Heisenberg's Hand-Written Manuscript
– Heisenberg and Nishijima

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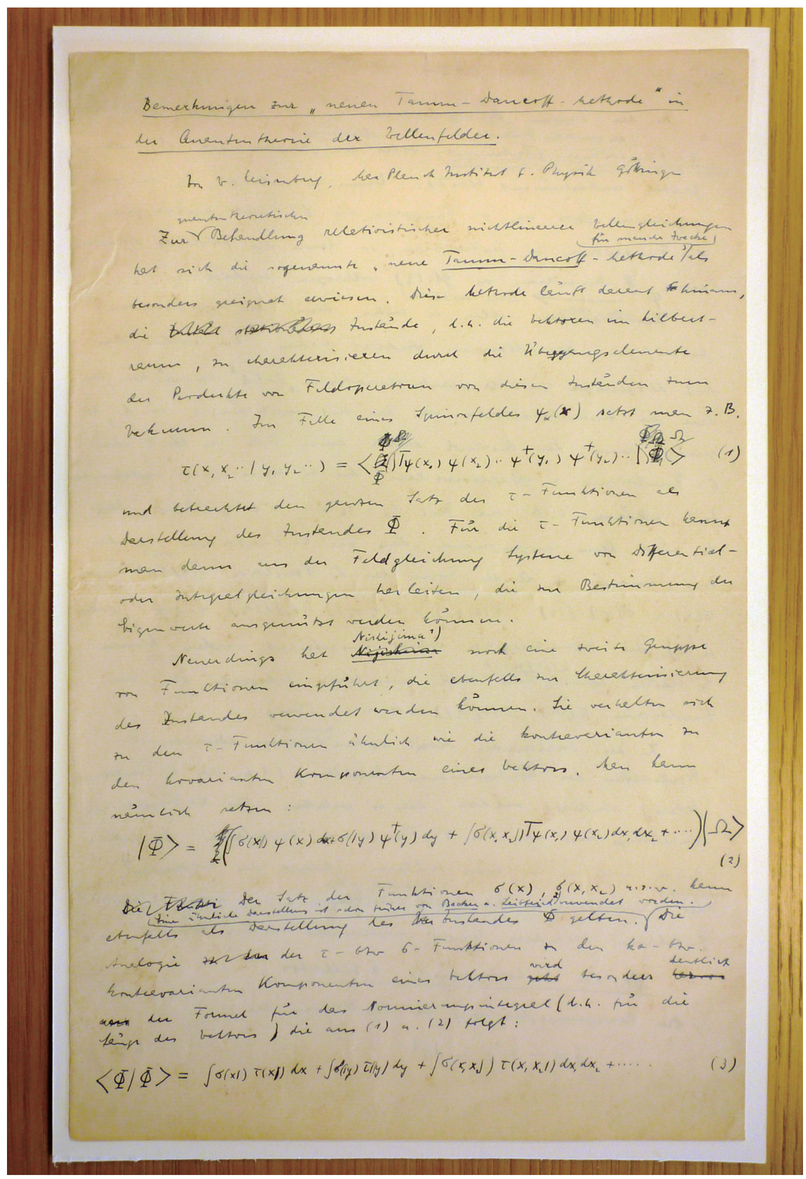
Straight through the entrance door of the IPMU library you will find the display of a handwritten manuscript of Werner Heisenberg, one of the two or three greatest physicists of the twentieth century.

All physicists agree that the two most revolutionary concepts to emerge in the last century were relativity and quantum mechanics. Today, one cannot think of physics without either of these. These two show a marked contrast in their development. The former was created and developed by a single genius, Albert Einstein, through the pursuit of logical consistency and aestheticism in the foundations of physics, without any pressing motivation from experiments. The latter, on the other hand, resulted from the work of a number of leading physicists in Europe, starting with Max Planck in 1900, forced to provide consistent explanations for newly discovered experimental results. During this development Heisenberg, in Göttingen, took the decisive steps towards the final theory of quantum mechanics.

Heisenberg discovered the rule that governs the motion of electrons in atoms and the emission of light from the atom. Immediately after his discovery (July 1925) his "strange"


Bemerkungen zur „neuen Tamm-Dancoff-Methode" in der Quantentheorie der Wellenfelder.

