Neutrino Models with Flavor Symmetry

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Plan of my talk

1 Tri-bi maximal mixing and Flavor Symmetry

2 Neutrino Flavor Models
with Non-Abelian Discrete Symmetry

3 Breaking of Flavor Symmetry

4 Related Phenomena of Flavor Symmetry

5 Summary

1 Tri-bimaximal mixing and Flavor symmetry

Recent experiments of the neutrino oscillations go into a new phase of precise determination of mixing angles and mass squared differences.

Neutrino Parameters

Global fit for 3 flavors **by Jose**

parameter	best fit	2σ	3σ	tri-bimaximal
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.59^{+0.23}_{-0.18}$	7.22–8.03	7.03–8.27	*
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75	*
$\sin^2\theta_{12}$	$0.318^{+0.019}_{-0.016}$	0.29–0.36	0.27–0.38	$1/3$
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67	$1/2$
$\sin^2\theta_{13}$	$0.013^{+0.013}_{-0.009}$	≤ 0.039	≤ 0.053	0

T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 10, 113011 (2008)

Three Flavor analysis strongly suggests Tri-bimaximal Mixing of Neutrinos

Harrison, Perkins, Scott (2002)

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

indicates Non-Abelian Flavor Symmetry ?

Consider the structure of Neutrino Mass Matrix, which gives Tri-bi maximal mixing

$$M_\nu^{\rm exp} \simeq V_{\rm tri-bi}^{*} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_{\rm tri-bi}^{\dagger}$$

$$= \boxed{\frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}}$$

- integer (inter-family related) matrix elements

$\Longleftrightarrow$ non-abelian discrete flavor sym

Mixing angles are independent of mass eigenvalues.

Those seem different from quark mixing angles $\left(\theta_{ij} \times \sqrt{\frac{m_i}{m_j}}\right)$

Let us consider Flavor Symmetry.

- abelian or non-abelian ?

  abelian : discriminate between generations
  non-abelian : connect different generations

- continuous or discrete ?

  continuous : free rotation between generations
  discrete : definite meaning of generations

Non-Abelian Discrete Symmetry is appropriate for Neutrino Flavor Physics if TBM is not accidental.

$$M_\nu^{\rm exp} = \frac{m_1+m_3}{2}\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1-m_3}{2}\cancel{\begin{pmatrix}1 & & \\ & & 1\\ & 1 & \end{pmatrix}}$$

- The 1st and 2nd terms are S_3 symmetric
  (the most general S_3–invariant forms)
- The 3rd is not (S_3 flavor sym breaking)

Need some ideas to realize Tri-bi maximal mixing by S_3 flavor symmetry

Extra property of neutrinos :

- $m_1 \simeq m_3$ — Chen–Wolfenstein
  - * the 3rd term negligible
  - * degenerate neutrino masses
- Magic matrix $\sum_i M_{\nu_{ij}} = \sum_j M_{\nu_{ij}}$ — Lam
- Twisted flavors — Haba–Watanabe–KY
- Extra higgs contributions — Mohapatra–Nasri–Yu

A_4 Symmetry may be hidden.

Tetrahedral Symmetry

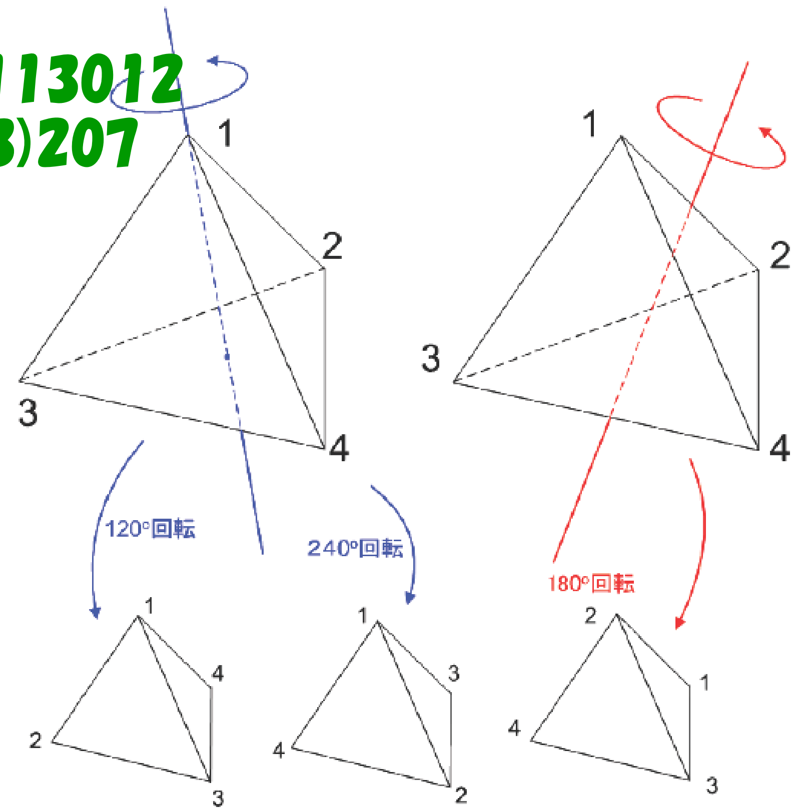
Four irreducible representations $\quad 1 \quad 1' \quad 1'' \quad 3$

A_4 is minimal symmetry including triplet $\quad$ 12 elements

E. Ma and G. Rajasekaran, PRD64(2001)113012
K.S.Babu, E.Ma, J.W.F.Valle, PLB 552(2003)207

the even permutation of 4 objects

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	-1



Suppose A_4 triplet $(\nu_e, \nu_\mu, \nu_\tau)_L$

$$M_\nu^{\rm exp} = \frac{m_1+m_3}{2}\begin{pmatrix}1&&\\&1&\\&&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&&\\&&1\\&1&\end{pmatrix}$$

- The 3rd term is A_4 symmetric
- A 3-dim higgs gives the general A_4-symmetric Majorana mass term: $3_L \times 3_L \times 3_H$ $3_L \times 3_L \times 1_H$

$$M_\nu^{A_4} = \begin{pmatrix}a&&\\&b&\\&&c\end{pmatrix} - \frac{1}{3}\begin{pmatrix}a&c&b\\c&b&a\\b&a&c\end{pmatrix} + x\begin{pmatrix}1&&\\&&1\\&1&\end{pmatrix}$$

$$\boxed{a = b = c \iff V_{\rm tri-bi}}$$

A_4 should be broken !

A_4 flavor sym breaking $\rightarrow Z_2 : \frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix}$

T', S_4, $\Delta(27)$, $\Delta(54)$ also give (near) Tri-bi maximal mixing !

Non-Abelian Discrete Flavor Symmetry is related with other Physical Phenomena.

- **$U_{e3}=0$ in Tri-bimaximal mixing!**

  There are hints Non-zero U_{e3} in experiments.
  How can one predict U_{e3} ?

- **CKM mixing in Quarks ? Cabibbo angle?**

  We need Quark-lepton unification in a GUT.

- **SUSY Flavor Sector, SUSY FCNC , EDM**

2 Neutrino Flavor Models with Non-Abelian Discrete Symmetry

Let us understand how to get the tri-bimaximal mixing in the example of A_4 flavor model.

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

A_4 Flavor model

$A_4 \times Z_3$ charge assignment

	(L_e, L_μ, L_τ)	R_e^c	R_μ^c	R_τ^c	$H_{u,d}$	χ_ℓ	χ_ν	χ
A_4	3	1	$1'$	$1''$	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω

χ_ℓ, χ_ν, χ are new scalars of gauge singlets.

A_4 invariant superpotential can be written by:

for charged leptons

$$W_L = \frac{y_e}{\Lambda}(L_e\chi_{\ell_1} + L_\mu\chi_{\ell_3} + L_\tau\chi_{\ell_2})R_e H_d \qquad \mathbf{3_L \times 3_{flavon} \to 1}$$

$$+\frac{y_\mu}{\Lambda}(L_e\chi_{\ell_2} + L_\mu\chi_{\ell_1} + L_\tau\chi_{\ell_3})R_\mu H_d \qquad \mathbf{3_L \times 3_{flavon} \to 1''}$$

$$+\frac{y_\tau}{\Lambda}(L_e\chi_{\ell_3} + L_\mu\chi_{\ell_2} + L_\tau\chi_{\ell_1})R_\tau H_d + h.c.,$$

$$\mathbf{3_L \times 3_{flavon} \to 1'}$$

for neutrinos

$$W_\nu = \frac{y_1}{\Lambda^2}(L_eL_e + L_\mu L_\tau + L_\tau L_\mu)H_uH_u\chi \qquad \mathbf{3_L \times 3_L \times 1_{flavon} \to 1}$$

$$+\frac{y_2}{3\Lambda^2}[(2L_eL_e - L_\mu L_\tau - L_\tau L_\mu)\chi_{\nu_1}$$

$$+(-L_eL_\tau + 2L_\mu L_\mu - L_\tau L_e)\chi_{\nu_2} \qquad \mathbf{3_L \times 3_L \times 3_{flavon} \to 1}$$

$$+(-L_eL_\mu - L_\mu L_e + 2L_\tau L_\tau)\chi_{\nu_3}]H_uH_u + h.c.,$$

$$1' \times 1'' \to 1$$

After $A_4 \times Z_3$ symmetry is spontaneously broken by VEVs of χ_ℓ, χ_ν, and χ, mass matrices are obtained as

$$M_l = \frac{v_d}{\Lambda} \begin{pmatrix} y_e \langle \chi_{\ell_1} \rangle & y_e \langle \chi_{\ell_3} \rangle & y_e \langle \chi_{\ell_2} \rangle \\ y_\mu \langle \chi_{\ell_2} \rangle & y_\mu \langle \chi_{\ell_1} \rangle & y_\mu \langle \chi_{\ell_3} \rangle \\ y_\tau \langle \chi_{\ell_3} \rangle & y_\tau \langle \chi_{\ell_2} \rangle & y_\tau \langle \chi_{\ell_1} \rangle \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{3\Lambda} \begin{pmatrix} 3y_1 \langle \chi \rangle + 2y_2 \langle \chi_{\nu_1} \rangle & -y_2 \langle \chi_{\nu_3} \rangle & -y_2 \langle \chi_{\nu_2} \rangle \\ -y_2 \langle \chi_{\nu_3} \rangle & 2y_2 \langle \chi_{\nu_2} \rangle & 3y_1 \langle \chi \rangle - y_2 \langle \chi_{\nu_1} \rangle \\ -y_2 \langle \chi_{\nu_2} \rangle & 3y_1 \langle \chi \rangle - y_2 \langle \chi_{\nu_1} \rangle & 2y_2 \langle \chi_{\nu_3} \rangle \end{pmatrix}$$

where $v_d = \langle H_d \rangle$, $v_u = \langle H_u \rangle$.

These mass matrices do not yet predict tri-bimaximal mixing !

We need $< \chi_\ell >= (V_\ell, 0, 0)$ $< \chi_\nu >= (V_\nu, V_\nu, V_\nu)$

Can one get Desired Vacuum
in Spontaneous Symmetry Breaking ?

If vacuum expectation values are aligned,
$\langle\chi_\ell\rangle = (V_\ell, 0, 0)$ and $\langle\chi_\nu\rangle = (V_\nu, V_\nu, V_\nu)$,
which are obtained by potential analysis, then

$$M_l = \frac{v_d v_T}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a-b/3 \\ -b/3 & a-b/3 & 2b/3 \end{pmatrix}.$$

where $a = y_1 V/\Lambda$, $b = y_2 V_\nu/\Lambda$.

$\mathbf{3_L \times 3_L \times 3_{flavon}}$ $\qquad$ $\mathbf{3_L \times 3_L \times 1_{flavon}}$

$$M_\nu = \frac{v_u^2 b}{\Lambda}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{v_u^2 b}{3\Lambda}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v_u^2 a}{\Lambda}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Therefore, mixing matrix is tri-bimaximal matrix, and masses are

$$m_1 = \frac{v_u^2(a+b)}{\Lambda}, \quad m_2 = \frac{v_u^2 a}{\Lambda}, \quad m_3 = -\frac{v_u^2(a-b)}{\Lambda}.$$

See-Saw Realization

Introduce A_4 triplet ν^c

	(l_e, l_μ, l_τ)	$(\nu^c_e, \nu^c_\mu, \nu^c_\tau)$	e^c	μ^c	τ^c	h_u	h_d	ξ	$\tilde{\xi}$	$(\phi_{T_1}, \phi_{T_2}, \phi_{T_3})$	$(\phi_{S_1}, \phi_{S_2}, \phi_{S_3})$	Φ
A_4	3	3	1	$1''$	$1'$	1	1	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	1	ω^2	1
$U(1)_{FN}$	0	0	$2q$	q	0	0	0	0	0	0	0	-1

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) + (x_A \xi + \tilde{x}_A \tilde{\xi})(\nu^c \nu^c) + x_B(\varphi_S \nu^c \nu^c)$$

Dirac $3_R \times 3_R \times 3_{flavon}$ $3_R \times 3_R \times 1_{flavon}$

$$m_\nu^D = y v_u \mathbf{1} \quad , \qquad M = \begin{pmatrix} A+2B/3 & -B/3 & -B/3 \\ -B/3 & 2B/3 & A-B/3 \\ -B/3 & A-B/3 & 2B/3 \end{pmatrix} u$$

$$A \equiv 2x_A \quad , \qquad B \equiv 2x_B \frac{v_S}{u}$$

Dirac Mass Matrix is diagonal one.

