Topological Phases of Eternal Inflation

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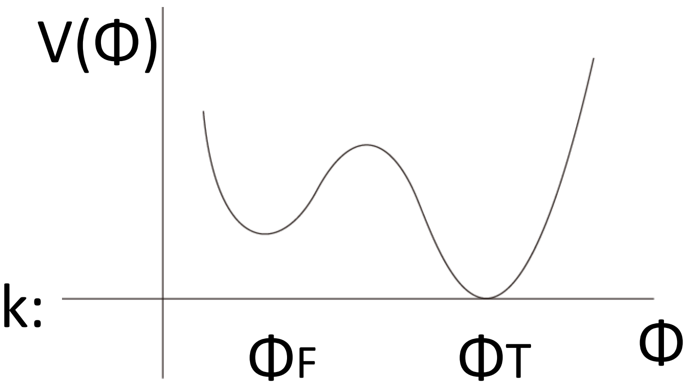
Setup: Gravity coupled to a theory with metastable (false) vacuum



- Scalar field with the potential which has two minima
  (Motivation: string landscape; Today's talk: analysis using semi-classical gravity)

- (If we ignore gravity,) first order phase transition:
  - False vacuum decays by quantum tunneling (bubble nucleation); The nucleation rate Γ: given by the action of the "bounce".
  - The whole space eventually turns into the true vacuum.

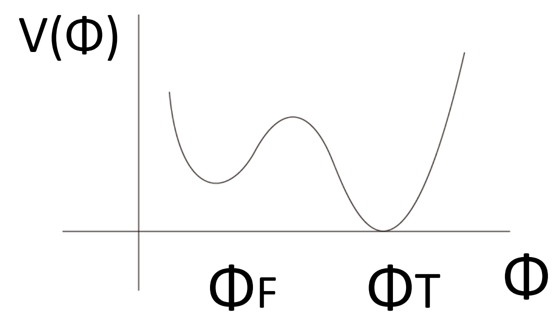
- What happens when there is gravity?

Coupling to gravity

- We consider the case

  $V(\Phi_F) > 0$ (False vacuum: de Sitter with Hubble parameter $H^2 = G\,V(\Phi_F)$)

  $V(\Phi_T) = 0$ (True vacuum: flat spacetime)



- Initial condition: The space (or at least a region larger than horizon volume) was filled with false vacuum.

- The decay of false vacuum: semi-classically described by Coleman-De Luccia instanton (bounce)
- Bubble nucleation rate per unit physical 4-volume:

$$\Gamma \sim e^{-(S_{\rm cl} - S_{\rm deSitter})}$$

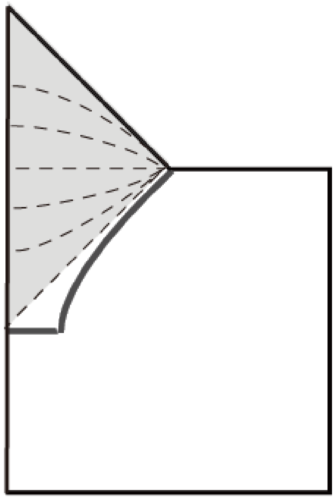
(No “up transitions” from true to false vacuum)

Bubble of true vacuum

- Analytic continuation of CDL instanton: Open FRW universe inside a bubble

$$ds^2 \sim -dt^2 + t^2(dR^2 + \sinh^2 R d\Omega^2)$$

  - At late time, flat spacetime in open slicing (Milne universe)



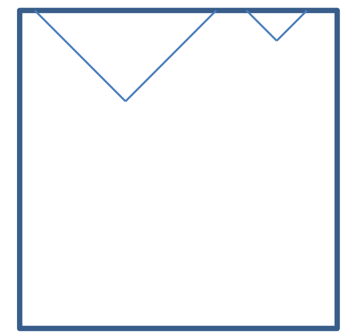
- Many bubbles are nucleated in the de Sitter region.
- If $\Gamma < c\, H^4$, bubble nucleation cannot catch up the expansion of space. ("Eternal Inflation")
  - False vacuum exists forever (dominates the physical volume).
  - Infinite number of bubbles are nucleated eventually.
  - Bubble collisions are inevitable [Guth-Weinberg, '83]

Outline

- There are three phases of eternal inflation depending on the nucleation rate. (“Phases”: the way the universe evolves)
- The phases are characterized by the existence of percolating structures (lines, sheets) of bubbles in the global de Sitter space. (First proposed by Winitzki, ’01.)
- The cosmology of the true vacuum region is qualitatively different in each phase.