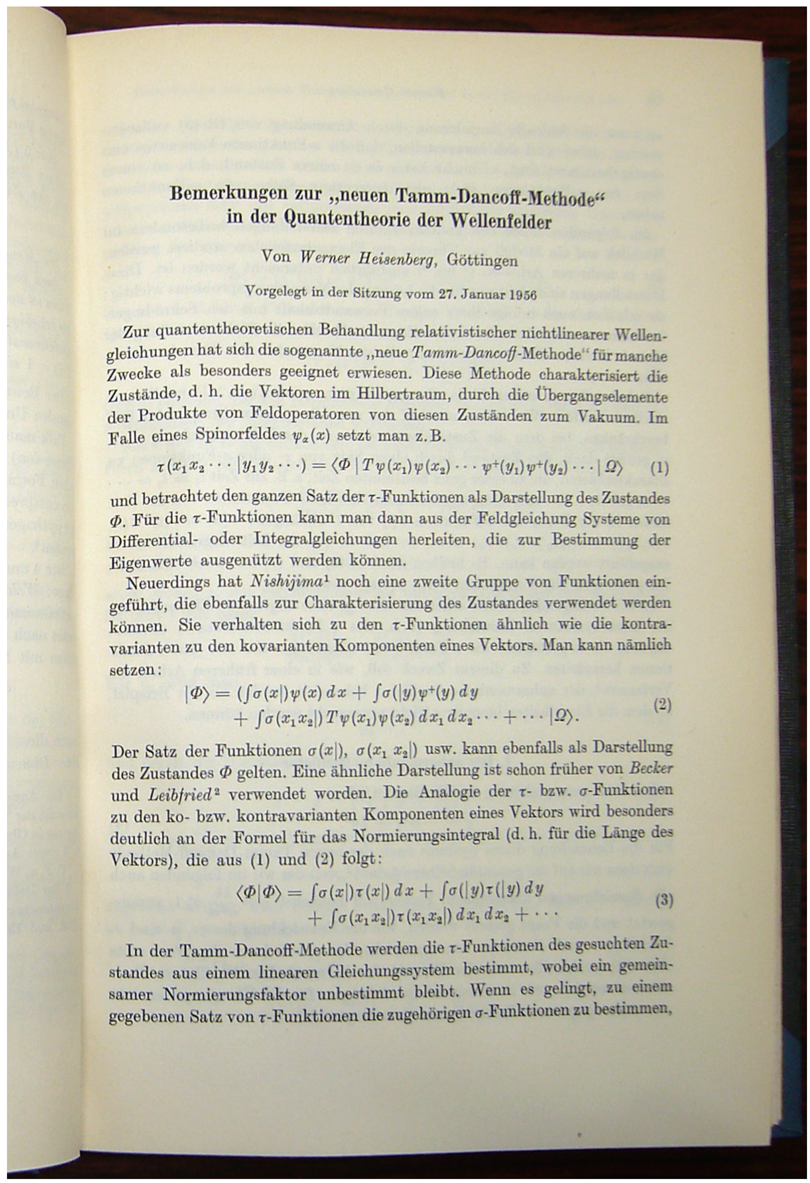
von W. Heisenberg, Max Planck Institut f. Physik Göttingen

Figure 1. The first page of a Heisenberg's handwritten manuscript, displayed at the IPMU library.

rule turned out to be the very one that mathematicians called matrix multiplication (Born, Heisenberg & Jordan, November 1925). Physicists were absolutely unfamiliar with this mathematical concept, in which the multiplication of two numbers does not commute, and it should be stressed that Heisenberg did not know this mathematics, but discovered it by pondering deeply on the empirical characteristics of atomic physics. A year later, Erwin Schrödinger discovered a wave equation that seemed to govern the dynamics of atoms; Schrödinger's equation was apparently completely different from Heisenberg's matrix mechanics, but the two methods yielded the same answers. It took less than a year for physicists to realize that these two formulations are mathematically equivalent (it is said that David Hilbert knew this all along). This work formed the theoretical basis not only for atomic physics but also for nuclear and elementary-particle physics. (Incidentally, Einstein's theory of general relativity was based on Riemannian geometry, another mathematical concept that was entirely new to physicists at the time. When Einstein started to develop general relativity, he was ignorant of Riemannian geometry until his colleague-friend Marcel Grossmann told him about it. These are two examples in which new mathematics was crucial in the development of physics.)