Tri-bimaximal mixing comes from Majorana Mass matrix !

S_4 Flavor Model can also give Tri-bimaximal mixing

S_4 group is the symmetry group of octahedron or permutation of four elements. Number of elements is 24.

- Irreducible representations of S_4 are 3_1, 3_2, 2, 1_1, and 1_2.
- Multiplication rules are

$$
\begin{aligned}
&3_1 \times 3_1 = 1_1 + 2 + 3_1 + 3_2 \\
&3_2 \times 3_2 = 1_1 + 2 + 3_1 + 3_2 \\
&3_1 \times 3_2 = 1_2 + 2 + 3_1 + 3_2 \\
&2 \times 3_1 = 3_1 + 3_2 \\
&2 \times 3_2 = 3_1 + 3_2 \\
&2 \times 2 = 1_1 + 1_2 + 2 \\
&\quad \vdots
\end{aligned}
$$

  etc.
- S_4 invariant representation is 1_1.

$$\mathbf{3_1}, \quad \mathbf{3_2}, \quad \mathbf{2}, \quad \mathbf{1_1}, \quad \mathbf{1_2}$$

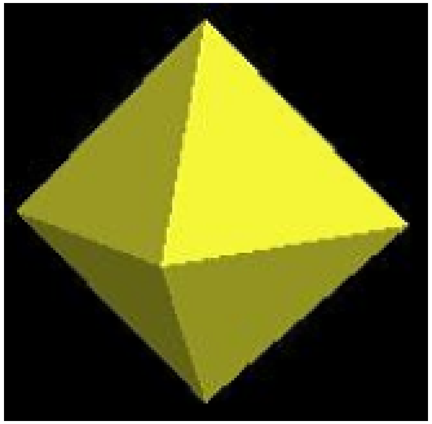


Figure: S_4 symmetry: Octahedron

Other Successful Models

$\Delta(54)$ Flavor Symmetry two 1 four 2 two 3

which is a series of $\Delta(6n^2)$

JHEP 0904: 011, 2009 JHEP 0912: 054, 2009

$\Delta(6)$ is S_3 $\Delta(24)$ is isomorphic to S_4

Simple and non-trivial example is $\Delta(54)$

$\Delta(54)$ is a nonAbelian discrete symmetry from stringy origin.
T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby, M. Ratz,
Nucl. Phys. B768: 135, 2007

$\Delta(27)$ Flavor Symmetry nine 1 and two 3

which is a series of $\Delta(3n^2)$

$\Delta(3)$ is Z_3 $\Delta(12)$ is isomorphic to A_4

S_4 flavor model

F. Bazzocchi, L. Merlo, and S. Morisi, Phys. Rev. D 80 053003 (2009).

- The assignment of the model:

neutrinos

	ℓ	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	ψ	η	Δ	φ	ξ'
S_4	$\mathbf{3}_1$	$\mathbf{1}_2$	$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{2}$	$\mathbf{1}_2$
Z_5	ω^4	1	ω^2	ω^4	ω	1	1	ω^2	ω^2	ω^3	ω^3	1
$U(1)_{FN}$	0	1	0	0	0	0	-1	0	0	0	0	0

- The superpotential in the lepton sector is as follows:

$$w_\ell = \sum_{i=1}^{4} \frac{\theta}{\lambda} \frac{y_{e,i}}{\Lambda^3} e^c (\ell X_i)' h_d + \frac{y_\mu}{\Lambda^2} \mu^c (\ell\psi\eta)' h_d + \frac{y_\tau}{\Lambda} \tau^c (\ell\psi) h_d + h.c + \cdots$$

$$X = \{\psi\psi\eta, \psi\eta\eta, \Delta\Delta\xi', \Delta\varphi\xi'\}$$

$$w_\nu = x(\nu^c\ell)h_u + x_d(\nu^c\nu^c\varphi) + x_t(\nu^c\nu^c\Delta) + h.c. + \cdots$$

3×3 $\quad 3\times3\times2_{\text{flavon}}$ $\quad 3\times3\times3_{\text{flavon}}$

- Vacuum alignment:

Neutrinos: $\langle\varphi\rangle = \begin{pmatrix}1\\1\end{pmatrix} v_\varphi, \quad \langle\Delta\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix} v_\Delta$

Charged leptons: $\langle\eta\rangle = \begin{pmatrix}0\\1\end{pmatrix} v_\eta, \quad \langle\psi\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix} v_\psi$

- Magnitude of VEVs:

$$\frac{\langle\theta\rangle}{\Lambda} = t, \quad \frac{\langle\phi\rangle}{\Lambda} = u \ (\phi = \psi, \eta, \Delta, \varphi, \xi')$$

- The charged lepton mass matrix: almost diagonal

$$m_\ell = \begin{pmatrix} y_e^{(1)} u^2 t & y_e^{(2)} u^2 t & y_e^{(3)} u^2 t \\ 0 & y_\mu u & 0 \\ 0 & 0 & y_\tau \end{pmatrix} u v_d$$

- The Dirac and Majorana neutrino mass matrix: $(b = 2x_d v_\varphi,\ c = 2x_t v_\Delta)$

$$m_\nu^D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} x v_u, \quad M_N = \begin{pmatrix} 2c & b-c & b-c \\ b-c & b+2c & -c \\ b-c & -c & b+2c \end{pmatrix}$$

- The left-handed Majorana neutrino mass matrix:

$$m_\nu = -(m_\nu^D)^T M_N^{-1} m_\nu^D =$$

$$\frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$m_1 = -\frac{x^2 v_u^2}{3c-b}, \quad m_2 = -\frac{x^2 v_u^2}{2b}, \quad m_3 = -\frac{x^2 v_u^2}{3c+b}$$

Can Non-abelian discrete symmetry predict quark mixing angles as well as Tri-bimaximal mixing of neutrinos ?

Yes, it is possible in S_4 !

H. Ishimori, K. Saga, Y. Shimizu, M. Tanimoto, arXiv:1004.5004
PRD 2010

$S_4 \times Z_4 \times U(1)_{FN}$ with SUSY SU(5) GUT

Use S_4 doublet for left-handed quarks !

	(T_1,T_2)	T_3	(F_1,F_2,F_3)	(N_e^c,N_μ^c)	N_τ^c	H_5	$H_{\bar 5}$	H_{45}	Θ
$SU(5)$	10	10	$\bar 5$	1	1	5	$\bar 5$	45	1
S_4	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_4	$-i$	-1	i	1	1	1	1	-1	1
$U(1)_{FN}$	ℓ	0	0	m	0	0	0	0	-1

	(χ_1,χ_2)	(χ_3,χ_4)	(χ_5,χ_6,χ_7)	$(\chi_8,\chi_9,\chi_{10})$	$(\chi_{11},\chi_{12},\chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$

Up quarks **M_R** **Dirac Neutrinos** **Charged leptons Down quarks**

$10\ (q_1, u^c, e^c)$ $\quad \bar 5\ (d^c, l_e)$

We take l=m=1, n=2.

Right-handed neutrinos are $SU(5)$ gauge singlets

S_4 invariant superpotential for leptons

$3_L \times 2_R \times 3_{flavon}$

$$w_l = -3y_1\left[\frac{e^c}{\sqrt{2}}(l_\mu\chi_9 - l_\tau\chi_{10}) + \frac{\mu^c}{\sqrt{6}}(-2l_e\chi_8 + l_\mu\chi_9 + l_\tau\chi_{10})\right]h_{45}\Theta^\ell/(\Lambda\bar{\Lambda}^\ell)$$
$$+ y_2\tau^c(l_e\chi_{11} + l_\mu\chi_{12} + l_\tau\chi_{13})h_d/\Lambda.$$

$3_L \times 1_R \times 3_{flavon}$

$2_R \times 2_R$ $\quad$ $1_R \times 1_R$

$$w_N = y_1^N(N_e^cN_e^c + N_\mu^cN_\mu^c)\Theta^{2m}/\bar{\Lambda}^{2m-1}$$
$$+ y_2^N\left[(N_e^cN_\mu^c + N_\mu^cN_e^c)\chi_3 + (N_e^cN_e^c - N_\mu^cN_\mu^c)\chi_4\right]\Theta^{2m-n}/\bar{\Lambda}^{2m-n} + MN_\tau^cN_\tau^c,$$

$2_R \times 2_R \times 2_{flavon}$

$3_L \times 2_R \times 3_{flavon}$

$$w_D = y_1^D\left[\frac{N_e^c}{\sqrt{6}}(2l_e\chi_5 - l_\mu\chi_6 - l_\tau\chi_7) + \frac{N_\mu^c}{\sqrt{2}}(l_\mu\chi_6 - l_\tau\chi_7)\right]h_u\Theta^m/(\Lambda\bar{\Lambda}^m)$$
$$+ y_2^DN_\tau^c(l_e\chi_5 + l_\mu\chi_6 + l_\tau\chi_7)h_u/\Lambda.$$

$3_L \times 1_R \times 3_{flavon}$

We take VEV's

$$\langle h_u \rangle = v_u, \quad \langle h_d \rangle = v_d, \quad \langle h_{45} \rangle = v_{45}, \quad \langle \Theta \rangle = \theta,$$
$$\langle (\chi_3, \chi_4) \rangle = (u_3, u_4), \quad \langle (\chi_5, \chi_6, \chi_7) \rangle = (u_5, u_6, u_7),$$
$$\langle (\chi_8, \chi_9, \chi_{10}) \rangle = (u_8, u_9, u_{10}), \quad \langle (\chi_{11}, \chi_{12}, \chi_{13}) \rangle = (u_{11}, u_{12}, u_{13}),$$

$$\alpha_i \equiv u_i/\Lambda \text{ and } \lambda \equiv \theta/\bar{\Lambda}$$

We get Lepton Mass Matrices

$$M_l = -3y_1\lambda^\ell v_{45} \begin{pmatrix} 0 & \alpha_9/\sqrt{2} & -\alpha_{10}/\sqrt{2} \\ -2\alpha_8/\sqrt{6} & \alpha_9/\sqrt{6} & \alpha_{10}/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2 v_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{11} & \alpha_{12} & \alpha_{13} \end{pmatrix}$$

$$M_N = \begin{pmatrix} \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda) & y_2^N \lambda^{2m-n} \alpha_3 \Lambda & 0 \\ y_2^N \lambda^{2m-n} \alpha_3 \Lambda & \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda) & 0 \\ 0 & 0 & M \end{pmatrix}$$

Due to $m-n<0$

$$M_D = y_1^D \lambda^m v_u \begin{pmatrix} 2\alpha_5/\sqrt{6} & -\alpha_6/\sqrt{6} & -\alpha_7/\sqrt{6} \\ 0 & \alpha_6/\sqrt{2} & -\alpha_7/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & \alpha_6 & \alpha_7 \end{pmatrix}$$

Define VEVs : $\langle \chi_i \rangle \equiv u_i$ and $\alpha_i \equiv u_i/\Lambda$

Vacuum alignment

take vacuum alignment $(u_8, u_9, u_{10}) = (0, u_9, 0)$ and $(u_{11}, u_{12}, u_{13}) = (0, 0, u_{13})$

$$M_l = \begin{pmatrix} 0 & -3y_1\lambda^\ell \alpha_9 v_{45}/\sqrt{2} & 0 \\ 0 & -3y_1\lambda^\ell \alpha_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2\alpha_{13}v_d \end{pmatrix}$$

$$M_l^\dagger M_l = v_d^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6|\bar{y}_1\lambda^\ell\alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2\alpha_{13}^2 \end{pmatrix}$$

$$m_e^2 = 0 \ , \quad m_\mu^2 = 6|\bar{y}_1\lambda^\ell\alpha_9|^2 v_d^2 \ , \quad m_\tau^2 = |y_2|^2\alpha_{13}^2 v_d^2$$

No mixing in the left-hand !

$\theta_{12}=60^\circ$ in the right-hand !