View from the future infinity of de Sitter

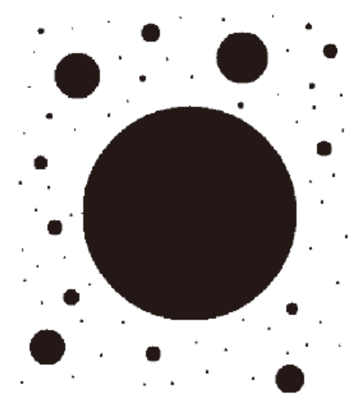
- Consider conformal future infinity.
- A bubble: represented as a sphere cut out from de Sitter.
  (intersection of future light cone from the nucleation and the future infinity; radius = horizon size)



- Bubbles nucleated earlier: larger sphere.
- Larger spheres: rarer (less volume for nucleation).
  - Parametrization of de Sitter

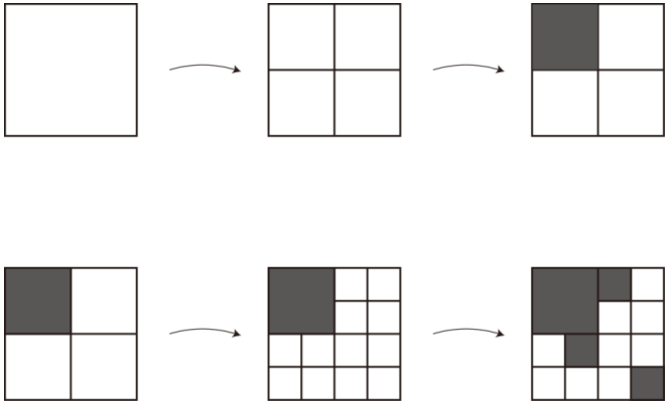
$$ds^2 \sim \frac{-d\eta^2 + d\vec{x}^2}{H^2\eta^2} \qquad (-\infty < \eta < 0)$$

  - (comoving) horizon volume $\sim H^{-3}|\eta|^3$
  - Physical volume for nucleation sites $\sim |\eta|^{-3}$



Model for eternal inflation

- Mandelbrot model (Fractal percolation model)
  - Start from a white cell.
    (One horizon volume of inflating region)
  - Divide the cell into cells with half its linear size.
    (The space grows by a factor of 2.)



Picture for the 2D version

  - Paint each cell in black with probability P.
    (Bubble is nucleated and takes up a horizon volume. P ~ Γ)
  - Subdivide the surviving (white) cells, and paint cells in black w/ probability P. Repeat this infinite times.

White region could be non-empty

- If $P > 1 - (1/2)^3 = 7/8$, the whole space turns black. (No eternal inflation).

- If $P < 7/8$, white region has non-zero fractal dimension d_F (rate of growth of the cells).

$$N_{\rm cells} = 2^{n d_F}, \quad d_F = 3 - |\log(1-P)|/\log 2$$

$$(n : \# \text{ of steps})$$

Non-zero fractal dimension d_F : Physical volume of de Sitter region grows. (Eternal inflation)

Three phases of eternal inflation

From the result on the 3D Mandelbrot model

[Chayes et al ('91)]

In order of increasing P (or Γ), there are
(white = inflating, black = non-inflating)

- Black island phase: Black regions form isolated clusters;
  ∴ percolating white sheets.
- Tubular phase: Both regions form tubular network;
  ∴ percolating black and white lines.
- White island phase: White regions are isolated;
  ∴ percolating black sheets.

Geometry of the true vacuum region

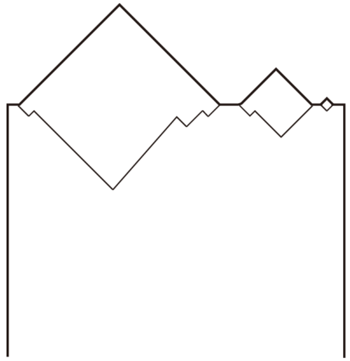
- Mandelbrot model: the picture of the de Sitter side. (de Sitter region outside the light cone of the nucleation site is not affected by the bubble.)

