuations Hamiltonian is of the form

$$H_{int} = \int (\mathcal{A}\zeta^3 + \mathcal{B}\Pi\zeta^2 + \mathcal{C}\Pi^2\zeta + \mathcal{D}\Pi^3)$$

→ The coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ depend on the theory under consideration.

- Then

$$\begin{aligned}\mathcal{X}(q_i) &= 3\mathcal{A}_{123} + \mathcal{B}_{123}\Omega_2(q_1) + \mathcal{B}_{213}\Omega_2(q_2) + \mathcal{B}_{312}\Omega_2(q_3) + \mathcal{C}_{123}\Omega_2(q_2)\Omega_2(q_3) \\ &\quad + \mathcal{C}_{213}\Omega_2(q_1)\Omega_2(q_3) + \mathcal{C}_{312}\Omega_2(q_1)\Omega_2(q_2) + 3\mathcal{D}_{123}\Omega_2(q_1)\Omega_2(q_2)\Omega_2(q_3)\end{aligned}$$

where $\mathcal{C}_{213} = \mathcal{C}(q_2, q_1, q_3)$, *etc.*

# Solution

The equations for the response functions can be solved:

- $\Omega_2(q) = 2a^3\epsilon\dot{\zeta}_q/\zeta_q$
- $\Omega_3(z, q_i) = -\left(\prod_i 1/\zeta_{q_i}(z)\right) \int_{z_0}^{z} dz' \mathcal{X}(z', q_i) \prod_i \zeta_{q_i}(z'),$

where $\zeta_q$ is a solution of the *linearised* equation of motion

$$0 = \ddot{\zeta}_q + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta}_q - a^{-2}q^2\zeta_q,$$

# Response functions and 2- and 3-point functions

- One can show that

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega_2(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E_2(q)].$$

  so the power spectra can be expressed in terms of the late time behavior of the response functions.

- One can also show that

$$B(q_1, q_2, q_3) \sim \frac{\mathrm{Im}[\Omega_3(q_1, q_2, q_3)]}{\prod_{i=1}^{3} \mathrm{Im}[\Omega_2(q_i)]}$$

  evaluated at late times.

We will next show that $\Omega_2(q)$, $E_2(q)$ and $\Omega_3(q_1, q_2, q_3)$ are related to two- and three-point functions of a **strongly coupled** $3d$ **QFT**.

# Outline

# Domain-wall/cosmology correspondence

The springboard for our discussion is a correspondence between cosmologies and domain-wall spacetimes.

- Domain-wall spacetime:

$$
\begin{aligned}
ds^2 &= dr^2 + e^{2A(r)} dx^i dx^i \\
\Phi &= \Phi(r)
\end{aligned}
$$

- This solves the field equations that follow from

$$
S_{DW} = \frac{1}{2\bar{\kappa}^2} \int d^4x \sqrt{g} \left[ -R + (\partial\Phi)^2 + 2\bar{\kappa}^2 \bar{V}(\Phi) \right],
$$

# Domain-wall/cosmology correspondence

- One can prove the following:

## Domain-wall/Cosmology correspondence

For **every** domain-wall solution of a model with potential $\bar{V}$ there is a FRW solution for a model with potential $(V = -\bar{V})$. [Cvetic, Soleng (1994)], [KS, Townsend (2006)]

- The correspondence can be understood as analytic continuation. The flip in the sign of $V$ guarantees that the metric remains real.
- An equivalent way to state the correspondence is

$$\bar{\kappa}^2 = -\kappa^2$$

# Domain-walls and holography

Domain-wall spacetimes enter prominently in holography. They describe holographic RG flows.

- The $AdS_{d+1}$ metric is the unique metric whose isometry group is the same as the conformal group in $d$ dimensions. This is the main reason why the bulk dual of a $CFT$ is $AdS$.
- The domain-wall spacetimes are the most general solutions whose isometry group is the Poincaré group in $d$ dimensions. Thus, if a QFT has a holographic dual the bulk solution must be of the domain-wall type.

# Holographic RG flows

There are two different types of domain-wall spacetimes whose holographic interpretation is fully understood.