Taking vacuum alignment $(u_3, u_4) = (0, u_4)$ and $(u_5, u_6, u_7) = (u_5, u_5, u_5)$

$$M_N = \begin{pmatrix} \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda) & 0 & 0 \\ 0 & \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda) & 0 \\ 0 & 0 & M \end{pmatrix}$$

$$M_D = y_1^D \lambda^m v_u \begin{pmatrix} 2\alpha_5/\sqrt{6} & -\alpha_5/\sqrt{6} & -\alpha_5/\sqrt{6} \\ 0 & \alpha_5/\sqrt{2} & -\alpha_5/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & \alpha_5 & \alpha_5 \end{pmatrix}$$

After seesaw, we get the tri-bimaximal mixing

$$M_\nu = \frac{b+c}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{3a-b}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{b-c}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$a = \frac{(y_2^D \alpha_5 v_u)^2}{M}, \qquad b = \frac{(y_1^D \alpha_5 v_u \lambda^m)^2}{\lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda)}, \qquad c = \frac{(y_1^D \alpha_5 v_u \lambda^m)^2}{\lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda)}.$$

$$m_1 = b\ , \qquad m_2 = 3a\ , \qquad m_3 = c\ .$$

Flavor Symmetry predicts θ 13

Higher dimensional mass operators, which predict Deviation from the Tri-bimaximal mixing

Superpotential of next-to-leading order

$$
\begin{aligned}
\Delta w_l = {} & y_{\Delta_a}(T_1,T_2)\otimes(F_1,F_2,F_3)\otimes(\chi_1,\chi_2)\otimes(\chi_{11},\chi_{12},\chi_{13})\otimes H_{\bar 5}/\Lambda^2 \\
& + y_{\Delta_b}(T_1,T_2)\otimes(F_1,F_2,F_3)\otimes(\chi_5,\chi_6,\chi_7)\otimes\chi_{14}\otimes H_{\bar 5}/\Lambda^2 \\
& + y_{\Delta_c}(T_1,T_2)\otimes(F_1,F_2,F_3)\otimes(\chi_1,\chi_2)\otimes(\chi_5,\chi_6,\chi_7)\otimes H_{45}/\Lambda^2 \\
& + y_{\Delta_d}(T_1,T_2)\otimes(F_1,F_2,F_3)\otimes(\chi_{11},\chi_{12},\chi_{13})\otimes\chi_{14}\otimes H_{45}/\Lambda^2 \\
& + y_{\Delta_e}T_3\otimes(F_1,F_2,F_3)\otimes(\chi_5,\chi_6,\chi_7)\otimes(\chi_8,\chi_9,\chi_{10})\otimes H_{\bar 5}\otimes/\Lambda^2 \\
& + y_{\Delta_f}T_3\otimes(F_1,F_2,F_3)\otimes(\chi_8,\chi_9,\chi_{10})\otimes(\chi_{11},\chi_{12},\chi_{13})\otimes H_{45}\otimes/\Lambda^2
\end{aligned}
$$

$$
\begin{aligned}
\Delta w_N = {} & y^N_{\Delta_1}(N^c_e,N^c_\mu)\otimes(N^c_e,N^c_\mu)\otimes(\chi_1,\chi_2)\otimes\chi_{14}/\Lambda \\
& + y^N_{\Delta_2}(N^c_e,N^c_\mu)\otimes N^c_\tau\otimes(\chi_5,\chi_6,\chi_7)\otimes(\chi_{11},\chi_{12},\chi_{13})\otimes\Theta/(\Lambda\bar\Lambda) \\
& + y^N_{\Delta_3}(N^c_e,N^c_\mu)\otimes N^c_\tau\otimes(\chi_8,\chi_9,\chi_{10})\otimes(\chi_8,\chi_9,\chi_{10})\otimes\Theta/(\Lambda\bar\Lambda) \\
& + y^N_{\Delta_4}N^c_\tau\otimes N^c_\tau\otimes(\chi_8,\chi_9,\chi_{10})\otimes(\chi_8,\chi_9,\chi_{10})/\Lambda.
\end{aligned}
$$

$$
\Delta w_D = y^D_\Delta(N^c_e,N^c_\mu)\otimes(F_1,F_2,F_3)\otimes(\chi_8,\chi_9,\chi_{10})\otimes(\chi_{11},\chi_{12},\chi_{13})\otimes H_5\otimes\Theta/(\Lambda^2\bar\Lambda)
$$

$$M_l^\dagger M_l \simeq \begin{pmatrix} |\epsilon_{11}|^2+|\epsilon_{21}|^2+|\epsilon_{31}|^2 & \frac{1}{2}(\sqrt{3}\epsilon_{11}^*+\epsilon_{21}^*)m_\mu & \epsilon_{31}^* m_\tau \\ \frac{1}{2}(\sqrt{3}\epsilon_{11}+\epsilon_{21})m_\mu & m_\mu^2 & \frac{1}{2}(\sqrt{3}\epsilon_{13}+\epsilon_{23})m_\mu \\ \epsilon_{31}m_\tau & \frac{1}{2}(\sqrt{3}\epsilon_{13}^*+\epsilon_{23}^*)m_\mu & m_\tau^2 \end{pmatrix}$$

$$\epsilon_{ij} = \mathcal{O}(\alpha_i\alpha_j v_d) = \mathcal{O}(m_e)$$

$$\Delta M_N = \Lambda \times$$

$$\begin{pmatrix} y^N_{\Delta_1}\alpha_1\alpha_{14} & y^N_{\Delta_1}\alpha_1\alpha_{14} & -\frac{\lambda}{\sqrt{6}}y^N_{\Delta_2}\alpha_5\alpha_{13}+\frac{\lambda}{\sqrt{2}}y^N_{\Delta_3}\lambda\alpha_9^2 \\ y^N_{\Delta_1}\alpha_1\alpha_{14} & -y^N_{\Delta_1}\alpha_1\alpha_{14} & -\frac{\lambda}{\sqrt{2}}y^N_{\Delta_2}\alpha_5\alpha_{13}+\frac{\lambda}{\sqrt{6}}y^N_{\Delta_3}\alpha_9^2 \\ -\frac{\lambda}{\sqrt{6}}y^N_{\Delta_2}\alpha_5\alpha_{13}+\frac{\lambda}{\sqrt{2}}y^N_{\Delta_3}\alpha_9^2 & -\frac{\lambda}{\sqrt{2}}y^N_{\Delta_2}\alpha_5\alpha_{13}+\frac{\lambda}{\sqrt{6}}y^N_{\Delta_3}\alpha_9^2 & y^N_{\Delta_4}\alpha_9^2 \end{pmatrix}$$

$$\Delta M_D = \begin{pmatrix} * & * & * \\ y^D_\Delta \lambda\alpha_9\alpha_{13}v_u & * & * \\ * & * & * \end{pmatrix}$$

we have non-zero U_{e3} of order $\alpha_i = \langle \chi_i \rangle / \Lambda$

Determination of magnitudes α_i

Desired Vacuum Alignments **FN charges l=m=1, n=2**

$$(\chi_1, \chi_2) = (1, 1), \quad (\chi_3, \chi_4) = (0, 1),$$
$$(\chi_5, \chi_6, \chi_7) = (1, 1, 1), \quad (\chi_8, \chi_9, \chi_{10}) = (0, 1, 0), \quad (\chi_{11}, \chi_{12}, \chi_{13}) = (0, 0, 1),$$

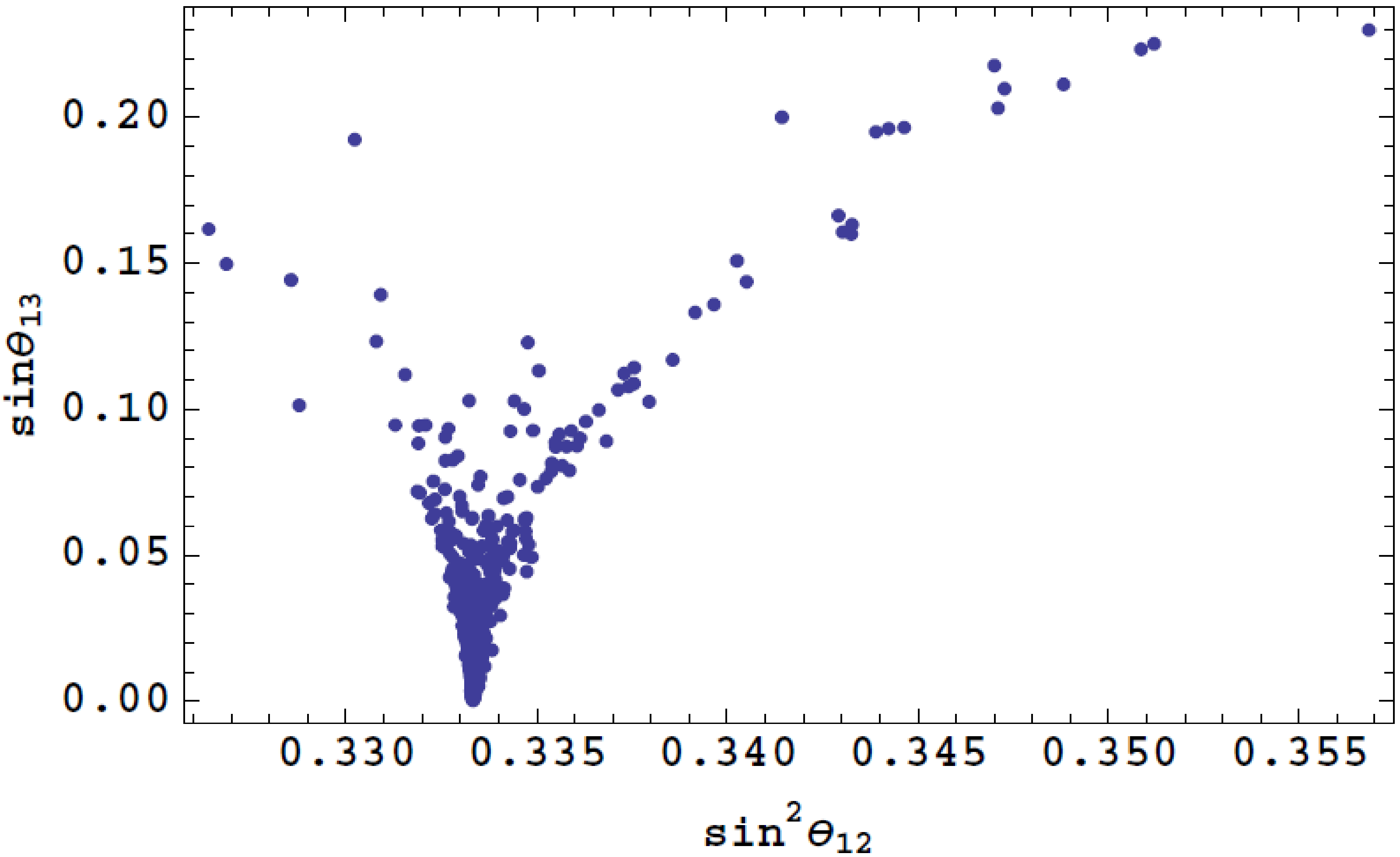
$$\alpha_3 = \alpha_8 = \alpha_{10} = \alpha_{11} = \alpha_{12} = 0,$$
$$\alpha_1 = \alpha_2 \simeq \sqrt{\left|\frac{y_2^u m_c}{2 y_1^{u2} v_u}\right|},$$
$$\alpha_4 = \frac{(y_1^D \lambda)^2 (m_3 - m_1) m_2 M}{6 y_2^N y_2^{D^2} m_1 m_3 \Lambda}, \qquad \alpha_5 = \alpha_6 = \alpha_7 = \frac{\sqrt{m_2 M}}{\sqrt{3} y_2^D v_u},$$
$$\alpha_9 = \frac{m_\mu}{\sqrt{6} |\bar{y_1}| \lambda v_d}, \qquad \alpha_{13} = \frac{m_\tau}{y_2 v_d}\ .$$

tanβ=3

Putting observed masses and M=10^{12} GeV, we get

$$\alpha_1 \sim 3.0 \times 10^{-2}, \qquad \alpha_4 \sim 10^{-2},$$
$$\alpha_5 \sim 10^{-2}, \qquad \alpha_9 \sim 5.1 \times 10^{-3}, \qquad \alpha_{13} \sim 2.1 \times 10^{-2}.$$

We can predict mixing angles.



The model predicts quark mixing angles

$S_4 \times Z_4$ with SUSY SU(5) GUT ⇒ Tri-bimaximal, Cabibbo angle

2 and **1** for SU(5) **10**, **3** for SU(5) $\bar{\mathbf{5}}$

Down quark sector is fixed
through the charged lepton sector.

Top quark mass is given without coupling of flavons.