This single discovery would be more than sufficient to raise Heisenberg to the top rank of physicists, but his first-rate contributions continued. These include the discovery of Heisenberg's uncertainty principle (key to understanding and the heart of quantum mechanics), that nuclei consist of protons and neutrons rather than protons and electrons, the concept of isospin, the principle of the ferromagnet, and (with Wolfgang Pauli) the foundation of quantum field theory, which is the fundamental theory that describes elementary particles and their dynamics. All of this was done before he became some 30 years of age. He later also proposed the fundamental concept of the scattering matrix to describe interactions of elementary particles.

During World War II, Germany and its science were annihilated not only by the Allied raids but also by its own regime. So was the Kaiser Wilhelm Institut für Physik in Berlin to which Heisenberg belonged. Heisenberg, who remained in Germany in the hope that he could help to rebuild post-war science in Germany,



Bemerkungen zur „neuen Tamm-Dancoff-Methode" in der Quantentheorie der Wellenfelder

Von *Werner Heisenberg*, Göttingen

Vorgelegt in der Sitzung vom 27. Januar 1956

Zur quantentheoretischen Behandlung relativistischer nichtlinearer Wellengleichungen hat sich die sogenannte „neue *Tamm-Dancoff*-Methode" für manche Zwecke als besonders geeignet erwiesen. Diese Methode charakterisiert die Zustände, d. h. die Vektoren im Hilbertraum, durch die Übergangselemente der Produkte von Feldoperatoren von diesen Zuständen zum Vakuum. Im Falle eines Spinorfeldes $\psi_\alpha(x)$ setzt man z. B.

$$\tau(x_1 x_2 \cdots | y_1 y_2 \cdots) = \langle \Phi \,|\, T\psi(x_1)\psi(x_2)\cdots\psi^+(y_1)\psi^+(y_2)\cdots \,|\, \Omega\rangle \tag{1}$$

und betrachtet den ganzen Satz der τ-Funktionen als Darstellung des Zustandes Φ. Für die τ-Funktionen kann man dann aus der Feldgleichung Systeme von Differential- oder Integralgleichungen herleiten, die zur Bestimmung der Eigenwerte ausgenützt werden können.

Neuerdings hat *Nishijima*[1] noch eine zweite Gruppe von Funktionen eingeführt, die ebenfalls zur Charakterisierung des Zustandes verwendet werden können. Sie verhalten sich zu den τ-Funktionen ähnlich wie die kontravarianten zu den kovarianten Komponenten eines Vektors. Man kann nämlich setzen:

$$\begin{aligned} |\Phi\rangle = (&\textstyle\int \sigma(x|)\psi(x)\,dx + \int \sigma(|y)\psi^+(y)\,dy \\ &+ \textstyle\int \sigma(x_1 x_2|)\, T\psi(x_1)\psi(x_2)\,dx_1\,dx_2 \cdots + \cdots |\Omega\rangle. \end{aligned} \tag{2}$$

Der Satz der Funktionen $\sigma(x|)$, $\sigma(x_1\, x_2|)$ usw. kann ebenfalls als Darstellung des Zustandes Φ gelten. Eine ähnliche Darstellung ist schon früher von *Becker* und *Leibfried*[2] verwendet worden. Die Analogie der τ- bzw. σ-Funktionen zu den ko- bzw. kontravarianten Komponenten eines Vektors wird besonders deutlich an der Formel für das Normierungsintegral (d. h. für die Länge des Vektors), die aus (1) und (2) folgt:

$$\begin{aligned} \langle\Phi|\Phi\rangle = &\textstyle\int \sigma(x|)\tau(x|)\,dx + \int \sigma(|y)\tau(|y)\,dy \\ &+ \textstyle\int \sigma(x_1 x_2|)\tau(x_1 x_2|)\,dx_1\,dx_2 + \cdots \end{aligned} \tag{3}$$

In der Tamm-Dancoff-Methode werden die τ-Funktionen des gesuchten Zustandes aus einem linearen Gleichungssystem bestimmt, wobei ein gemeinsamer Normierungsfaktor unbestimmt bleibt. Wenn es gelingt, zu einem gegebenen Satz von τ-Funktionen die zugehörigen σ-Funktionen zu bestimmen,

Figure 2. The first page of the published Heisenberg's paper corresponding to his handwritten manuscript shown in Fig. 1. (The photographed material is owned by Ochanomizu University.)