1. The domain-wall is asymptotically $AdS_{d+1}$,

$$A(r) \to r, \qquad \Phi(r) \to 0, \qquad \text{as} \quad r \to \infty$$

   This corresponds to a QFT that in the UV approaches a fixed point. The fixed point is the CFT which is dual to the $AdS$ spacetime approached as $r \to \infty$.

# Holographic RG flows

2. The domain-wall has the following asymptotics

$$A(r) \to n \log r, \qquad \Phi(r) \to \sqrt{2n} \log r, \qquad \text{as} \quad r \to \infty$$

This case has only been understood recently [Kanitscheider, KS, Taylor (2008)] [Kanitscheider, KS (2009)].

$\longrightarrow$ Specific cases of such spacetimes are ones obtained by taking the near-horizon limit of the non-conformal branes (D0, D1, F1, D2, D4).

$\longrightarrow$ These solutions describe QFTs with a "generalized conformal structure": all terms in the action have the same scaling and there is a dimensionful coupling constant.

# Domain-wall/cosmology correspondence

Let us see how the correspondence acts on the domain-walls describing holographic RG flows.

1. Asymptotically AdS domain-walls are mapped to inflationary cosmologies that approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

2. The second type of domain-walls is mapped to solutions that approach power-law scaling solutions at late times,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

# Outline

# Holography: a primer

The holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

1. There is 1-1 correspondence between local gauge invariant operators $\mathcal{O}$ of the boundary QFT and bulk supergravity modes $\Phi$.
   - $\rightarrow$ The bulk metric corresponds to the energy momentum tensor of the boundary theory.
2. Correlation functions of gauge invariant operators can be extracted from the asymptotics of bulk solutions.

# Asymptotic solutions

- The standard gauge/gravity duality is based on spacetimes that are asymptotically locally Anti-de Sitter.
- These spacetimes have a conformal boundary and *near the conformal boundary* Einstein equations (with negative cosmological constant) hold.
- This implies that the metric has the following asymptotic form (in 4 bulk dimensions) [Fefferman, Graham (1985)]

$$ds^2 = dr^2 + e^{2r} g_{ij}(x,r) dx^i dx^j$$

$$g_{ij}(x,r) = \mathbf{g}_{(0)\mathbf{ij}}(\mathbf{x}) + e^{-2r} g_{(2)ij}(x) + e^{-3r} g_{(3)ij}(x) + \ldots$$

- $\mathbf{g}_{(0)}(\mathbf{x})$ is the metric of the spacetime where the boundary theory lives and (as such) it is also the source of the boundary energy momentum tensor.

# Correlation functions

- Using the formalism of holographic renormalization, we then find a precise relation between correlation functions and asymptotics [de Haro, Solodukhin, KS (2000)]

$$\langle T_{ij} \rangle = \frac{3}{2\kappa^2} g_{(3)ij}.$$

- This formula only requires that Einstein equations hold near the conformal boundary. *In particular, it is also valid when curvatures are large in the interior.*
- Higher-point functions are obtained by differentiating the 1-point functions w.r.t. sources and then setting the sources to their background value

$$\langle T_{i_1 j_1}(x_1) T_{i_2 j_2}(x_2) \cdots T_{i_n j_n}(x_n) \rangle \sim \frac{\delta^{(n-1)} g_{(3) i_1 j_1}(x_1)}{\delta g_{(0) i_2 j_2}(x_2) \cdots \delta g_{(0) i_n j_n}(x_n)} \Big|_{g_{(0)} = \eta}$$

# Correlation functions

Thus to solve the theory we need to know $g_{(3)}$ as a function of $g_{(0)}$. This can be obtained perturbatively.

- $\longrightarrow$ From gravity to QFT
  2-point functions are obtained by solving linearized fluctuations, 3-point functions by solving quadratic fluctuations etc. Here it is crucial that the gravitational approximation is valid and this results in correlators of strongly coupled QFT.
- $\longrightarrow$ From QFT to gravity
  Given QFT correlators one obtains an asymptotic solution. If the QFT correlators are that of weakly coupled QFT then the bulk description has the prescribed asymptotic behavior and is strongly coupled in the interior.