Down Quarks

$$M_d = v_d \begin{pmatrix} 0 & 0 & 0 \\ \bar{y}_1 \lambda^\ell \alpha_9/\sqrt{2} & \bar{y}_1 \lambda^\ell \alpha_9/\sqrt{6} & 0 \\ 0 & 0 & y_2 \alpha_{13} \end{pmatrix}$$

$$\bar{y}_1 v_d = y_1 v_{45}$$

$$M_d^\dagger M_d = v_d^2 \begin{pmatrix} \frac{1}{2}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & \frac{1}{2\sqrt{3}}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ \frac{1}{2\sqrt{3}}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & \frac{1}{6}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2 \alpha_{13}^2 \end{pmatrix}$$

Left-handed mixing is given as

$$U_d^{(0)} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Including next-to-leading order, we get down quark mass matrix

$$M_d^\dagger M_d \simeq$$

$$\begin{pmatrix} |\frac{\sqrt{3}m_s}{2}+\bar{\epsilon}_{12}|^2+|\bar{\epsilon}_{11}|^2+|\bar{\epsilon}_{13}|^2 & (\frac{\sqrt{3}}{2}m_s+\bar{\epsilon}_{12}^*)(\frac{1}{2}m_s+\bar{\epsilon}_{22})+\bar{\epsilon}_{11}^*\bar{\epsilon}_{21}+\bar{\epsilon}_{13}^*\bar{\epsilon}_{23} & \bar{\epsilon}_{13}^* m_b \\ (\frac{\sqrt{3}}{2}m_s+\bar{\epsilon}_{12})(\frac{1}{2}m_s+\bar{\epsilon}_{22}^*)+\bar{\epsilon}_{11}\bar{\epsilon}_{21}^*+\bar{\epsilon}_{13}\bar{\epsilon}_{23}^* & |\frac{m_s}{2}+\bar{\epsilon}_{22}|^2+|\bar{\epsilon}_{21}|^2+|\bar{\epsilon}_{23}|^2 & \bar{\epsilon}_{23}^* m_b \\ \bar{\epsilon}_{13} m_b & \bar{\epsilon}_{23} m_b & m_b^2 \end{pmatrix}$$

$$U_d^{(0)\dagger} M_d^\dagger M_d U_d^{(0)} \simeq \begin{pmatrix} m_d^2 & \mathcal{O}(m_d m_s) & \frac{1}{2}(\bar{\epsilon}_{13}^* - \sqrt{3}\bar{\epsilon}_{23}^*)m_b \\ \mathcal{O}(m_d m_s) & m_s^2 & \frac{1}{2}(\sqrt{3}\bar{\epsilon}_{13}^* + \bar{\epsilon}_{23}^*)m_b \\ \frac{1}{2}(\bar{\epsilon}_{13} - \sqrt{3}\bar{\epsilon}_{23})m_b & \frac{1}{2}(\sqrt{3}\bar{\epsilon}_{13} + \bar{\epsilon}_{23})m_b & m_b^2 \end{pmatrix}$$

$$U_d^{(0)} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{12}^d = \mathcal{O}\left(\frac{m_d}{m_s}\right) = \mathcal{O}(0.05)\,, \quad \theta_{13}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}(0.005)\,, \quad \theta_{23}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}(0.005)$$

Up Quark Sector

$$w_u = y_1^u \left[(u^c \chi_1 + c^c \chi_2) q_3 + t^c (q_1 \chi_1 + q_2 \chi_2) \right] h_u / \Lambda + y_2^u t^c q_3 h_u$$

$$\langle (\chi_1, \chi_2) \rangle = (u_1, u_2)$$

Direct Yukawa coupling

$$M_u = v_u \begin{pmatrix} 0 & 0 & y_1^u \alpha_1 \\ 0 & 0 & y_1^u \alpha_2 \\ y_1^u \alpha_1 & y_1^u \alpha_2 & y_2^u \end{pmatrix}$$

We add the next-to-leading mass matrix

$$\Delta M_u = v_u \times \begin{pmatrix} y^u_{\Delta_{a1}}(\alpha_1^2+\alpha_2^2) + y^u_{\Delta_{a2}}(\alpha_1^2-\alpha_2^2) + y^u_{\Delta_b}\alpha_{14}^2 & y^u_{\Delta_{a2}}\alpha_1\alpha_2 & 0 \\ y^u_{\Delta_{a2}}\alpha_1\alpha_2 & y^u_{\Delta_{a1}}(\alpha_1^2+\alpha_2^2) - y^u_{\Delta_{a2}}(\alpha_1^2-\alpha_2^2) + y^u_{\Delta_b}\alpha_{14}^2 & 0 \\ 0 & 0 & y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}$$

Up Quarks

We take alignment $\alpha_1 = \alpha_2$ **, we get**

$$M_u = v_u \begin{pmatrix} 2y^u_{\Delta_{a1}}\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & y^u_{\Delta_{a2}}\alpha_1^2 & y^u_1\alpha_1 \\ y^u_{\Delta_{a2}}\alpha_1^2 & 2y^u_{\Delta_{a1}}\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & y^u_1\alpha_1 \\ y^u_1\alpha_1 & y^u_1\alpha_1 & y^u_2 + y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}$$

After rotating it by the orthogonal matrix,

$$U_u^{(0)} = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We obtain

$$\hat{M}_u = U_u^\dagger M_u U_u = v_u \begin{pmatrix} (2y^u_{\Delta_{a1}} - y^u_{\Delta_{a2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & 0 & 0 \\ 0 & (2y^u_{\Delta_{a1}} + y^u_{\Delta_{a2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & \sqrt{2}y^u_1\alpha_1 \\ 0 & \sqrt{2}y^u_1\alpha_1 & y^u_2 + y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}$$

We obtain CKM matrix elements

$$V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix}, \qquad r_c = \sqrt{\frac{m_c}{m_c + m_t}}\,, \qquad r_t = \sqrt{\frac{m_t}{m_c + m_t}}\,,$$

$$U_u \simeq U_u^{(0)} P V_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix},$$

$$U_d \simeq \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d\theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d\theta_{23}^d & -\theta_{23}^d - \theta_{12}^d\theta_{13}^d & 1 \end{pmatrix}.$$

In the leading order, we predict

$$V_{us} \simeq \sin 15^\circ \simeq 0.26$$
$$V_{cb} \simeq \sqrt{m_c/m_t} \simeq 0.048$$
$$V_{ub} \simeq 0$$

Including next-to-leading order corrections, we get

$$V^0_{us} \simeq \theta^d_{12}\cos 15^\circ + \sin 15^\circ,$$
$$V^0_{ub} \simeq \theta^d_{13}\cos 15^\circ + \theta^d_{23}\sin 15^\circ,$$

$$V^0_{cb} \simeq -r_t\theta^d_{13}e^{i\rho}\sin 15^\circ + r_t\theta^d_{23}e^{i\rho}\cos 15^\circ - r_c\ ,$$
$$V^0_{td} \simeq -r_c\sin 15^\circ e^{i\rho} - r_c(\theta^d_{12} + \theta^d_{13}\theta^d_{23})e^{i\rho}\cos 15^\circ + r_t(-\theta^d_{13} + \theta^d_{12}\theta^d_{23})$$

$$\theta^d_{12} = \mathcal{O}\left(\frac{m_d}{m_s}\right) = \mathcal{O}\,(0.05)\,,\quad \theta^d_{13} = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}\,(0.005)\,,\quad \theta^d_{23} = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}\,(0.005)$$

The parameter set

$$\rho = 123^\circ,\quad \theta^d_{12} = -0.0340,\quad \theta^d_{13} = 0.00626,\quad \theta^d_{23} = -0.00880$$

reproduces observed values very well.

Values of parameters are consistent with our mass matrices.

CP violation can be discussed !

As seen in these examples,
in order to reproduce the tri-bi maximal mixing, we need

Non-Abelian Discrete Symmetry
(A_4, S_4, $\Delta(27)$, $\Delta(54)$...)

and

Symmetry Breaking
(Vacuum Alignment of flavons).

Spontaneous Symmetry Breaking ? (Scalar potential)
Or Explicit Breaking
through Boundary condition in extra-dim.

3 Breaking of Flavor Symmetry

(1) Spontaneous Symmetry Breaking of Flavons

Realization of Vacuum Alignment for S_4 model

Introduce driving fields with R charge 2

We can generate the vacuum alignment through F-terms by coupling flavons fields, which carry the R charge $+2$ under $U(1)_R$ symmetry.

	(χ_1, χ_2)	(χ_3, χ_4)	(χ_5, χ_6, χ_7)	$(\chi_8, \chi_9, \chi_{10})$	$(\chi_{11}, \chi_{12}, \chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$
$U(1)_R$	0	0	0	0	0	0

	$(\chi_{15}, \chi_{16}, \chi_{17})$	χ_1^0	χ_2^0	χ_3^0	(χ_4^0, χ_5^0)
$SU(5)$	1	1	1	1	1
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
Z_4	-1	-1	i	-1	$-i$
$U(1)_{FN}$	$-z$	$2\ell + n$	0	2ℓ	z
$U(1)_R$	0	2	2	2	2

$$
\begin{aligned}
w' = \kappa_1 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes (\chi_3, \chi_4) \otimes \chi_1^0/\Lambda \\
+ \eta_1 (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_2^0 \\
+ \eta_2 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes \chi_3^0 + \eta_3 \chi_{14} \otimes \chi_{14} \otimes \chi_3^0 \\
+ \eta_4 (\chi_5, \chi_6, \chi_7) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0) \, ,
\end{aligned}
$$

Scalar potential

$$
\begin{aligned}
V = \left| \frac{\kappa_1}{\Lambda} \left[2\chi_1\chi_2\chi_3 + (\chi_1^2 - \chi_2^2)\, \chi_4 \right] \right|^2 + \left| \eta_1 (\chi_8\chi_{11} + \chi_9\chi_{12} + \chi_{10}\chi_{13}) \right|^2 \\
+ \left| \eta_2 (\chi_1^2 + \chi_2^2) + \eta_3 \chi_{14}^2 \right|^2 + \left| \frac{1}{\sqrt{2}} \eta_4 (\chi_6\chi_{16} - \chi_7\chi_{17}) \right|^2 \\
+ \left| \frac{1}{\sqrt{6}} \eta_4 (-2\chi_5\chi_{15} + \chi_6\chi_{16} + \chi_7\chi_{17}) \right|^2 \; .
\end{aligned}
$$

We obtain Desired Vacuum Alignment

$$
\begin{aligned}
\chi_1 = \chi_2, \quad \chi_3 = 0, \quad \chi_5 = \chi_6 = \chi_7, \quad \chi_8 = \chi_{10} = \chi_{11} = \chi_{12} = 0, \\
\chi_{14}^2 = -\frac{2\eta_2}{\eta_3} \chi_1^2, \quad \chi_{15} = \chi_{16} = \chi_{17} \; .
\end{aligned}
$$

3 Breaking of Flavor Symmetry

(2) Symmetry Breaking at boundary conditions

without flavons !

H. Ishimori, Y. Shimizu, M. Tanimoto and A.Watanabe,
Neutrino masses and mixing from S4 flavor twisting,'
arXiv:1010.3805 [hep-ph].

Flavor Twisting

N. Haba, A.Watanabe, K.Yoshioka, Phys. Rev. Lett.97, 041601 (2006)

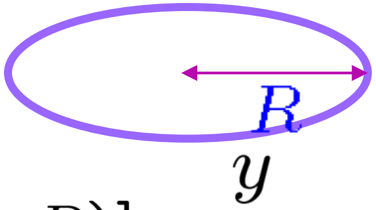
Compactification of 5th-Dim (S^1/Z_2)

Scherk and Schwarz,'79

Compactification

Scherk-Schwarz compactification

translation: $y \to y + 2\pi R$



identification of points: $\mathcal{L}[\Psi(y)] = \mathcal{L}[\Psi(y + 2\pi R)]$

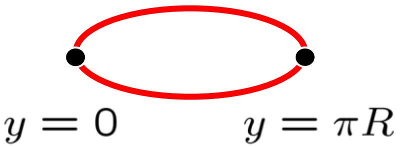
$$\Psi(y + 2\pi R) = T\,\Psi(y)$$

a represetation matrix of symmetry group

[Scherk and Schwarz,'79]

Orbifolding

reflection: $y \to -y$



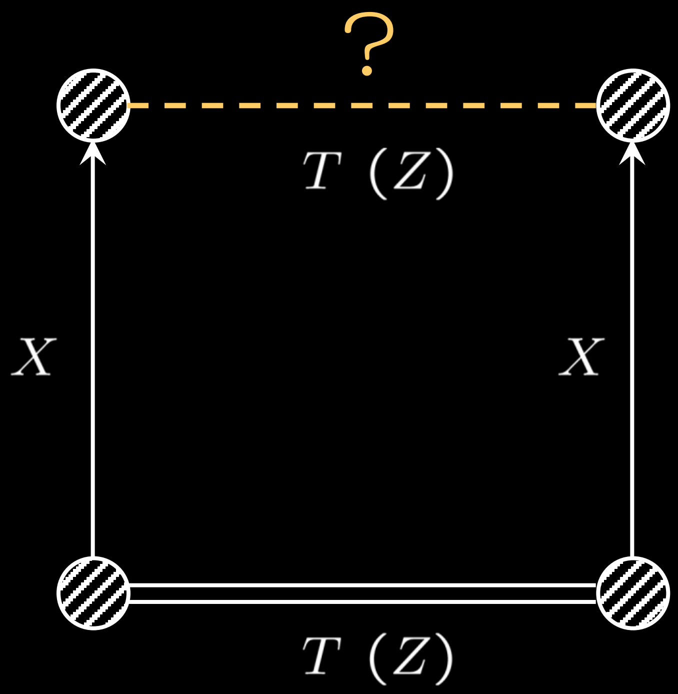
$$\Psi(-y) = Z \otimes \gamma_5 \Psi(y)$$