- To find the spacetime in the non-inflating region inside (the cluster of) bubbles, we need to understand the dynamics of bubble collisions.

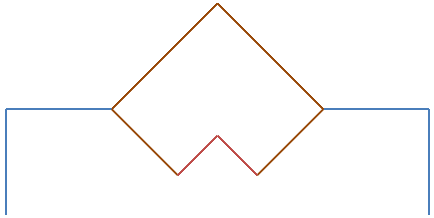
- We will use intuition gained from simple examples of bubble collisions.

Black island phase (isolated cluster of bubbles)

Small deformations of open FRW universe.



- Basic fact: A collision of two bubbles (of the same vacuum) [c.f. Bousso, Freivogel, Yang, '07]
  - Spatial geometry approaches smooth H^3 at late time.
  - Residual symmetry SO(2,1): spatial slice has H^2 factor
  - Negative curvature makes the space expand.

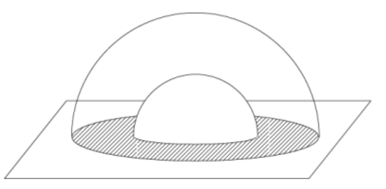


- In the black island phase, local geometry near the collision should be similar to the two bubble case. Spatial geometry will approach smooth H^3 .

Tubular phase (tube-like structure of bubbles)

Spatial slice in the late time limit: negatively curved space whose boundary has infinite genus. (The whole space is accessible to a single observer.)

- Late time geometry: $ds^2 = -dt^2 + t^2 ds^2_{H/\Gamma}$
- Spatial geometry: H^3 modded out by discrete elements of isometry
- Boundary genus = # of elements



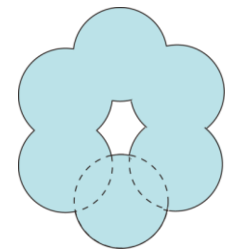
Genus 1 case

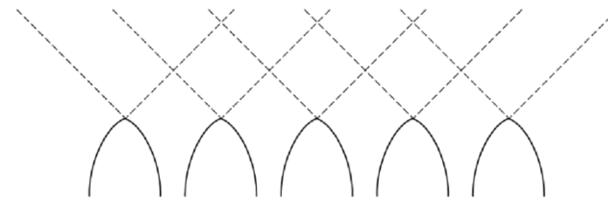
- Modding out makes the causal contact easier (than in Milne universe).

- Simpler example: true vacuum with toroidal boundary

  [Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, '08]

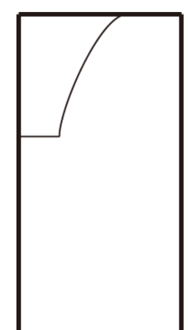
  - Start from a ring-like initial configuration of bubbles
  - Solve a sequence of junction conditions





  - The spatial geometry approaches negatively curved space with toroidal boundary.

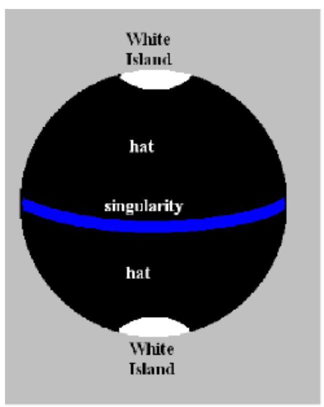
  - There is also a "coarse grained" solution (flat space patched with de Sitter across a toroidal domain wall)



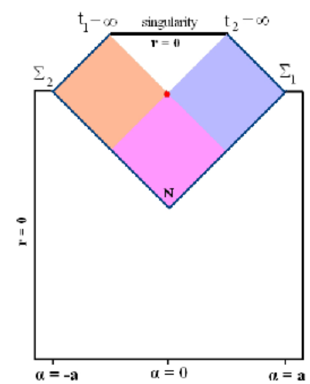
White island phase (isolated inflating region)

<u>An observer in the black region is "surrounded" by the white region (contrary to the intuition from Mandelbrot model).</u>

- Simple case: two white islands (with S^2 symmetry) [Kodama et al '82, BFSSSYY '08]



Global slicing (S^3)