# Outline

# Correlation functions for holographic RG flows

- To compute correlation functions we perturb around the domain-wall. The linearized equations are given by [Bianchi, Freedman, KS (2001)], [Papadimitriou, KS (2004)],

$$\begin{aligned} 0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \bar{q}^2 e^{-2A}\zeta \\ 0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \bar{q}^2 e^{-2A}\gamma_{ij}, \end{aligned}$$

- Comparing with the cosmological perturbations, we find that the equations are mapped to each other provided

$$\bar{q} = -iq$$

- The same holds to all order: the fluctuation equations are mapped to each other provided the momenta are continued as above.

# Correlation functions for holographic RG flows

We now want to extract 2- and 3-point functions.

- Schematically, we must expand the perturbed solution near $r \to \infty$ and extract the piece that scales like $e^{-3r}$.
  - The part linear in fluctuation gives the 2-point function.
  - The part quadratic in fluctuation gives the 3-point function.
- It is convenient to work in terms of response functions [Papadimitriou, KS (2004)]

$$\bar{\Pi} = -\bar{\Omega}_2 \zeta - \bar{\Omega}_3 \zeta^2 + \cdots , \quad \bar{\Pi}^{\gamma}_{ij} = -\bar{E}_2 \gamma_{ij} + \cdots ,$$

where $\bar{\Pi}, \bar{\Pi}^{\gamma}_{ij}$ are radial canonical momenta.

# 2-point functions for holographic RG flows

The 2-point function of the energy momentum tensor is then given by

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

where $\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}), \quad \pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2.$

$$A(\bar{q}) = 4 \left[\bar{E}_2(\bar{q})\right]_{(0)}, \qquad B(\bar{q}) = \frac{1}{4} \left[\bar{\Omega}_2(\bar{q})\right]_{(0)}.$$

The subscript indicates that one should pick the term with appropriate scaling in the asymptotic expansion.

# 3-point functions for holographic RG flows

- Similarly, one can derive a holographic formula for the 3-point function

$$\langle T_{i_1 j_1}(\bar{q}_1) T_{i_2 j_2}(\bar{q}_2) T_{i_3 j_3}(\bar{q}_3)\rangle = \ldots$$

in terms of response functions.

- The 3-point function for the trace of stress energy tensor, $T = T_i^i$, is related to the response function $\Omega_3$ by

$$[\Omega_3(\bar{q}_1, \bar{q}_2, \bar{q}_3)]_{(0)} \sim \langle T(\bar{q}_1) T(\bar{q}_2) T(\bar{q}_3)\rangle + \sum_i \langle T(\bar{q}_i) T(-\bar{q}_i)\rangle$$
$$-2\big[\langle T(\bar{q}_1)\Upsilon(\bar{q}_2, \bar{q}_3))\rangle + \langle T(\bar{q}_2)\Upsilon(\bar{q}_1, \bar{q}_3))\rangle + \langle T(\bar{q}_3)\Upsilon(\bar{q}_1, \bar{q}_2))\rangle\big].$$

where

$$\Upsilon(\vec{x}_1, \vec{x}_2) = \frac{\delta T_{ij}(\vec{x}_1)}{\delta g^{kl}(\vec{x}_2)}\Big|_0 \delta^{ij}\delta^{kl}.$$

# Outline

# Holography for cosmology

We are now ready to present the holographic dictionary for cosmology.

- The DW/cosmology correspondence maps the near boundary region to the late time region.
- Under the analytic continuation

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq$$

the response functions continue as follows

$$\bar{\Omega}_2(\bar{q}) = \Omega_2(-iq), \quad \bar{E}_2(\bar{q}) = E_2(-iq),$$
$$\bar{\Omega}_3(\bar{q}_1, \bar{q}_2, \bar{q}_3) = \Omega_3(-iq_1, -iq_2, -iq_3).$$

- The analytic continuations translate in QFT language to

$$\bar{N} \to -iN, \qquad \bar{q} \to -iq$$

# Holographic dictionary: Power spectrum

- We have shown earlier that

$$\Delta_S^2(q) = \frac{-q^3}{4\pi^2 \mathrm{Im}\Omega_{(0)}(q)}, \quad \Delta_T^2(q) = \frac{-q^3}{2\pi^2 \mathrm{Im}E_{(0)}(q)},$$