Boundary conditions are

$$\Psi(y + 2\pi R) = T\Psi(y)$$

$$\Psi(-y) = Z \otimes \gamma_5 \Psi(y)$$

Symmetry Breaking



$$[T\,(Z),\ X] \neq 0$$

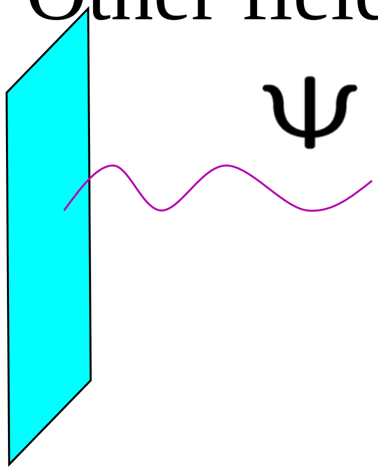
Symmetry breaking

X is a representaion matrix of symmetry group.

a slide by K. Yoshioka

Neutrino flavor twisting

- 5-dim model (for simplicity)
- 5-dim Dirac fermion (gauge singlet)
- Other fields are confined on 4-dim



$$\Psi(x, y) \sim \sum_n a^{(n)}(x) e^{i\frac{n}{R}y}$$

[K.Dienes, E.Dudas, T.Gherghetta,'99]

S_4 Flavor symmetry

24 elements

$$Q = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{llll}
a_1 = Q^4, & a_2 = Q^2, & a_3 = PQ^2P^2, & a_4 = Q^2PQ^2P^2, \\
b_1 = P, & b_2 = Q^2P, & b_3 = QPQP^2, & b_4 = Q^2PQ^2, \\
c_1 = P^2, & c_2 = Q^2P^2, & c_3 = QPQ, & c_4 = Q^3PQ, \\
d_1 = PQPQ^2, & d_2 = PQP, & d_3 = Q^3, & d_4 = Q, \\
e_1 = Q^2PQ, & e_2 = PQ, & e_3 = Q^3P^2, & e_4 = QP^2, \\
f_1 = QPQ^2, & f_2 = PQP^2, & f_3 = P^2Q, & f_4 = QP.
\end{array}$$

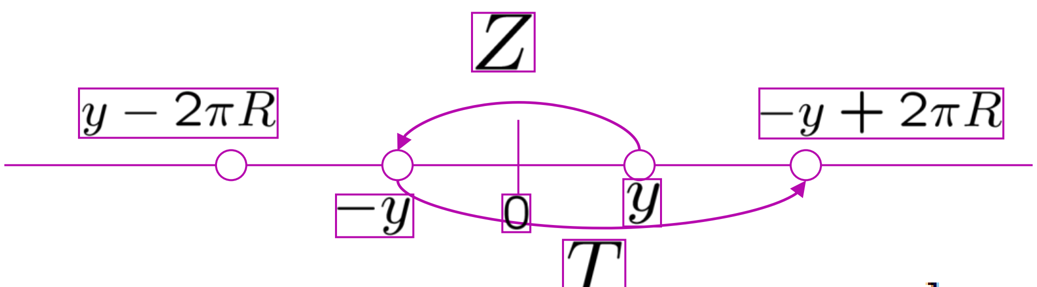
Consistency conditions

$$Z: \quad y \to -y$$

$$\Rightarrow \quad Z^2 = 1$$

$$T: \quad y \to y + 2\pi R \qquad\qquad Z$$

$$TZ = ZT^{-1}$$



another parity $Z' = TZ$

Possible boundary conditions

$$Z^2 = 1 \text{ and } Z'^2 = 1$$

Z'

Z	a_1	a_2	a_3	a_4	d_1	d_2	f_1	f_3	e_1	e_4
a_1										
a_2							$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
a_3					$\mathcal{C}_5$	$\mathcal{C}_6$			$\mathcal{C}_7$	$\mathcal{C}_8$
a_4					$\mathcal{C}_9$	$\mathcal{C}_{10}$	$\mathcal{C}_{11}$	$\mathcal{C}_{12}$		
d_1			$\mathcal{B}_1$	$\mathcal{B}_2$			$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
d_2			$\mathcal{B}_3$	$\mathcal{B}_4$			$\mathcal{A}_5$	$\mathcal{A}_6$	$\mathcal{A}_7$	$\mathcal{A}_8$
f_1		$\mathcal{B}_5$		$\mathcal{B}_6$	$\mathcal{A}_9$	$\mathcal{A}_{10}$			$\mathcal{A}_{11}$	$\mathcal{A}_{12}$
f_3		$\mathcal{B}_7$		$\mathcal{B}_8$	$\mathcal{A}_{13}$	$\mathcal{A}_{14}$			$\mathcal{A}_{15}$	$\mathcal{A}_{16}$
e_1		$\mathcal{B}_9$	$\mathcal{B}_{10}$		$\mathcal{A}_{17}$	$\mathcal{A}_{18}$	$\mathcal{A}_{19}$	$\mathcal{A}_{20}$		
e_4		$\mathcal{B}_{11}$	$\mathcal{B}_{12}$		$\mathcal{A}_{21}$	$\mathcal{A}_{22}$	$\mathcal{A}_{23}$	$\mathcal{A}_{24}$		

Table 1: The possible boundary conditions in terms of the two parities Z and Z'. The symbols $a_1, a_2, a_3, \cdots, e_4$ stand for the group elements (see Appendix B). The calligraphic characters represent the boundary conditions with which the theory becomes viable for neutrino phenomenology. Out of 100 general possibilities, 48 patterns are useful.

S_4 symmetry twisting

Taking $Z=f_1$, $Z'=d_1$

$$d_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad f_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

now we can write down the mode expansion.

Set Up

SM fields are at y=πR .

$$\mathcal{L} = i\overline{\Psi}_j \Gamma^M \partial_M \Psi_j - \frac{1}{2}(\overline{\Psi^c_i}(M_{ij})\Psi_j + \text{h.c.}) - \frac{1}{\sqrt{\Lambda}}\left(\overline{\Psi}_i(Y_{\nu_{ij}})L_j H + \overline{\Psi^c_i}(Y^c_{\nu_{ij}})L_j H + \text{h.c.}\right)\delta(y-\pi R)$$

Bulk fermions

$$\Psi_i(x,y) = \begin{pmatrix} \sum_{n=0}^{\infty} \chi^n_{R_{ij}}(y)\psi^n_{R_j}(x) \\ \sum_{n=0}^{\infty} \chi^n_{L_{ij}}(y)\psi^n_{L_j}(x) \end{pmatrix}$$

$$\int_0^{\pi R} dy \left[\chi^m_{R,L}{}^\dagger \chi^n_{R,L}\right]_{ij} = \delta_{mn}\delta_{ij}$$

We take Symmetry invariant mass parameters

S_4 triplet for Bulk fermion and L_i

$$M_{ij} = M\delta_{ij}, \quad m_{ij} = m\delta_{ij}, \quad m^c_{ij} = m^c\delta_{ij}.$$

$$m_{ij} = Y_{\nu_{ij}} v \text{ and } m^c_{ij} = Y^c_{\nu_{ij}} v$$

4-Dim Effective Lagrangian

$$\mathcal{L}_4 = iN^\dagger \sigma^\mu \partial_\mu N - \frac{1}{2}\left(N^{\rm T}\epsilon \otimes M_N N + \text{h.c.}\right)$$

$$M_N = \left(\begin{array}{c|c} 0 & M_D^{\rm T} \\ \hline M_D & M_H \end{array}\right) = \left(\begin{array}{c|cccccc} & M_0^{\rm T} & M_0^{c\rm T} & M_1^{\rm T} & M_1^{c\rm T} & \cdots \\ \hline M_0 & -M_{R_{00}}^* & M_{K_{00}} & & & \cdots \\ M_0^c & M_{K_{00}}^{\rm T} & M_{L_{00}} & & & \cdots \\ M_1 & & & -M_{R_{11}}^* & M_{K_{11}} & \cdots \\ M_1^c & & & M_{K_{11}}^{\rm T} & M_{L_{11}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right), \quad N = \begin{pmatrix} \nu_L \\ \epsilon\psi_R^{0\,*} \\ \psi_L^0 \\ \epsilon\psi_R^{1\,*} \\ \psi_L^1 \\ \vdots \end{pmatrix}$$

$$M_{K_{mn}} = \int_0^{\pi R} dy\; \chi_R^{m\,\dagger}(-\partial_y)\chi_L^n,$$

$$M_{R_{mn}} = \int_0^{\pi R} dy\; \chi_R^{m\,\rm T} M \chi_R^n, \quad M_{L_{mn}} = \int_0^{\pi R} dy\; \chi_L^{m\,\rm T} M \chi_L^n,$$

$$M_n = \frac{1}{\sqrt{\Lambda}}\chi_R^{n\,\dagger}(\pi R)m, \quad M_n^c = \frac{1}{\sqrt{\Lambda}}\chi_L^{n\,\rm T}(\pi R)m^c.$$

KK expansion satisfies the boundary conditioin

$$\chi_R^0(y)=\frac{1}{\sqrt{\pi R}}V\begin{pmatrix}\frac{1}{\sqrt{2}}e^{i\frac{y}{3R}} & 0 & 0\\ \frac{1}{\sqrt{2}}e^{-i\frac{y}{3R}} & 0 & 0\\ 0 & 0 & 1\end{pmatrix},\quad \chi_L^0(y)=\frac{1}{\sqrt{\pi R}}V\begin{pmatrix}\frac{1}{\sqrt{2}}e^{i\frac{y}{3R}} & 0 & 0\\ \frac{-1}{\sqrt{2}}e^{-i\frac{y}{3R}} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$\chi_R^n(y)=\sqrt{\frac{2}{\pi R}}V\begin{pmatrix}\frac{1}{2}e^{i\left(n+\frac{1}{3}\right)\frac{y}{R}} & \frac{1}{2}e^{-i\left(n-\frac{1}{3}\right)\frac{y}{R}} & 0\\ \frac{1}{2}e^{-i\left(n+\frac{1}{3}\right)\frac{y}{R}} & \frac{1}{2}e^{i\left(n-\frac{1}{3}\right)\frac{y}{R}} & 0\\ 0 & 0 & \cos\left(\frac{n}{R}y\right)\end{pmatrix}\qquad (n\geq 1),$$

$$\chi_L^n(y)=\sqrt{\frac{2}{\pi R}}V\begin{pmatrix}\frac{1}{2}e^{i\left(n+\frac{1}{3}\right)\frac{y}{R}} & \frac{-1}{2}e^{-i\left(n-\frac{1}{3}\right)\frac{y}{R}} & 0\\ \frac{-1}{2}e^{-i\left(n+\frac{1}{3}\right)\frac{y}{R}} & \frac{1}{2}e^{i\left(n-\frac{1}{3}\right)\frac{y}{R}} & 0\\ 0 & 0 & \sin\left(\frac{n}{R}y\right)\end{pmatrix}\qquad (n\geq 1),$$

where V is the unitary matrix

$$V=\frac{1}{\sqrt{3}}\begin{pmatrix}\omega & \omega^2 & 1\\ 1 & 1 & 1\\ \omega^2 & \omega & 1\end{pmatrix}$$

Taking $Z=f_1$, $Z'=d_1$ ***S_4 is broken !***

We can obtain desired neutrino mass matrix by explicit S_4 breaking without flavon.

