

Workshop on “Primitive Forms and Related Subjects”

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A workshop on Primitive Forms and Related Subjects was held at the Kavli IPMU from February 10 to 14, 2014 (organized by K. Hori, C. Li, S. Li, and K. Saito).

Let us give a brief historical overview on the subjects.

Modeled on the classical theory of elliptic integrals on a family of elliptic curves, the theory of period integrals of primitive forms over vanishing cycles of a function with isolated critical points was introduced in a work of K. Saito 1983. Here, a primitive form is defined to be a class of relative top degree differential forms on an open complex manifolds equipped with a deformation family of a function F having only isolated critical points, where the relative de Rham cohomology class of the primitive form should satisfy some infinite system of bilinear equations of higher residue pairings defined on a semi-infinite Hodge structure.

Primitive forms have been one of the subjects of the Math-String seminar and Topological Strings seminar at Kavli IPMU for the reason that the theory of primitive forms is relevant for the complex geometric aspect (“B-model”) of the Landau-Ginzburg model

whose superpotential is given by the function F . According to a postulate, called mirror symmetry in topological string theory, the theory should correspond to the symplectic geometric aspect (“A-model”) of another theory, such as the Gromov-Witten theory of a compact Kähler manifold or the Fan-Jarvis-Ruan-Witten theory (2007, 2013) of a Landau-Ginzburg orbifold.

As a consequence of the theory, a primitive form induces the flat structure on the deformation parameter space (i.e., a flat metric together with a ring structure on the tangent bundle of the parameter space satisfying some integrability conditions. The structure was later axiomatized as the Frobenius manifold structure by B. Dubrovin 1990). Then that structure defines a potential function called the prepotential on the parameter space. One mathematically rigorous formulation of the mirror symmetry conjecture asks that **the prepotential function obtained from a primitive form should coincides with that of the mirror side** (i.e., of Gromov-Witten theory or of FJRW theory) **after a suitable identification called the mirror map, of parameter**

spaces equipped with the flat coordinates.

However, the verification of this mirror symmetry conjecture was not achieved until recently, since even though primitive forms are theoretically known to exist, their explicit expression was not known except for two cases: universal unfoldings of simple singularities and simple elliptic singularities (1983 K. Saito). Mirror symmetry for these two cases has been confirmed rather recently (simple singularity case, Fan-Jarvis-Ruan 2007, simple elliptic singularity case, Krawitz-Shen 2011, Milanov-Shen 2012).

In the last year, there has been new progresses:

- 1) Towards a construction of primitive forms over Novikov rings for toric cases (Fukaya-Oh-Ohta-Ono),
- 2) Unified approach to primitive forms and to BCOV-theory via polyvector fields (Li-Li-Saito),
- 3) Perturbative construction of primitive forms (Li-Li-Saito). In particular, as an application of 3), we obtain
- 4) Verification of mirror symmetry for wide classes of singularities including cases with central charge being larger than 1, using

the perturbative formula of prepotentials (Li-Li-Saito-Shen).

The workshop was inspired by these recent developments, and consisted of

(A) three basic courses

- Lecture I (given by K. Ono, H. Ohta, and K. Fukaya) : Frobenius manifold structure and Lagrangean Foer theory for toric manifolds
- Lecture II (given by T. Jarvis) : Introduction to FJRW-theory and a mathematical approach to the Gauged Linear Sigma Model
- Lecture III (given by Si Li) : LG-model via Kodaira-Spencer gauge theory

(B) 10 research talks

- S. Barannikov: On the noncommutative Batalin-Vilkovisky formalism and EA matrix integrals
- A. Takahashi: From Calabi-Yau dg categories to Frobenius manifolds via primitive forms

- K. Hori, M. Romo: The parameter delta
- H. Fan: Analytic construction of quantum invariant of singularity
- Y. Zhang: On the genus two free energies for semi simple Frobenius manifolds
- A. Losev: K. Saito theory of primitive form, generalized harmonic theory and mirror symmetry
- H. Iritani: Gamma Conjecture for Fano manifolds
- Y. Shen: Mirror symmetry for exceptional unimodular singularities
- T. Milanov: The phase form in singularity theory
- D. Pomerleano: Deformation theory of affine symplectic manifolds

(C) 7 short communications

- Y. Shiraishi: On Weyl group and Artin group associated to orbifold projective lines
- N. Priddis: A Landau-Ginzburg/ Calabi-Yau correspondence for the mirror quintic

- M.R. Rahmati: Hodge theory of isolated hypersurface singularities
- B. Bychkov: On the number of coverings of the sphere ramified over given points
- M. van Garrel: Integrality of relative BPS state counts of toric Del Pezzo surfaces
- S. Sugiyama: On the Fukaya-Seidel categories of surface Lefschetz fibrations
- A. Bondal and I. Zhdanovskiy: Critical points of a functional and orthogonal pairs of Cartan subalgebras.

There were over 80 participants from all over the world, including Asia, Russia, Europe and America, and the workshop was quite active and successful by showing the current status of the research and inspiring further study of primitive forms including 1) geometric understanding of mirror symmetry and 2) towards a categorical construction primitive forms.