It follows

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}\left(\frac{-1}{8\mathrm{Im}B(-iq)}\right), \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}\left(\frac{-1}{\mathrm{Im}A(-iq)}\right),$$

where the holographic 2-point function is

$$\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl},$$

# Holographic dictionary: Non-Gaussianity

- We have seen earlier that

$$\langle \zeta_{q_1}\zeta_{q_2}\zeta_{q_3}\rangle = (2\pi)^3\delta(q_1+q_2+q_3)B(q_1,q_2,q_3)$$

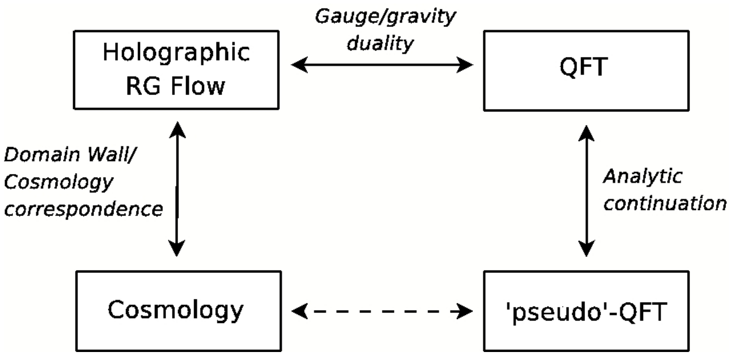
and $B(q_1,q_2,q_3) \sim \mathrm{Im}[\Omega_3(q_1,q_2,q_3)]/\prod_{i=1}^{3}\mathrm{Im}[\Omega_2(q_i)]$.

- It follows

$$B(q_1,q_2,q_3) = -\frac{1}{4}\frac{1}{\prod_i \mathrm{Im}\langle T(\bar{q}_i)T(-\bar{q}_i)\rangle}\cdot \mathrm{Im}\Big[\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle + \sum_i \langle T(\bar{q}_i)T(-\bar{q}_i)\rangle - 2\big(\langle T(\bar{q}_1)\Upsilon(\bar{q}_2,\bar{q}_3)\rangle + \text{cyclic perms}\big)\Big],$$

where the imaginary part is taken after the analytic continuation.

# Summary


Holographic RG Flow
Gauge/gravity duality
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

# Outline

# New holographic models

- We are now going to obtain new models by using weakly coupled QFT. This correspond to the gravitational theory being strongly coupled at early times.
- The boundary theory will be a combination of gauge fields, fermions and scalars and it should admit a large $N$ expansion.
- To extract predictions we need to compute $n$-point functions of the stress energy tensor analytically continue the result and insert them in the holographic formulae.

# The holographic model

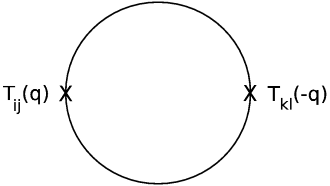
- As a model one can consider the strong coupling version of asymptotically dS cosmologies and power-law cosmology.
- In this work we focus on QFTs dual to the latter. These are super-renormalizable QFTs that depend on a single dimensionful coupling:

$$S = \frac{1}{g_{\mathrm{YM}}^2} \int d^3x \mathrm{tr} \left[ \frac{1}{2} F_{ij}^I F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \frac{1}{2} (D\chi^K)^2 + \bar{\psi}^L \not{D} \psi^L \right.$$
$$\left. + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu_{M L_1 L_2}^{\alpha\beta} \Phi^M \psi_\alpha^{L_1} \psi_\beta^{L_2} \right].$$

- All terms in this Lagrangian have dimension 4.

# A new mechanism for scale invariant spectrum

We need to compute the 2-point function of $T_{ij}$. The leading order computation is at 1-loop:




The answer follows from general considerations:

- The stress energy tensor has dimension 3 in three dimensions.
- 1-loop amplitudes are independent of $g_{YM}^2$
- There is a factor of $\bar{N}^2$ because of the trace over the gauge indices.

$$\langle T_{ij} T_{kl} \rangle \sim \bar{N}^2 \bar{q}^3$$

# A new mechanism for scale invariant spectrum

Recalling the holographic map:

$$\Delta_S^2 \sim \frac{\bar{q}^3}{\langle TT \rangle} \sim \frac{1}{N^2}$$

- Spectrum is scale invariant to leading order, independent of the details of the holographic theory.