$$M_N = \left(\begin{array}{c|c} 0 & M_D^{\mathrm{T}} \\ \hline M_D & M_H \end{array}\right) = \left(\begin{array}{c|cccccc} & M_0^{\mathrm{T}} & M_0^{c\mathrm{T}} & M_1^{\mathrm{T}} & M_1^{c\mathrm{T}} & \cdots \\ \hline M_0 & -M_{R_{00}}^{*} & M_{K_{00}} & & & \cdots \\ M_0^c & M_{K_{00}}^{\mathrm{T}} & M_{L_{00}} & & & \cdots \\ M_1 & & & -M_{R_{11}}^{*} & M_{K_{11}} & \cdots \\ M_1^c & & & M_{K_{11}}^{\mathrm{T}} & M_{L_{11}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right)$$

$$\begin{aligned} M_\nu \;=\; \frac{1}{\Lambda R}\Bigg[& \frac{s|M|R}{c+1/2}\frac{m^2}{M^*}\begin{pmatrix} \frac{4}{6} & \frac{-2}{6} & \frac{-2}{6} \\ \frac{-2}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{-2}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} + \frac{|M|R}{\tanh(\pi|M|R)}\frac{m^2}{M^*}\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \\ & -\frac{s|M|R}{c+1/2}\frac{(m^c)^2}{M}\begin{pmatrix} & & \\ & \frac{1}{2} & \frac{-1}{2} \\ & \frac{-1}{2} & \frac{1}{2} \end{pmatrix} - \frac{|M|R}{c+1/2}\frac{mm^c}{|M|}\begin{pmatrix} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{2} & \\ \frac{-1}{2} & & \frac{1}{2} \end{pmatrix}\Bigg] \end{aligned}$$

$$c \equiv \cosh(2\pi|M|R) \ \text{ and } \ s \equiv \sinh(2\pi|M|R)$$

$$M_\nu = \frac{-|M|}{\Lambda} V_{\text{tri-bi}} \begin{pmatrix} \frac{-2s}{2c+1}\frac{m^2}{M^*} & 0 & \frac{\sqrt{3}}{2c+1}\frac{mm^c}{|M|} \\ 0 & \frac{-1}{\tanh(\pi|M|R)}\frac{m^2}{M^*} & 0 \\ \frac{\sqrt{3}}{2c+1}\frac{mm^c}{|M|} & 0 & \frac{2s}{2c+1}\frac{(m^c)^2}{M} \end{pmatrix} V_{\text{tri-bi}}^{\mathrm{T}}$$

$$m_1 = \frac{|m|^2}{\Lambda}\frac{1}{2c+1}\left| s(1-r^2) + \sqrt{s^2(1+r^2)^2 + 3r^2} \right|,$$

$$m_2 = \frac{|m|^2}{\Lambda}\frac{1}{2c+1}\left[\frac{2c+1}{\tanh(\pi|M|R)} \right],$$

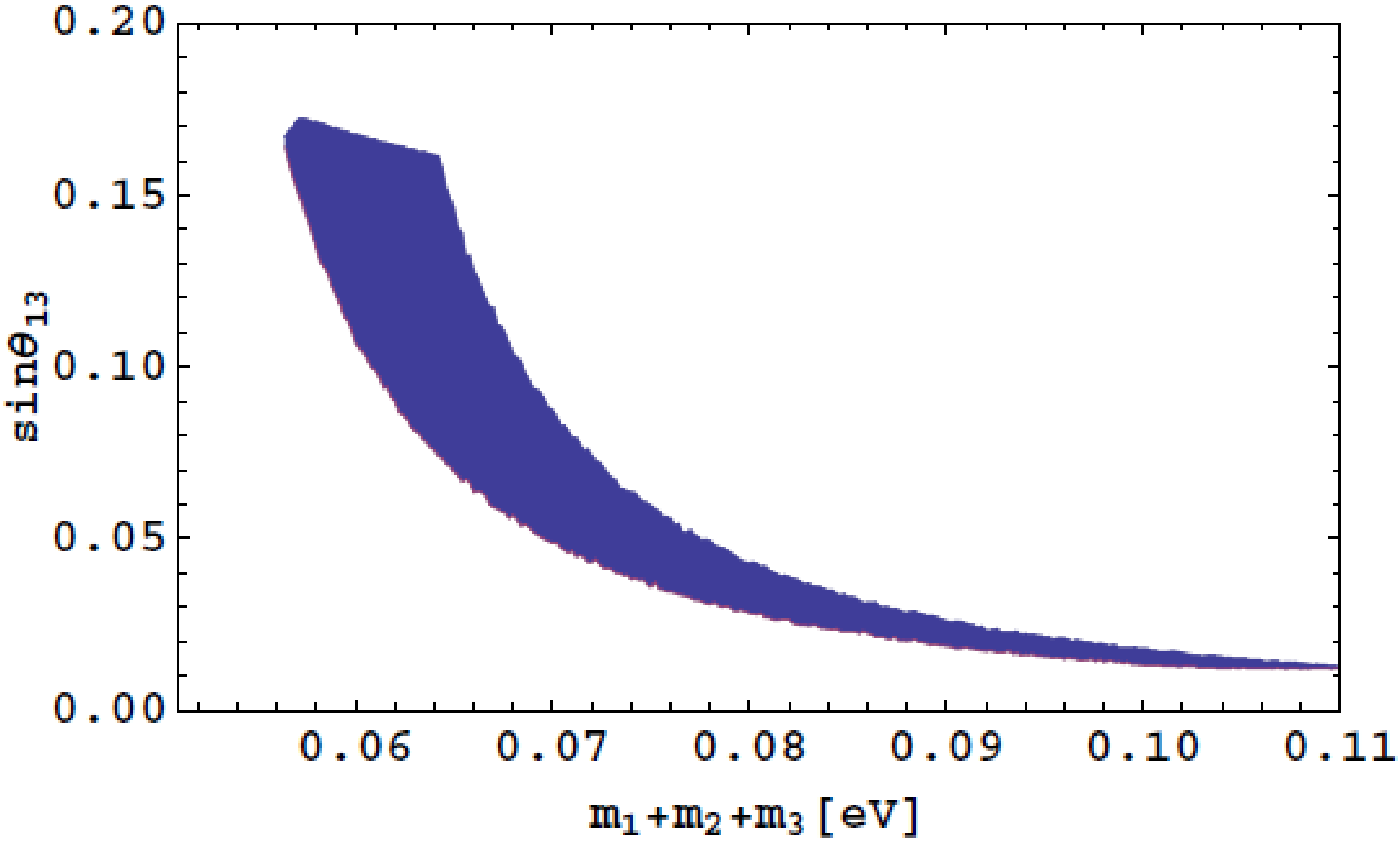
$$m_3 = \frac{|m|^2}{\Lambda}\frac{1}{2c+1}\left| s(1-r^2) - \sqrt{s^2(1+r^2)^2 + 3r^2} \right|,$$

$$U_{e2} = \frac{1}{\sqrt{3}}e^{i\rho}, \quad U_{e3} = \frac{2i}{\sqrt{6}}\sin\theta e^{i\rho}, \quad U_{\mu_3} = -i\left(\frac{1}{\sqrt{2}}\cos\theta e^{i\sigma} + \frac{1}{\sqrt{6}}\sin\theta e^{i\rho}\right)$$

If $m^c = 0$ (inverted), *the tri-bimaximal mixing is realized.*

If $m^c \gg m$ (normal), *large θ_{13} is predicted.*

normal mass hierarchy



How to get the diagonal charged-lepton mass matrix

	e_R	(μ_R,τ_R)	(L_e,L_μ,L_τ)	H	(ϕ_1,ϕ_2,ϕ_3)
S_4	$\underline{1}$	$\underline{2}$	$\underline{3}$	$\underline{1}$	$\underline{3}$

$$M_\ell = vY_s \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + vY_d \begin{pmatrix} 0 & 0 & 0 \\ \alpha_1 & \omega^2\alpha_2 & \omega\alpha_3 \\ \alpha_1 & \omega\alpha_2 & \omega^2\alpha_3 \end{pmatrix} ; \quad \alpha_i \equiv \langle\phi_i\rangle/\Lambda$$

$$M_\ell^\dagger M_\ell = v^2 \begin{pmatrix} (|Y_s|^2+2|Y_d|^2)\,\alpha_1^2 & (|Y_s|^2-|Y_d|^2)\,\alpha_1\alpha_2 & (|Y_s|^2-|Y_d|^2)\,\alpha_1\alpha_3 \\ (|Y_s|^2-|Y_d|^2)\,\alpha_1\alpha_2 & (|Y_s|^2+2|Y_d|^2)\,\alpha_2^2 & (|Y_s|^2-|Y_d|^2)\,\alpha_2\alpha_3 \\ (|Y_s|^2-|Y_d|^2)\,\alpha_1\alpha_3 & (|Y_s|^2-|Y_d|^2)\,\alpha_2\alpha_3 & (|Y_s|^2+2|Y_d|^2)\,\alpha_3^2 \end{pmatrix}$$

By assuming that $\alpha_1 v \sim m_e,\ \alpha_2 v \sim m_\mu,\ \alpha_3 v \sim m_\tau$

we obtain small mixing for left-handed direction.

4. Related Phenomena of Flavor Symmetry

Flavor symmetry constrains not only quark/lepton mass matrices, but also mass matrices of their superpartner, i.e. squark/slepton.

Specific patterns of squark/slepton mass matrices could be tested in future experiments.

Let us discuss lepton FCNC in our S_4 model with flavon.

Consider Soft SUSY Breaking Term in Supergravity.

we assume chiral superfields Φ_k to cause SUSY breaking

	(T_1, T_2)	T_3	(F_1, F_2, F_3)	(N_e^c, N_μ^c)	N_τ^c	H_5	$H_{\bar{5}}$	H_{45}	Θ
$SU(5)$	10	10	$\bar{5}$	1	1	5	$\bar{5}$	45	1
S_4	**2**	**1**	**3**	**2**	**1**$'$	**1**	**1**	**1**	**1**
Z_4	$-i$	-1	i	1	1	1	1	-1	1
$U(1)_{FN}$	ℓ	0	0	m	0	0	0	0	-1

	(χ_1, χ_2)	(χ_3, χ_4)	(χ_5, χ_6, χ_7)	$(\chi_8, \chi_9, \chi_{10})$	$(\chi_{11}, \chi_{12}, \chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	**2**	**2**	**3**$'$	**3**	**3**	**1**
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$

Second order Kähler potential of left-handed and right-handed leptons,

$$K = Z^{(L)}(\Phi) \sum_{i=e,\mu,\tau} |L_i|^2 + Z^{(R)}_{(1)}(\Phi) \sum_{i=e,\mu} |e_i|^2 + Z^{(R)}_{(2)}(\Phi)|e_\tau|^2$$

Slepton mass matrices are derived

$$(m^2_{\tilde{L}})_{ij} = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix}, \qquad (m^2_{\tilde{R}})_{ij} = \begin{pmatrix} m^2_{R(1)} & 0 & 0 \\ 0 & m^2_{R(1)} & 0 \\ 0 & 0 & m^2_{R(2)} \end{pmatrix}$$

3 **2 + 1**

For the left-handed sector, higher dimensional terms are given as

$$\Delta K_L = \sum_{i=1,3} Z^{(L)}_{\Delta_{a_i}}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi_i^c, \chi_{i+1}^c)/\Lambda^2$$
$$+ \sum_{i=5,8,11} Z^{(L)}_{\Delta_{b_i}}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2$$
$$+ Z^{(L)}_{\Delta_c}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes \chi_{14} \otimes \chi_{14}^c/\Lambda^2$$
$$+ Z^{(L)}_{\Delta_d}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.$$

Left-handed Slepton mass matrix is

$$(m^2_{\tilde{L}})_{ij} = \begin{pmatrix} m_L^2 + \tilde{\alpha}_{L1}^2 m_{3/2}^2 & k_L \alpha_5^2 m_{3/2}^2 & k_L \alpha_5^2 m_{3/2}^2 \\ k_L \alpha_5^2 m_{3/2}^2 & m_L^2 + \tilde{\alpha}_{L2}^2 m_{3/2}^2 & k_L \alpha_5^2 m_{3/2}^2 \\ k_L \alpha_5^2 m_{3/2}^2 & k_L \alpha_5^2 m_{3/2}^2 & m_L^2 + \tilde{\alpha}_{L3}^2 m_{3/2}^2 \end{pmatrix}$$

$\tilde{\alpha}$ is a linear combination of α_i's.

Right-handed Slepton mass matrix is

$$
\begin{aligned}
\Delta K_R = & \sum_{i=1,3} Z^{(R)}_{\Delta_{a_i}}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes(\chi_i, \chi_{i+1})\otimes(\chi_i^c, \chi_{i+1}^c)/\Lambda^2 \\
& + \sum_{i=5,8,11} Z^{(R)}_{\Delta_{b_i}}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes(\chi_i, \chi_{i+1}, \chi_{i+2})\otimes(\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2 \\
& + Z^{(R)}_{\Delta_c}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes\chi_{14}\otimes\chi_{14}^c/\Lambda^2 \\
& + Z^{(R)}_{\Delta_d}(\Phi)(R_e, R_\mu)\otimes R_\tau^c\otimes(\chi_1, \chi_2)/\Lambda^2 + Z^{(R)}_{\Delta_e}(\Phi)(R_e^c, R_\mu^c)\otimes R_\tau\otimes(\chi_1^c, \chi_2^c)/\Lambda^2 \\
& + \sum_{i=1,3} Z^{(R)}_{\Delta_{f_i}}(\Phi)R_\tau\otimes R_\tau^c\otimes(\chi_i, \chi_{i+1})\otimes(\chi_i^c, \chi_{i+1}^c)/\Lambda^2 \\
& + \sum_{i=5,8,11} Z^{(R)}_{\Delta_{g_i}}(\Phi)R_\tau\otimes R_\tau^c\otimes(\chi_i, \chi_{i+1}, \chi_{i+2})\otimes(\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2 \\
& + Z^{(R)}_{\Delta_h}(\Phi)R_\tau\otimes R_\tau^c\otimes\chi_{14}\otimes\chi_{14}^c/\Lambda^2 \\
& + Z^{(R)}_{\Delta_i}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes\Theta\otimes\Theta^c/\bar{\Lambda}^2 \\
& + Z^{(R)}_{\Delta_j}(\Phi)R_\tau\otimes R_\tau^c\otimes\Theta\otimes\Theta^c/\bar{\Lambda}^2.
\end{aligned}
$$

$$
(m^2_{\tilde{R}})_{ij} = \begin{pmatrix} m^2_{R(1)} + \tilde{\alpha}^2_{R11} m^2_{3/2} & \tilde{\alpha}^2_{R12} m^2_{3/2} & k_R \alpha_1 m^2_{3/2} \\ \tilde{\alpha}^2_{R12} m^2_{3/2} & m^2_{R(1)} + \tilde{\alpha}^2_{R22} m^2_{3/2} & k_R \alpha_1 m^2_{3/2} \\ k_R \alpha_1 m^2_{3/2} & k_R \alpha_1 m^2_{3/2} & m^2_{R(2)} + \tilde{\alpha}^2_{R33} m^2_{3/2} \end{pmatrix}
$$

Move to Super-CKM basis (Diagonal Basis of Charged Lepton) in order to estimate magnitudes of FCNC.