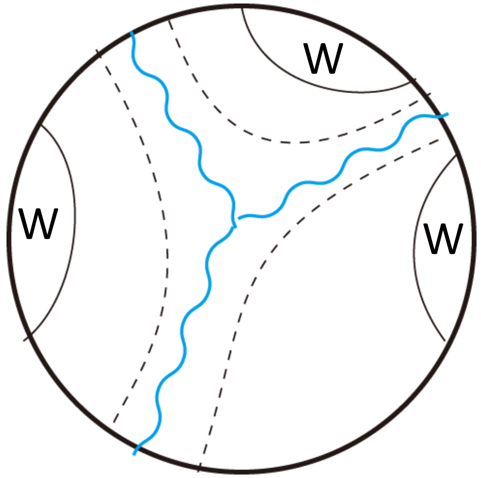


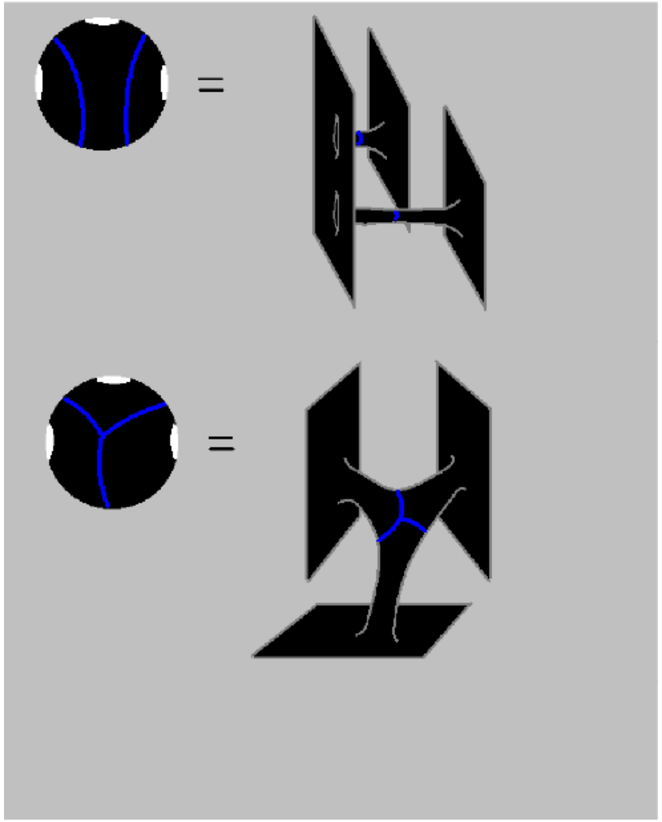
Penrose diagram

  – An observer can see only one boundary; the other boundary is behind the black hole horizon. [c.f. "non-traversability of a wormhole", "topological censorship"]

- In the white island phase, a white region will split.
  - Late time geometry for the three white island case:

[Kodama et al ’82]



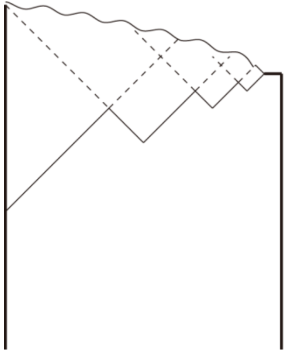


  - Singularity and horizons will form so that the boundaries are causally disconnected from each other.

- Consequence of constant splitting: size of a white island stays of order Hubble radius (= fixed size in unit of the minimum cell in Mandelbrot model).

- Distance from an observer to the boundary grows, but the area of the boundary remains finite. (Effectively a closed universe)
- Big black hole in the bulk.

- This universe will eventually collapse.
  - Simpler model: Shells of bubbles constantly colliding to a given bubble (a possible configuration in the white island phase)
  - Any given observer in the true vacuum will end up at singularity.



Summary

Three phases of eternal inflation and their cosmology:

- Black island phase:
  Small deformation of an open FRW
- Tubular phase:
  Negatively curved space with an infinite genus boundary
- White island:
  Observer sees one boundary and one or more black hole horizons (behind which there are other boundaries).

Future work

- The case with more than two vacua
  (e.g. False, Intermediate, True)

- Interpretation of phases in terms of FRW/CFT correspondence [Freivogel, YS, Susskind, Yeh, '06]
  (holographic dual theory defined on the boundary which is accessible to a single observer)

- Observational signature of each phase
  (Assuming true vacuum is our vacuum, and that there is slow-roll inflation after tunneling)