Furthermore,

- Amplitude of power spectrum $\mathcal{A} \sim 1/N^2$.
- Small $\mathcal{A} \sim 10^{-9} \Rightarrow$ large $N \sim 10^4$, justifying the large $N$ limit.

# Power spectra

The complete answer is

$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 + O(g_{\rm YM}^2), \qquad B(\bar{q}) = C_B \bar{N}^2 \bar{q}^3 + O(g_{\rm YM}^2),$$

where

$$C_A = (\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi)/256, \qquad C_B = (\mathcal{N}_A + \mathcal{N}_\phi)/256.$$

It follows

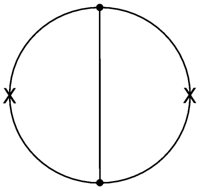
$$\Delta_S^2(q) = \frac{1}{16\pi^2 N^2 C_B} + O(g_{\rm YM}^2), \qquad \Delta_T^2(q) = \frac{2}{\pi^2 N^2 C_A} + O(g_{\rm YM}^2).$$

$\mathcal{N}_A$ : # of gauge fields, $\mathcal{N}_\phi$ : # of minimally coupled scalars,
$\mathcal{N}_\chi$ : # of conformally coupled scalars, $\mathcal{N}_\psi$ : # of fermions.

# Tensors-to-scalar ratio

- It follows that
$$r = \Delta_T^2/\Delta_S^2 = 32C_B/C_A,$$
- This is is not parametrically suppressed as in slow-roll inflation, nor does it satisfy the conventional slow-roll consistency condition $r = -8n_T$.
- An upper bound on $r$ translates into a constraint on the field content of the dual QFT.
- A smaller upper bound on $r$ requires increasing the number of conformal scalars and massless fermions and/or decreasing the number of gauge fields and minimal scalars.

# Subleading corrections



Subleading corrections give small deviations from scale invariance:

$$n_s - 1 \sim g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q.$$

The observational value $(n_s - 1) \sim 10^{-2}$ is then consistent with the QFT being weakly interacting.

- To determine the sign of $(n_s - 1)$ (positive: red-tilted spectrum, negative: blue-tilted spectrum) requires summing all 2-loop graphs, and will in general depend on the field content of the dual QFT.

*[Work in progress]*

# 2-loop details

Super-renormalizable theories often have infrared problems. The specific type of theories we consider however are well-defined: $g_{YM}^2$ acts as an infrared cut-off. [Jackiw, Templeton (1981)] [Appelquist, Pisarski (1981)].

The 2-loop integrals are indeed finite and one obtains:

$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 [1 + D_A g_{\rm eff}^2 \ln \bar{q}/\bar{q}_0 + O(g_{\rm eff}^4)],$$
$$B(\bar{q}) = C_B \bar{N}^2 \bar{q}^3 [1 + D_B g_{\rm eff}^2 \ln \bar{q}/\bar{q}_0 + O(g_{\rm eff}^4)],$$

where $g_{\rm eff}^2 = g_{\rm YM}^2 \bar{N}/\bar{q}$ and $D_A$ and $D_B$ are numerical constants.

This leads to

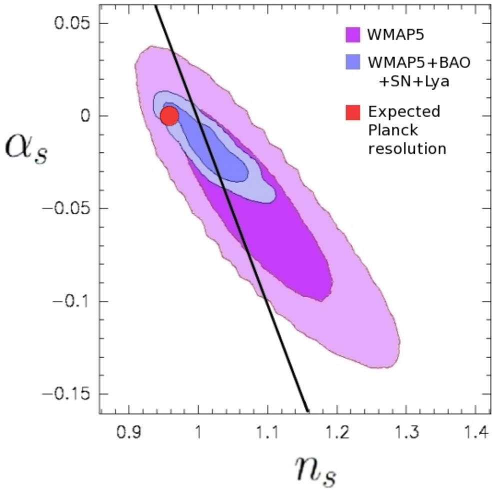
$$n_S(q) - 1 = -D_B g_{\rm eff}^2 + O(g_{\rm eff}^4), \qquad n_T(q) = -D_A g_{\rm eff}^2 + O(g_{\rm eff}^4).$$

# Running

- Independent of the details of the theory, the scalar spectral index runs as
$$\alpha_s = \frac{dn_s}{d \ln q} = -(n_s - 1) + O(g_{\rm eff}^4).$$
- This prediction is qualitatively different from slow-roll inflation, for which $\alpha_s/(n_s - 1)$ is of first-order in slow-roll.
- This prediction is consistent with current data and Planck should be able to either exclude or confirm this running.