Dominant term

$$(m^2_{L(R)})^{SCKM}_{12} \sim \theta^l_{L(R)12}(m^2_{L(R)11} - m^2_{L(R)22}) + m^2_{L(R)12}$$

where $\theta^\ell_{L12} \simeq \mathcal{O}(\frac{m_e}{m_\mu}), \quad \theta^\ell_{R12} \simeq \sin 60^\circ$

Mass Insertion Parameters

$$(\delta^l_{LL(RR)})_{ij} \equiv \frac{(m^2_{L(R)})^{SCKM}_{ij}}{m^2_{SUSY}}$$

Our prediction is $\quad (\delta^l_{LL(RR)})_{12} = \mathcal{O}(\tilde{\alpha}^2) = \mathcal{O}(10^{-4}).$

Experimental Constraint from μ→eγ

$$(\delta^l_{LL(RR)})_{12} \leq \mathcal{O}(10^{-3}) \text{ when } m_{SUSY} = \mathcal{O}(100\text{GeV})$$

F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B477(1996) 321

Numerical analyses are required.

A *terms are obtained as*

$$(m^2_{LR})^{SCKM}_{ij} = U^\dagger_E (m^2_{LR})_{ij} V_E \simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) \\ \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) \\ \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}(m_\tau) \end{pmatrix}$$

$$\simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) \\ \mathcal{O}\left(m_e\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(m_e\right) \\ \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) & \mathcal{O}(m_\tau) \end{pmatrix}$$

$$(\delta^l_{LR})_{12} = \mathcal{O}(\tfrac{m_e}{m_{SUSY}}) \simeq \mathcal{O}(5 \times 10^{-6}) \quad if \quad m_{SUSY} = 100\mathrm{GeV}$$

Dangerous !

Experimental Constraint

$$(\delta^l_{LR})_{12} \leq \mathcal{O}(10^{-6}) \text{ if } m_{SUSY} = \mathcal{O}(100\mathrm{GeV})$$

We need numerical analyses of $\mu \to e \gamma$.

μ⟶eγ Decay

$$\frac{\mathrm{BR}(\ell_i \to \ell_j \gamma)}{\mathrm{BR}(\ell_i \to \ell_j \nu_i \bar{\nu_j})} = \frac{48\pi^3 \alpha}{G_F^2}\left(|A_L^{ij}|^2 + |A_R^{ij}|^2\right)$$

$$\begin{aligned} A_L^{ij} \simeq \frac{\alpha_2}{4\pi}\frac{(\delta_\ell^{LL})_{ij}}{m_{\tilde{\ell}}^2} t_\beta \Bigg[& \frac{\mu M_2}{(M_2^2 - \mu^2)}\left(f_{2n}(x_2, x_\mu) + f_{2c}(x_2, x_\mu)\right) \\ & + \tan^2\theta_W \mu M_1 \left(\frac{f_{3n}(x_1)}{m_{\tilde{\ell}}^2} + \frac{f_{2n}(x_1, x_\mu)}{(\mu^2 - M_1^2)}\right)\Bigg] \\ & + \frac{\alpha_1}{4\pi}\frac{(\delta_\ell^{RL})_{ij}}{m_{\tilde{\ell}}^2}\left(\frac{M_1}{m_{\ell_i}}\right) 2 f_{2n}(x_1), \end{aligned}$$

$$A_R^{ij} \simeq \frac{\alpha_1}{4\pi}\left[\frac{(\delta_e^{RR})_{ij}}{m_{\tilde{\ell}}^2}\mu M_1 t_\beta \left(\frac{f_{3n}(x_1)}{m_{\tilde{\ell}}^2} - \frac{2 f_{2n}(x_1, x_\mu)}{(\mu^2 - M_1^2)}\right) + 2\frac{(\delta_e^{LR})_{ij}}{m_{\tilde{\ell}}^2}\left(\frac{M_1}{m_{\ell_i}}\right) f_{2n}(x_1)\right]$$

EDM of Electron

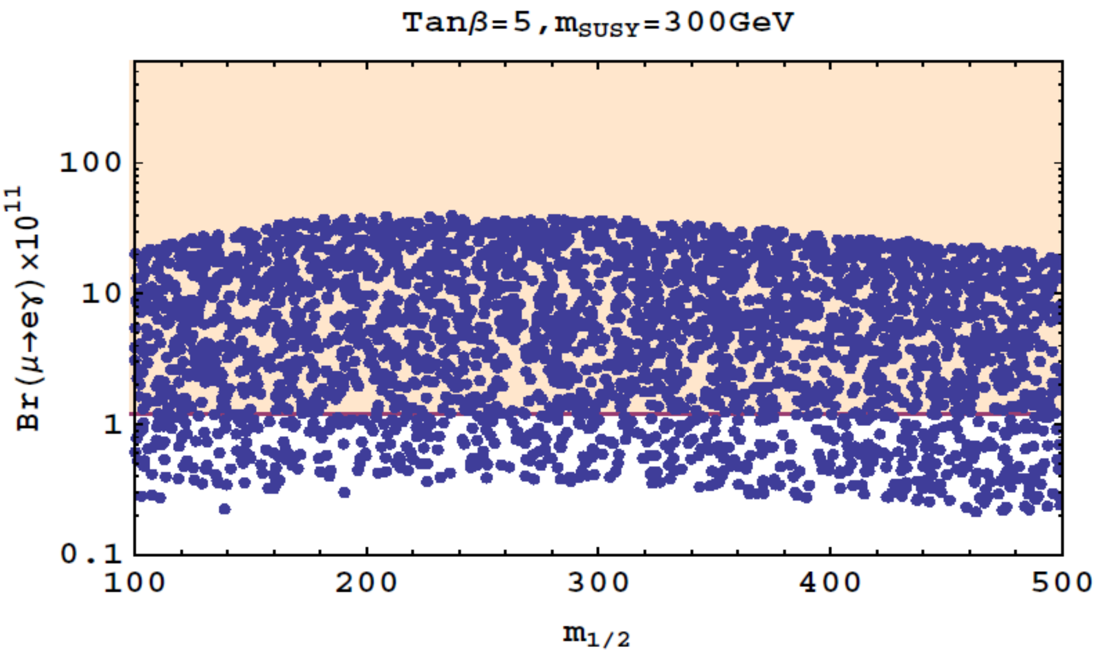
J.Hisano, M. Nagai, P. Paradisi, Phys.Rev.D80:095014,2009.

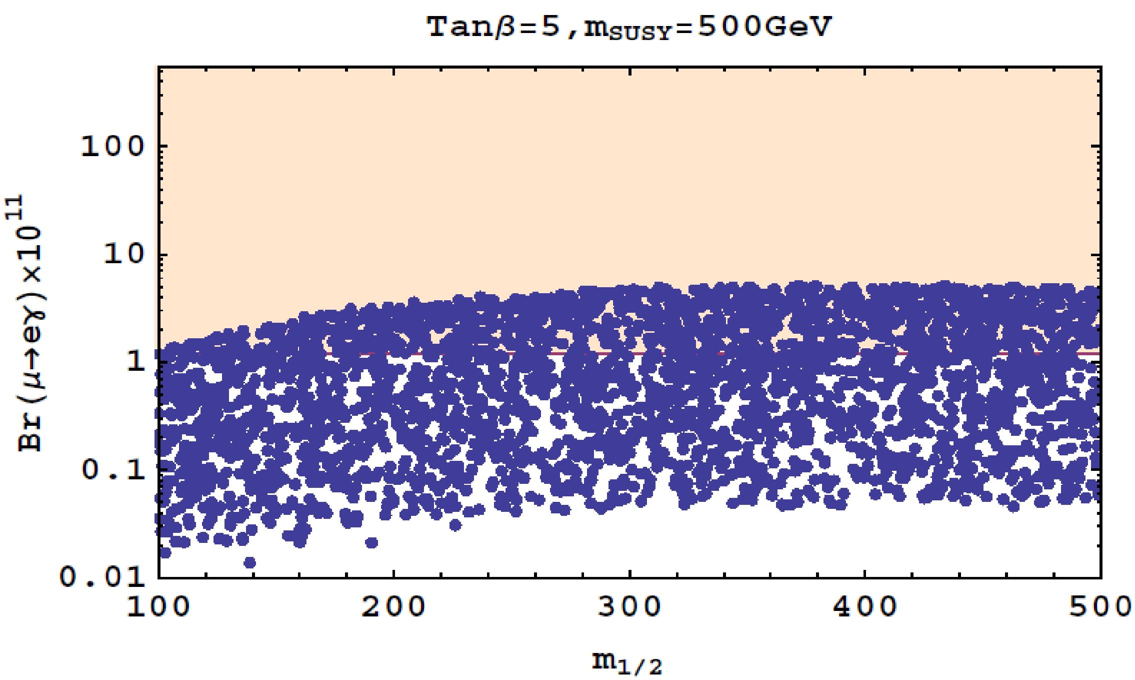
$$\frac{d_e}{e} = -\frac{\alpha_1}{4\pi}\frac{M_1}{m_{\tilde{\ell}}^2}\Big\{ \mathrm{Im}[(\delta_\ell^{LR})_{1k}(\delta_e^{RR})_{k1} + (\delta_\ell^{LL})_{1k}(\delta_\ell^{LR})_{k1}]\, f_{3n}(x_1) + \mathrm{Im}[(\delta_\ell^{LL})_{1k}(\delta_\ell^{LR})_{kl}(\delta_e^{RR})_{l1}$$

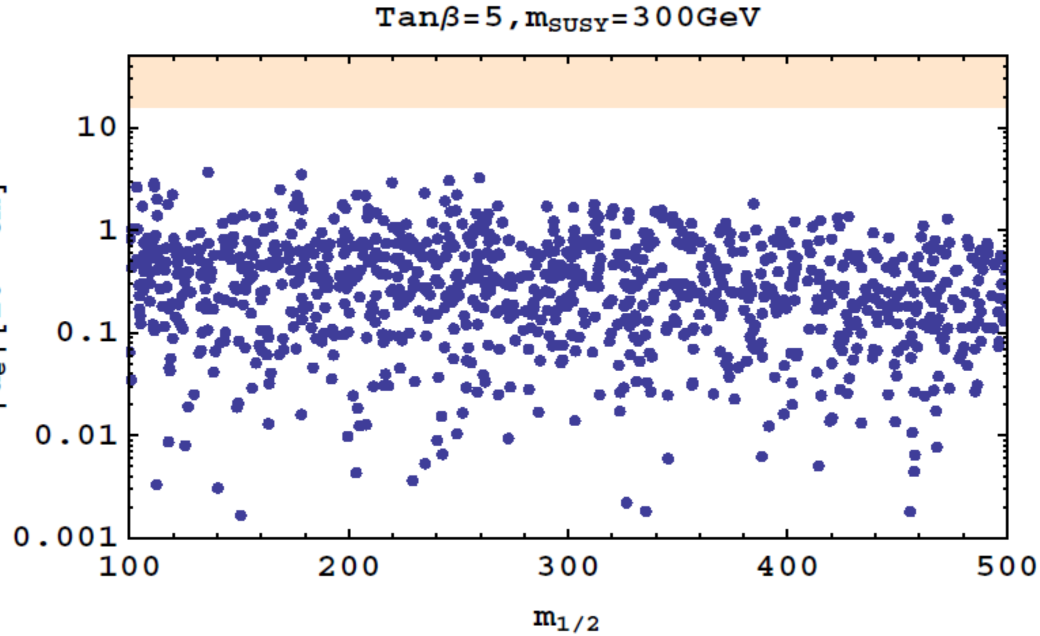
$$+ \quad (\delta_\ell^{LR})_{1k}(\delta_e^{RR})_{kl}(\delta_e^{RR})_{l1} + (\delta_\ell^{LL})_{1k}(\delta_\ell^{LL})_{kl}(\delta_\ell^{LR})_{l1}]\, f_{4n}(x_1)\Big\}$$