Special Contribution

endeavoured to reestablish the KWI in Göttingen, but the use of the name "Kaiser" was forbidden by the British occupation forces (Göttingen was in the British sector). Together with colleagues, among them Otto Hahn, Heisenberg succeeded in re-establishing the institute, with, however, the new name "Max-Planck-Institut."

Heisenberg devoted himself from 1953 onward to developing non-linear spinor field theory with the hope that it would describe the world of elementary particles and their interactions. During the course of this work he became impressed by the research done by Nishijima, and in August 1955 he enquired in a letter to Tomonaga, who was a disciple of Heisenberg just before the war, whether Nishijima would be interested in coming to Göttigen for a year. Nishijima happily accepted his offer and arrived in Göttingen in January 1956.

Kazuhiko Nishijima (b. 1926) was educated at the University of Tokyo and then recruited by Sachio Hayakawa to a group of particle physicists at the newly founded Osaka-City University upon the request of Yuzuru Watase. The new group included Yoichiro Nambu as professor, Hayakawa himself as associate professor, Yoshio Yamaguchi as lecturer, and Kazuhiko Nishijima and Tadao Nakano as research assistants. This was a powerful group that had great success in interpreting the new experimental results that were flooding the field. For example they explained the basic properties of the new particles now called strange particles by hypothesizing that they are always produced in pairs. Nishijima went further and boldly, but correctly, conjectured that the new particle, the K meson, should have isospin 1/2 rather than integer. He then introduced the quantum number of η charge (strangeness, in later terminology), and proposed what is now called the Nishijima-Gell-Mann rule. This is in fact a general rule when symmetry of a higher rank is broken to lower symmetry, so it applies to many concepts that were unknown in Nishijima's time. Nishijima eventually left work on the phenomenology of new particles to study the more mathematical elements of field theory.

While Heisenberg was developing field theory, the work of Nishijima published in *Progress of Theoretical Physics* (1953) came to his attention, and hence his invitation via Tomonaga in August 1955. Displayed in the library is the first page of Heisenberg's hand-written manuscript entitled "Bemerkungen zur 'neuen Tamm-Dancoff-Methode' in der Quantentheorie der Wellenfelder" published in *Nachrichten der Akademie der Wissenschaften in Göttingen*, *Math.-Phys. Klasse IIa*, S. 27-36 (1956). (The entire manuscript of this paper is retained at IPMU). This paper aims at a non-perturbative treatment in non-linear spinor theories with some truncation, which he called the 'Tamm-Dancoff method,' and a calculation of the fermion mass and, as a fermion bound state, the boson mass in the lowest approximation for a simplified model. Here the separate treatment of the covariant and contravariant representations of the state vector was important in treating the bound state in field theory; this treatment had been an invention in Nishijima's paper that dealt with the scattering matrix for composite particles.

We may presume that Heisenberg gave this manuscript to Nishijima to ask for his opinion before the presentation at the Academy. Nishijima kept it until he passed away at age 82 in February 2009 of leukaemia.

Nishijima extended his stay at Göttingen by half a year, as seen in a letter in which Heisenberg asked Watase for permission for this extension. He then went to the Institute for Advanced Study in Princeton; there his work with Rudolf Haag and Wolfhart Zimmermann on the scattering matrix of composite particles led to what is now called the HNZ construction. Frequent communications between the two scientists continued for the next two years, in particular about the role of γ_5 and the surprising recent discovery of parity violation in field theory. Nishijima returned to Osaka-City University in 1959, but then went to the University of Illinois at Urbana-Champaign where he remained until 1966. During that time he wrote a well-regarded textbook, *Fields and Particles* (W. A. Benjamin, 1969). He worked at the University of Tokyo until 1985 when he was asked to direct the Yukawa Institute of Kyoto University.