# WMAP data

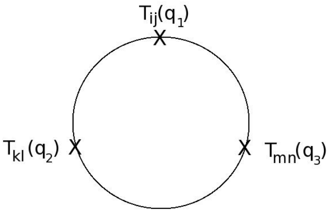
WMAP Cosmological Parameter Plotter




Solid line:

$$\alpha = -(n_s - 1)$$

# Non-Gaussianity




Direct computation gives

$$
\begin{aligned}
&\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle + \sum_i \langle T(\bar{q}_i)T(-\bar{q}_i)\rangle \\
&\quad - 2\big(\langle T(\bar{q}_1)\Upsilon(\bar{q}_2,\bar{q}_3)\rangle + \text{cyclic perms}\big) \\
&= 2C_B\bar{N}^2\Big(2\bar{q}_1\bar{q}_2\bar{q}_3 + \sum_i \bar{q}_i^3 - (\bar{q}_1\bar{q}_2^2 + 5\,\text{perms})\Big)
\end{aligned}
$$

- Using the holographic formula one finds

$$B(q_1,q_2,q_3) = B_{NL}^{\text{equil}}(q_1,q_2,q_3)$$

with

$$f_{NL}^{\text{equil}} = \mathbf{5/36}$$

- This is independent of all details of theory.
- This value is larger than the $f_{NL}$ for slow-roll inflation, but probably still too small to be detected by Planck.

# Outline

# Conclusions

- I have presented a holographic description of inflationary cosmology in terms of a 3-dimensional QFT (without gravity!)
- When gravity is weakly coupled, holography correctly reproduces standard inflationary predictions for cosmological observables.
- When gravity is strongly coupled, one finds new models that have a QFT description.

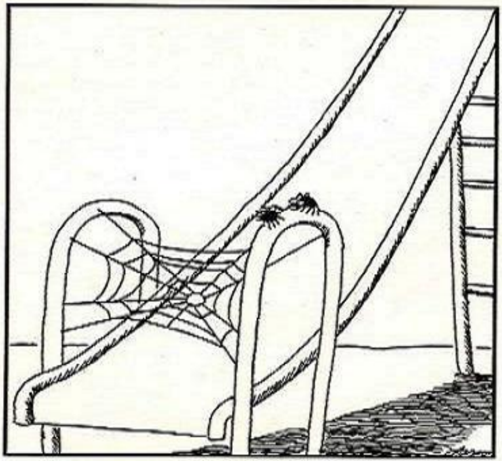
# Observational signatures

I presented models with the following universal features:

1. they have a nearly scale invariant spectrum of small amplitude primordial fluctuations.
2. the scalar spectral index runs as $\alpha_s = -(n_s - 1)$.
3. the three point function of curvature perturbations is exactly equal to the equilateral form with $f_{NL}^{\text{equil}} = 5/36$.

Both predictions 2 and 3 could easily be ruled out by the Planck data next year.

# Outlook



"If we pull this off, we'll eat like kings."

# DW/cosmology correspondence [KS, Townsend (2006)]

- FRW spacetime ($k = 0, -1, +1$)

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin\theta^2 d\Omega_{d-2}^2) \right)$$

- Curved domain-wall ($\kappa = 0, -1, +1$)

$$ds^2 = dz^2 + e^{2A(z)} \left( -\frac{d\tau^2}{1 + \kappa\tau^2} + \tau^2 (d\psi^2 + \sinh\psi^2 d\Omega_{d-2}^2) \right)$$

- The analytic continuation

$$(t, r, \theta) = -i(z, \tau, \psi)$$

maps the one solution to the other with

$$a(t) \leftrightarrow e^{A(z)}, \qquad k \leftrightarrow -\kappa$$