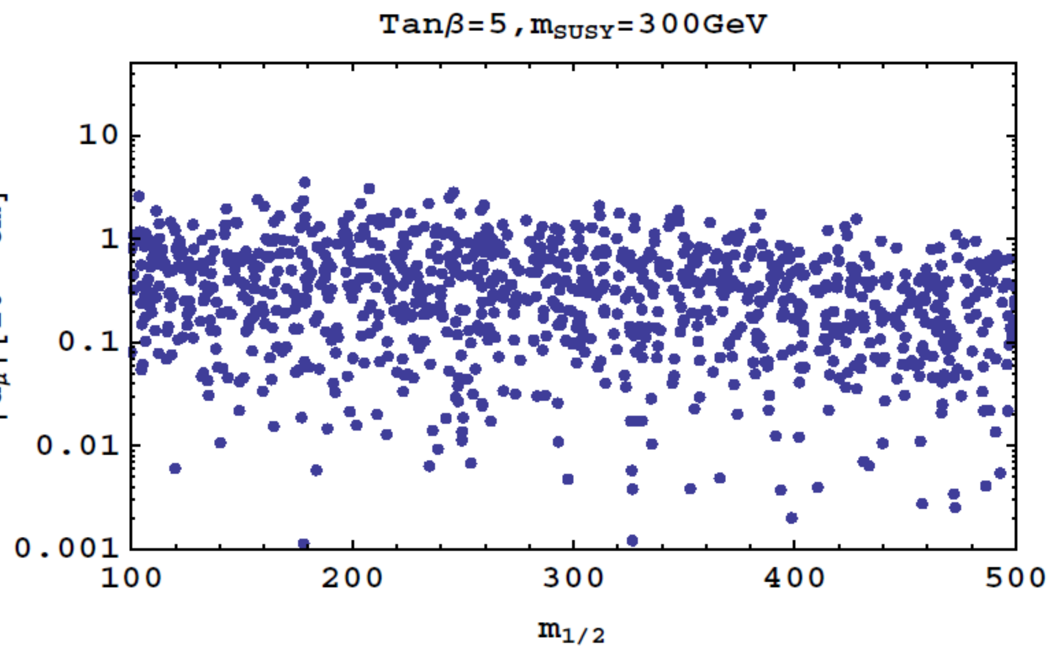
Taking Dominant Terms, we get

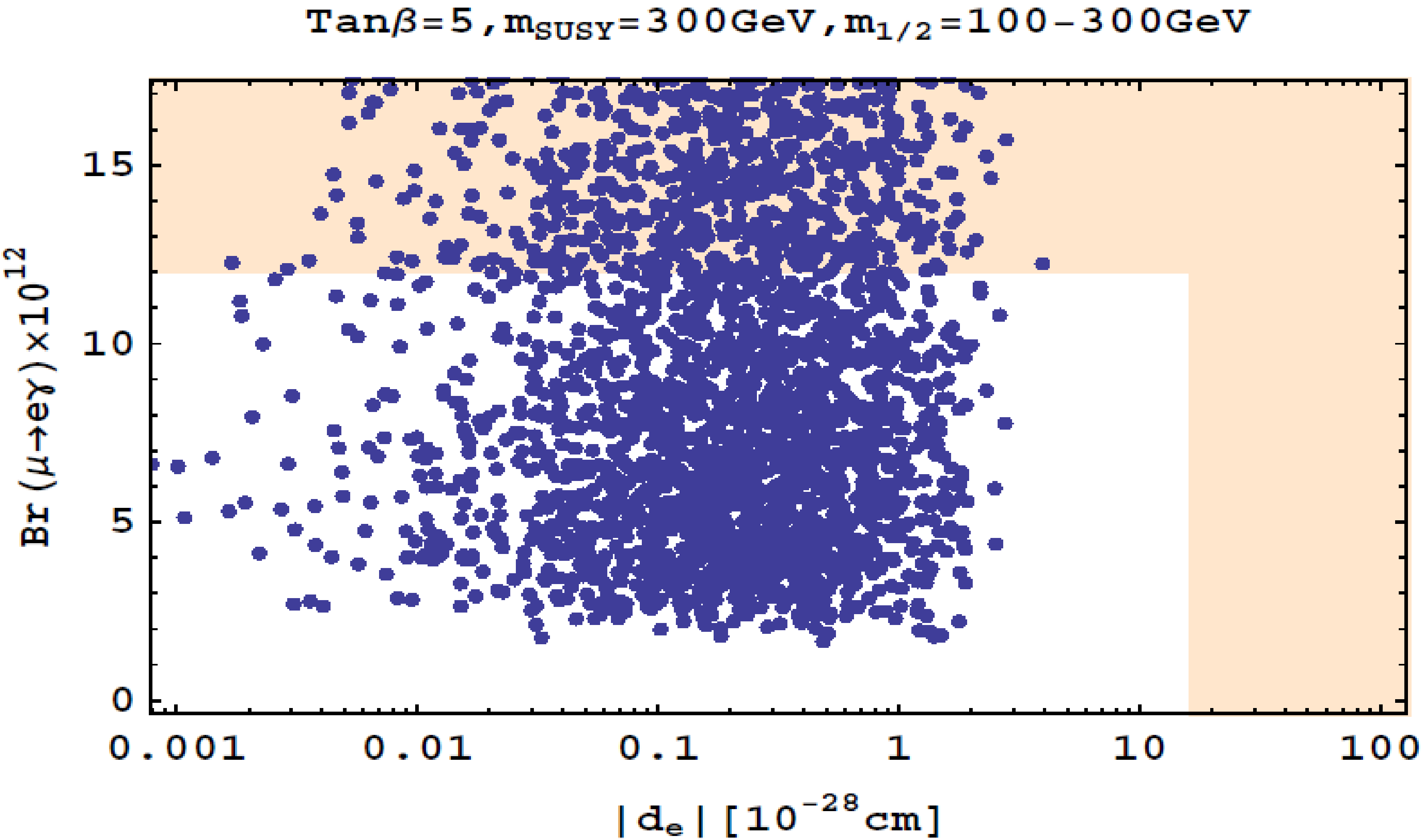
$$\frac{d_e}{e} \approx -\frac{\alpha_1}{4\pi}\frac{M_1}{m_{\tilde{\ell}}^2}\Big\{ \mathcal{O}(\frac{m_e}{m_{\tilde{\ell}}}\alpha_1)\, f_{3n}(x_1) + \mathcal{O}(\frac{m_\tau}{m_{\tilde{\ell}}}(1 + \frac{\mu\tan\beta}{m_{\tilde{\ell}}})\alpha_1\tilde{\alpha}^2)\, f_{4n}(x_1)\Big\}$$


Tanβ=5,m_SUSY=300GeV
Br(μ→eγ)×10^11
100
10
1
0.1
100
200
300
400
500
m_1/2


Tanβ=5,m_SUSY=500GeV
Br(μ→eγ)×10^11
100
10
1
0.1
0.01
100
200
300
400
500
m_1/2


Tanβ=5, m_SUSY=300GeV
|d_e| [10^-28 cm]
10
1
0.1
0.01
0.001
100
200
300
400
500
m_1/2


Tanβ=5, m_SUSY=300GeV
|d_μ| [10^-28 cm]
10
1
0.1
0.01
0.001
100
200
300
400
500
m_1/2



Tanβ=5, m_SUSY=300GeV, m_1/2=100-300GeV
Br (μ→eγ)×10^12
0
5
10
15
0.001
0.01
0.1
1
10
100
|d_e| [10^-28 cm]

5 Summary

☆ Non-Abelian Flavor Symmetry can give realistic lepton mixing matrices, but we need Symmetry Breaking

Tri-bimaximal mixing A_4, S_4 ....

☆ Symmetry Breaking requires new physics:

Vacuum alignments of flavons, Extra Dim. . . . .

☆ Non-Abelian Flavor Symmetry may also predict quark mixing angles.

☆ Non-Abelian Flavor Symmetry can be tested by related phenomena: FCNC, EDM ….

Problem in Flavor Symmetry

Can we predict Neutrino Masses?

$$\frac{\Delta m^2_{32}}{\Delta m^2_{21}} = 0.026 - 0.040 \sim \mathcal{O}(\lambda^2)$$

Normal mass hierarchy $m_3 \gg m_2 \geq m_1$

Inverted mass hierarchy $m_2 \geq m_1 \gg m_3$

T2K and NOνA !

Symmetry cannot predict mass spectrum.

Symmetry breaking gives mass spectrum.

More study of Symmetry Breaking !

S_4 invariant superpotential

$$
\begin{aligned}
w = & \; y_1^u (T_1, T_2) \otimes T_3 \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda + y_2^u T_3 \otimes T_3 \otimes H_5 \\
& + y_1^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes \Theta^{2m}/\bar{\Lambda}^{2m-1} \\
& + y_2^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_3, \chi_4) \otimes \Theta^{2m-n}/\bar{\Lambda}^{2m-n} + M N_\tau^c \otimes N_\tau^c \\
& + y_1^D (N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5 \otimes \Theta^m/(\Lambda \bar{\Lambda}^m) \\
& + y_2^D N_\tau^c \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5/\Lambda \\
& + y_1 (F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{45} \otimes \Theta^\ell/(\Lambda \bar{\Lambda}^\ell) \\
& + y_2 (F_1, F_2, F_3) \otimes T_3 \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda,
\end{aligned}
$$

Our Multiplication Rule of S_4

$$
\begin{aligned}
(a_1, a_2)_2 \times (b_1, b_2)_2 \quad &= \quad (a_1 b_1 + a_2 b_2)_{1_1} + (-a_1 b_2 + a_2 b_1)_{1_2} \\
&\quad +(a_1 b_2 + a_2 b_1, a_1 b_1 - a_2 b_2)_2
\end{aligned}
$$

$$
\begin{aligned}
(a_1, a_2, a_3)_{3_1} \times (b_1, b_2, b_3)_{3_1} \quad &= \quad (a_1 b_1 + a_2 b_2 + a_3 b_3)_{1_1} \\
&\quad +(\frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3), \frac{1}{\sqrt{6}}(-2 a_1 b_1 + a_2 b_2 + a_3 b_3))_2 \\
&\quad +(a_2 b_3 + a_3 b_2, a_1 b_3 + a_3 b_1, a_1 b_2 + a_2 b_1)_{3_1} \\
&\quad +(a_3 b_2 - a_2 b_3, a_1 b_3 - a_3 b_1, a_2 b_1 - a_1 b_2)_{3_2}
\end{aligned}
$$

$$
\begin{aligned}
(a_1, a_2)_2 \times (b_1, b_2, b_3)_{3_1} \quad &= \quad (a_2 b_1, -\frac{1}{2}(\sqrt{3} a_1 b_2 + a_2 b_2), \frac{1}{2}(\sqrt{3} a_1 b_3 - a_2 b_3))_{3_1} \\
&\quad +(a_1 b_1, \frac{1}{2}(\sqrt{3} a_2 b_2 - a_1 b_2), -\frac{1}{2}(\sqrt{3} a_2 b_3 + a_1 b_3))_{3_2}
\end{aligned}
$$

There are many models with Non-Abelian Discrete Symmetries.

If you are interested in Non-Abelian Discrete Symmetries,
See the review article

" Non-Abelian Discrete Symmetries in Particle Physics "

Hajime Ishimori, Tatsuo Kobayashi, Hiroshi Ohki,
Hiroshi Okada, Yusuke Shimizu, Morimitsu Tanimoto,

e-Print: arXiv:1003.3552 [hep-th]
Prog. Theor. Phys. Suppl. 183:1-163, 2010

We review pedagogically non-Abelian discrete groups and show some applications for physical aspects.

Origin of the non-Abelian Flavor symmetry ?

Tri-bimaximal neutrino mixing from orbifolding,
G.Altarelli, F.Feruglio, Y.Lin, NPB775, 31 (2007) hep-ph/0610165

Stringy origin of non-Abelian discrete flavor symmetries
T. Kobayashi, H. Niles, F. Ploeger, S. Raby, M. Ratz, NPB768,135(2007) hep-ph/0611020

Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models
H. Abe, K-S. Choi, T. Kobayashi, H. Ohki, NPB820, 317 (2009) 0904.2631

Non-Abelian Discrete Flavor Symmetry from T^2/Z_N Orbifolds
A.Adulpravitchai, A. Blum, M. Lindner, JHEP0907, 053 (2009), 0906.0468

Non-Abelian Discrete Groups from the Breaking of Continuous Flavor Symmetries
A.Adulpravitchai, A. Blum, M. Lindner, JHEP0909, 018 (2009), 0907.2332

Non-Abelian Discrete Flavor Symmetries on Orbifolds
H.Abe, K.S.Choi, T.Kobayashi, H.Ohki ,M.~Sakai, arXiv:1009.5284 [hep-th].